Contribution to the theory of superheat instability in semiconductors

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It is shown that a periodic superheat instability arises in semiconductors located in crossed electric and magnetic fields. Various types of finite amplitude stationary waves are found and their stability is investigated.

1. INTRODUCTION

In a plasma situated in a strong electric field $\mathbf{E}_0$, an instability is created by heating of the electron gas (the so called superheat instability[1]). In the case of a semiconductor plasma, this instability was investigated in[2-6]. Without a constant magnetic field, the superheat instability is as a rule aperiodic. We shall show that in an external magnetic field $\mathbf{H}_0 \perp \mathbf{E}_0$ there is produced a drift-wave instability. The physical mechanism of the phenomenon consists in the following. In the stationary state, the momentum and energy lost by the electrons in the collisions is compensated for by the action of the constant electric field. As a result, drift motion and a definite electron temperature (average energy) $\Theta_0(E_0)$ are produced in the sample. Owing to the quasielastic character of the scattering, this temperature exceeds the equilibrium lattice temperature T. The momentum and energy relaxation times τ_r and τ_e depend on the electric field. The conductivity σ in an isotropic plasma is then proportional to τ_p, and inversely proportional to τ_r in a magnetoactive plasma. Thus, the character of the dependence of the Joule heat on the electric field is altered in a magnetic field. For this reason, the superheat instability in a magnetic field can be used to develop oscillators of the Gunn type.

We consider here different types of stationary waves of finite amplitude, ascertain the conditions for their onset, and determine the stability criteria. It turns out that standard solutions such as solitons and periodic waves are unstable in the given-current regime to spatial perturbations. By choosing the proper external load, however, the soliton instability can be stabilized.

2. FUNDAMENTAL EQUATIONS. LINEAR APPROXIMATION

To describe the superheat instability of quasipotential oscillations (curl E ≈ 0) it is necessary to use the following equations: the possion equation

$$\varepsilon_0 \operatorname{div} \mathbf{E} = 4\pi e(N - N_0), \tag{2.1}$$

the continuity equation

$$e\,\partial N / \partial t + \operatorname{div} \mathbf{j} = 0 \tag{2.2}$$

and the energy transport equation

$$\frac{3}{2}\frac{\partial}{\partial t} N\Theta + \operatorname{div} \mathbf{Q} - \mathbf{jE} = -\frac{1}{\tau_э} N\Theta. \tag{2.3}$$

Here ϵ_0 is the dielectric constant of the lattice, E is the electric field, N is the electron density (N_0 is its equilibrium value, j and Q are the electric current and electron energy flux, and Θ is the electron temperature.

We choose a coordinate system such that the y axis is directed along the constant electric field and the z axis parallel to the external magnetic field. In the stationary state, the current has components j_{0x} and j_{0y}. The Hall current j_{0x} leads to the onset of a magnetic self-field

$$H^{\text{self}} = \frac{4\pi}{c}\int j_{0x}\,dy \approx \frac{4\pi}{c}\bar{j}_{0x}L,$$

where L is the characteristic dimension of the sample along the y axis. When the condition $L \ll cH_0/4\pi j_{0x}$ is satisfied, the magnetic self-field of the current can be neglected in comparison with the external field.

We consider the one-dimensional problem, in which all the variables depend only on the coordinate y and the time t. The connection between the current and the energy flux, on the one hand, with the electric field and electron density and temperature, on the other, is then

$$j_y = \frac{\sigma}{e}\left[eE_y - \frac{\Theta}{N}\frac{\partial N}{\partial y} + (q-1)\frac{\partial\Theta}{\partial y}\right], \tag{2.4}$$

$$Q_y = \frac{\sigma\Theta}{e^2}\left(\frac{5}{2} - q\right)\left[eE_y - \frac{\Theta}{N}\frac{\partial N}{\partial y} + (q-2)\frac{\partial\Theta}{\partial y}\right], \tag{2.5}$$

where $\sigma = e^2N/m\omega_H^2\tau_p$, $\omega_H = eH_0/mc$ is the cyclotron frequency,

$$\tau_r = \tau_{r0}\frac{\Gamma(5/2)}{\Gamma(5/2 - q)}\left(\frac{\Theta}{T}\right)^q$$

is the momentum relaxation time, and $\tau_e = \tau_{e0}(\Theta/G)^{1-r}$ (see (2.3)) is the energy relaxation time; q and r are numbers characterizing the scattering mechanisms[7]. These relations were derived in an approximation in which the effective electron temperature $\Theta \gg T$. In addition, it is assumed that $\omega_H\tau_r \gg 1$, and the characteristic times in which the variable quantities change are long in comparison with the momentum relaxation time.

Linearizing the system (2.1—2.5) and changing over to the Fourier representation (all the variable quantities are proportional to expi (ky − ωt)), we obtain the dependence of the frequency ω on the wave vector k. The dispersion equation takes the form

$$\omega^2 + i\omega\Omega_1 - \Omega_2^2 = 0, \tag{2.6}$$

where

$$\Omega_1 = \frac{4\pi\sigma}{\varepsilon_0} + \frac{2}{3}(r+q)\frac{\sigma E_0^2}{N_0\Theta_0} + \frac{2}{3}(5 - 3q + q^2)\left(ikv_0 + \frac{\sigma k^2\Theta_0}{e^2N_0}\right), \tag{2.7}$$

$$\begin{aligned}\Omega_2^2 = {} & \frac{4\pi\sigma}{\varepsilon_0}\left[\frac{2}{3}(r-q)\frac{\sigma E_0^2}{N_0\Theta_0} + ikv_0 + \frac{2}{3}\left(\frac{5}{2} - q\right)\frac{\sigma k^2\Theta_0}{e^2N_0}\right] \\ & - \frac{2}{3}ikv_0\left[(r+q)\frac{\sigma E_0^2}{N_0\Theta_0} + (5-2q)\frac{\sigma k^2\Theta_0}{e^2N_0}\right] + \frac{2}{3}k^2v_0^2\left(\frac{7}{2} - 2q - r\right) \\ & - \frac{2}{3}\left(\frac{5}{2} - q\right)\frac{\sigma^2k^4\Theta_0^2}{e^4N_0^2},\end{aligned} \tag{2.8}$$

$$v_0 = j_{0y} / eN_0.$$

In the case of short Maxwellian relaxation times

$$\frac{4\pi\sigma}{\varepsilon_0} \gg \frac{2}{3}(r+q)\frac{\sigma E_0^2}{N_0\Theta_0} \tag{2.9}$$

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the long-wave ($4\pi\sigma/\epsilon_0 \gg kv_0$) perturbations of the temperature and of the electric field are unstable if $r - q < 0$:

$$\omega_1 \approx -4\pi\sigma i / \varepsilon_0, \tag{2.10}$$

$$\omega_2 \approx kv_0 - \frac{2}{3} i (r-q) \frac{\sigma E_0^2}{N_0 \Theta_0} - \frac{2}{3} i \left(\frac{5}{2} - q\right) \frac{\sigma k^2 \Theta_0}{e^2 N_0}. \tag{2.11}$$

In the opposite limiting case, the solution of (2.6) is

$$\omega_1 \approx -\frac{2}{3} i (r+q) \frac{\sigma E_0^2}{N_0 \Theta_0}, \tag{2.12}$$

$$\omega_2 \approx kv_0 - i \frac{4\pi\sigma}{\varepsilon_0} \frac{r-q}{r+q} \left[1 + k^2 r_d^2 \left(1 + \frac{q^2}{r-q}\right)\right], \tag{2.13}$$

where $r_d - (\epsilon_0 \Theta_0 / 4\pi e^2 N_0)^{1/2}$ is the electron Debye screening radius. Thus, a periodic drift-wave instability can develop in crossed fields[1)].

Even from the dispersion relations (2.11) and (2.13) themselves it is apparently possible to predict, in most general outlines, the character of the stationary motions that are established as a result of instability development. In fact, owing to nonlinear effect, generation of higher harmonics sets in as a result of nonlinear effects, and these harmonics cause the wave front to become steeper. As a result, the role of the spatial gradients increases, and the process of growth of the oscillation amplitudes stops. In this case, stationary periodic oscillations are apparently produced in the system.

At the same time, for sufficiently long-wave perturbation, the diffusion effects (the terms proportional to k^2 in (2.11) and (2.13)) do not play an important role. The amplitude can stop growing only when other energy and momentum scattering mechanisms are turned on. Obviously, a stationary solution of the shock-wave type can arise in this case. We can thus expect some of the indicated types of stationary wave motion to be ultimately established as $t \to \infty$. Assuming that such motions exist, we obtain them by solving the initial system of equations.

3. NONLINEAR STATIONARY WAVES IN THE QUASINEUTRALITY APPROXIMATION

We first investigate the case of short Maxwellian-relaxation times. The dispersion equation (2.11) can then be obtained from the equations div $\mathbf{j} = 0$, curl $\mathbf{E} \approx 0$, and the linearized energy transport equation in which we put $N = N_0$, i.e., we assume the quasineutrality condition to be satisfied. Integrating the continuity equation, we obtain the dependence of the electric field on the temperature and on the total current $j_0 = \sigma E_y(y) + (\sigma/e)(q-1) \times \partial\Theta/\partial y$. The balance equation after changing over to dimensionless quantities is

$$W_{\xi\xi} + \alpha[(2-q)W]^{(q-1)/(2-q)}(u-u_0)W_\xi + \frac{j_0^2}{j^2(W_{02})}\left(1 - \frac{j^2(W)}{j_0^2}\right) \times [(2-q)W]^{q/(2-q)} = 0. \tag{3.1}$$

Here

$$W = \frac{1}{2-q}\left(\frac{\Theta}{T}\right)^{2-q}, \quad \xi = \frac{y - st}{l}, \quad l = \frac{\sigma_0 T}{ej(W_{02})}\left(\frac{5}{2} - q\right)^{1/2},$$

$$\sigma = \sigma_0 \left(\frac{\Theta}{T}\right)^{-q}, \quad \alpha = \frac{3}{2}\frac{\sigma_0 NT}{\tau_{e0} j^2(W_{02})}, \quad u = \frac{s\tau_{e0}}{l},$$

$$u_0 = v_0 \tau_{e0}/l, \quad j(W) = [(2-q)W]^{(r-q)/2(2-q)}(\sigma_0 NT/\tau_{e0})^{1/2}.$$

Relation (3.1) is expressed in a coordinate system moving together with the wave, the phase velocity of which is s. The current j(W) determines the amount of heat $j^2(W)/\sigma$ transferred by the electrons to the lattice as a result of the collisions. It can be shown[7] that when the various energy and momentum scattering mechanisms are taken into account, the j(W) plot has the shape of the letter N (Fig. 1)[2)]. At a fixed external current, the values of W_{01}, W_{02}, and W_{03} for a stationary and homogeneous distribution of the electron temperature are determined from the condition $j(W) = j_0$. The possible forms of the stationary temperature waves can be established by investigating Eq. (3.1) in the phase space W_ξ, W[8]. We note that (3.1) is similar to the equation for the electric field in the theory of the Gunn effect.

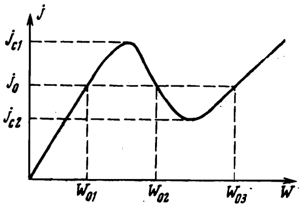

FIG. 1.

The points W_{01}, W_{02}, and W_{03} on the phase plane are singular points of Eq. (3.1)[3)]. The behavior of the system near the singular points is determined by the roots of the characteristic equation (2.11). If we make the change of variables $s = \omega/k$ and $k = -i\lambda/l$, and change over to dimensionless quantities, then (2.11) takes the form

$$\lambda^2 + \alpha[(2-q)W_0]^{(q-1)/(2-q)}(u-u_0)\lambda - \frac{2}{j_0}\frac{\partial j}{\partial W}\bigg|_{W_0}[(2-q)W_0]^{q/(2-q)} = 0, \tag{3.2}$$

where W_0 is one of the singular points. Solving this equation, we obtain

$$\lambda_{1,2} = -\frac{1}{2}\alpha(u-u_0)[(2-q)W_0]^{(q-1)/(2-q)}$$
$$\pm\left\{\frac{1}{4}\alpha^2(u-u_0)^2[(2-q)W_0]^{2(q-1)/(2-q)} + \frac{2}{j_0}\frac{\partial j}{\partial W}\bigg|_{W_0}[(2-q)W_0]^{q/(2-q)}\right\}^{1/2}. \tag{3.3}$$

We see therefore that the points W_{01} and W_{03} at which $\partial j/\partial W|_{W_0} > 0$ are saddle points. If $\partial j/\partial W|_{W_{02}} < 0$, then the type of the singular point depends on the sign of the discriminant Δ:

$$\Delta < 0 \begin{cases} u = u_0, & \text{center} \\ u > u_0, & \text{stable focus} \\ u < u_0, & \text{unstable focus} \end{cases}$$
$$\Delta \geqslant 0 \begin{cases} u > u_0, & \text{stable node} \\ u < u_0, & \text{unstable node} \end{cases}$$

If $\partial j/\partial W|_{W_0} = 0$, corresponding in Fig. 1 to the external currents j_{c1} and j_{c2}, then only two singular points exist, one of which is a saddle point and the other a saddle-node boundary curve which is stable at $u > u_0$ and unstable at $u < u_0$.

The expression (3.1) is analogous to the equation of motion of a particle in a field with a potential

$$V = \frac{j_0^2}{j^2(W_{02})}(2-q)^{q/(2-q)} \times \int^W dW' W'^{q/(2-q)}\left(1 - \frac{j^2(W')}{j_0^2}\right). \tag{3.4}$$

At $u = u_0$ there is no friction, periodic oscillations or domains set in, and the total energy of the system is conserved

$$\mathscr{H}_0 = V + \frac{1}{2}\left(\frac{dW}{d\xi}\right)^2 = \text{const.} \tag{3.5}$$

The possible trajectories on the phase plane are shown in Fig. 2. The reciprocal period of the oscillation is

$$\xi=\int_{W_1}^{W_2}\frac{dW}{\sqrt{2(\mathscr{H}_0-V)}}. \tag{3.6}$$

If $j_0 > j_{0c}$ (j_{0c} is the critical value of the field at which $V(W_0) = V(W_{03})$), a "hot" domain appears (the separatrix Γ in Fig. 2a), and a "cold" domain at $j_0 < j_{0c}$ (Fig. 2b); Γ_1 are the phase trajectories of the periodic waves. For energy levels lying near the bottom of the potential well V, the oscillations are almost harmonic, and the connection between the period of the oscillations and the amplitude can be obtained analytically by the Bogolyubov method[10].

If $u \neq u_0$, the separatrix (see Figs. 3 and 4) joining the saddle with the node (focus) corresponds to a shock wave (shock wave with oscillations). We note that shock waves can be produced also at $u = u_0$ if $j_0 = j_{0c}$, and correspond to separatrices that join saddles[5,9].

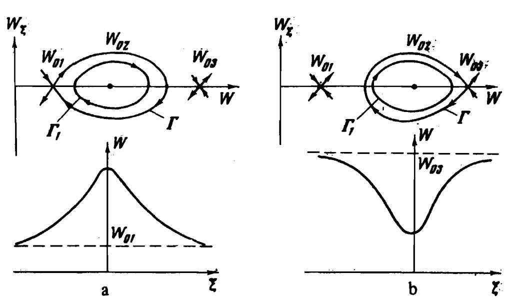

FIG. 2

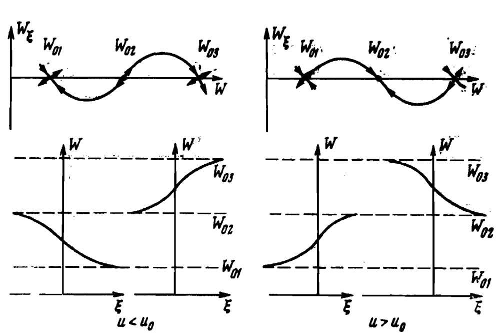

FIG. 3

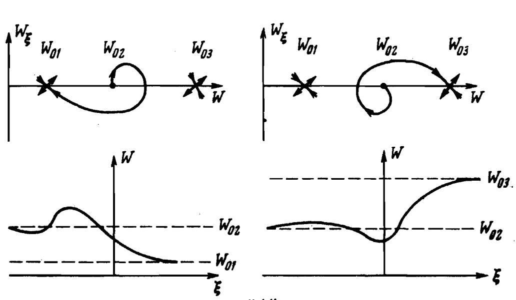

FIG. 4

4. TEST FOR STABILITY

In this section we investigate the stationary solutions obtained above for stability to space-time perturbations. We use a quasiclassical method[11]. We consider the case of a fixed external current. Let

$$W = W_0(\xi) + W_1(\xi,\tau), \tag{4.1}$$

where $W_0(\xi)$ is the stationary solution and $W_1(\xi, \tau)$ is a small perturbation ($\tau = t/\tau_{e0}$). The equation for $W_1(\xi, \tau)$ is

$$\left(\frac{\partial^2}{\partial\xi^2}+D_1(\xi)\frac{\partial}{\partial\xi}+D_2(\xi)\right)W_1(\xi,\tau)=P(\xi)\frac{\partial W_1(\xi,\tau)}{\partial\tau},$$
$$P(\xi)=\alpha[(2-q)W_0(\xi)]^{(q-1)/(2-q)},\qquad D_1(\xi)=(u-u_0)P(\xi),$$
$$D_2(\xi)=\alpha(q-1)(u-u_0)[(2-q)W_0(\xi)]^{(2q-3)/(2-q)}W_{0\xi} \tag{4.2}$$
$$+q\left[1-\frac{j^2(W_0(\xi))}{j_0^2}\right][(2-q)W_0(\xi)]^{2(q-1)/(2-q)}$$
$$-2\frac{j(W_0(\xi))}{j_0^2}\frac{\partial j}{\partial W}\bigg|_{W_0(\xi)}[(2-q)W_0(\xi)]^{q/(2-q)}.$$

We seek the time and coordinate dependence in the form

$$W_1(\xi,\tau)\sim\exp\left[\gamma\tau+\int^{\xi}\varkappa_1(\xi')d\xi'\right], \tag{4.3}$$

where

$$\varkappa_1(\xi)=-{}^1/_2D_1\pm\sqrt{{}^1/_4D_1^2+\Gamma-D_2},\qquad \Gamma=\gamma P(\xi). \tag{4.4}$$

Equation (4.2) describes the motion of a particle in a potential well, the dependence of which on the coordinate ξ is determined by the form of the stationary solution. The function $W_1(\xi, \tau)$ should be bounded as $\xi \to \pm\infty$. If such a solution can be constructed at $\Gamma > 0$, then the corresponding stationary solution is unstable, and if there are no solutions banded in ξ at $\Gamma < 0$, then the stationary solution is stable in time.

Let us assess the possibility of the existence of finite solutions for (4.2). By substituting

$$\Psi=W_1(\xi,\tau)\exp\left[\frac{1}{2}\int D_1(\xi')d\xi'\right] \tag{4.5}$$

we reduce (4.2) to the form

$$\hat{\mathscr{H}}\Psi=-\Gamma\Psi, \tag{4.6}$$

where

$$\hat{\mathscr{H}}=-\frac{d^2}{d\xi^2}+\tilde{V}(\xi),\qquad \tilde{V}(\xi)=\frac{D_1^2}{4}-D_2. \tag{4.7}$$

We see therefore that negative eigenvalues of the operator correspond to positive values of Γ. The form of the potential $\tilde{V}(\xi)$ near the singular points is determined by the sign and magnitude of the last term in D_2 (see (4.2)).

Stationary shock waves that transform the system from the state $W_{02,1}$ into $W_{01,2}$ or from $W_{02,3}$ into $W_{03,2}$ are unstable[11]. Indeed, let the system go over from the state W_{02} to W_{01} ($u < u_0$); we then have $D_2 = D_2(W_{02}) > 0$ in the region $-\infty < \xi < 0$ and $D_3 = D_2(W_{01}) > 0$ for $\infty > \xi > 0$. Thus, when $\Gamma > 0$ there exist wave numbers that ensure a finite solution in all of ξ space.

Let us investigate the stability of the shock waves when the phase trajectory is a separatrix passing from the saddle W_{01} to the saddle W_{03}. The point W_{02} is a focus. A plot of $\tilde{V}(\xi)$ is shown in Fig. 5. In the regions $\xi < -\xi_1$ and $\xi > \xi_1$ ($D_2(\xi_1) - {}^1/_4D_1^2(\xi_1) = \Gamma$) we can construct two exponentially-decreasing solutions at $\xi = \pm\infty$. Oscillating solutions exist inside the region $-\xi_1 < \xi < \xi_1$. The number of zeroes of the solution is determined by the corresponding eigenvalue $-\Gamma$. It can be shown that when $\Gamma = 0$ the eigenfunction $\hat{\mathscr{H}}$ is proportional to $W_{0\xi}(\xi)$ and has no zeroes (ground state). Thus, all the eigenvalues

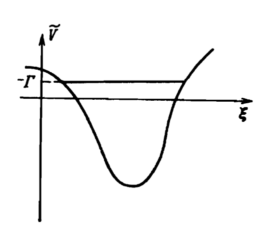

FIG. 5

of the operator $\hat{\mathscr{H}}$ are positive ($-\Gamma > 0$), and consequently this stationary solution is stable.

In the case of a domain, the function $\tilde{V}(\xi)$ has two minima. The level $\Gamma = 0$ corresponds to an eigenfunction having one zero. This means that there exists one negative eigenvalue of the operator $\hat{\mathscr{H}}$, and the domain is unstable. In other words, when the second minimum of $\tilde{V}(\xi)$ appears, a splitting of the level $\Gamma = 0$ takes place.

For periodic stationary waves ($u = u_0$), the function $\tilde{V}(\xi)$ is periodic and the number of negative levels is infinite. A solution of this type is therefore unstable.

5. STABILIZATION OF INSTABILITY BY AN EXTERNAL LOAD

Let us determine the effect of an external load of resistance R. The total current now depends on the time

$$j_0(\tau)=\frac{\mathscr{E}}{RS}-\frac{l}{RS}\int_0^{L/l}E(\xi,\tau)d\xi, \tag{5.1}$$

where $\mathscr{E}$ is the source emf and S the cross section area of the sample. Hence

$$\delta j_0(\tau)=-\frac{l}{RS}\int_0^{L/l}E_1(\xi,\tau)d\xi. \tag{5.2}$$

At the same time, using (2.4) with $N = N_0$ we can express $E_1(\xi, \tau)$ in terms of $\Theta(\xi, \tau)$ and $\delta j_0(\tau)$. Then $\delta j_0(\tau)$ in (5.2) takes the form

$$\delta j_0(\tau)=\left\{\frac{RS}{l}+\int_0^{L/l}\frac{d\xi'}{\sigma(W_0)}\right\}^{-1}\int_0^{L/l}d\xi'\left\{\frac{q-1}{e}\frac{\partial\Theta_1}{\partial\xi'}-j_{00}\frac{\partial}{\partial W_0}\left(\frac{1}{\sigma(W_0)}\right)W_1\right\}, \tag{5.3}$$

where $\sigma(W_0)$ and j_{00} are the values of the conductivity and of the current in the stationary state. We use the cyclicity condition to determine Θ_1 on the boundary of the sample: $\Theta_1(0) = \Theta_1(L)$. Then

$$\delta j_0(\tau)=-j_{00}\left\{\frac{RS}{l}+\int_0^{L/l}\frac{d\xi'}{\sigma(W_0)}\right\}^{-1}\int_0^{L/l}d\xi'\frac{\partial}{\partial W_0}\left(\frac{1}{\sigma(W_0)}\right)W_1(\xi',\tau). \tag{5.4}$$

We write down an equation for the determination of $W_1(\xi, \tau)$:

$$\begin{gathered}\left(\frac{\partial^2}{\partial\xi^2}+D_1(\xi)\frac{\partial}{\partial\xi}+D_2(\xi)\right)W_1-\Gamma W_1\\ =\delta j_0\left\{\frac{\tau_{00}}{elN}P(\xi)W_{0\xi}-\frac{2}{j_{00}}[(2-q)W_0]^{q/(2-q)}\right\}.\end{gathered} \tag{5.5}$$

If the system is in a homogeneous state $W = W_{02}$ and the perturbation is also homogeneous, then

$$\delta j_0(\tau)=-j_{00}\left[1+\frac{RS}{L}\sigma(W_0)\right]^{-1}\sigma(W_0)\frac{\partial}{\partial W_0}\left(\frac{1}{\sigma(W_0)}\right)W_1(\tau). \tag{5.6}$$

From (5.5) we obtain

$$\gamma=-\frac{P(W_{02})}{\alpha^2}\left\{r-q+\frac{2q}{1+RS\sigma/L}\right\}. \tag{5.7}$$

It is easily seen that the instability of homogeneous perturbations can be stabilized if the following inequality is satisfied:

$$R<\frac{L}{S\sigma}\frac{r+q}{q-r}. \tag{5.8}$$

This inequality has a simple physical meaning. It means that the total conductivity of the circuit $L/RS + \sigma(r-q)/(r+q)$ is positive (the second term is the differential conductivity of the sample).

Let us investigate the effect of the load on the instability of stationary waves such as solitons or periodic oscillations. We note that no other type of stationary waves can be realized in a bounded volume, since they do not satisfy the stationarity conditions.

Equation (5.5) reduces with the aid of the transformation (4.5) to the form

$$\hat{\mathscr{H}}\Psi+P(\xi)\gamma\Psi=\Phi(\xi)\int_0^{L/l}K(\xi')\Psi(\xi')d\xi', \tag{5.9}$$

where

$$\begin{gathered}\Phi(\xi)=-\frac{\tau_{00}}{elN}P(\xi)W_{0\xi}(\xi)+\frac{2}{j_{00}}[(2-q)W_0(\xi)]^{q/(2-q)},\\ \delta j_0=\int_0^{L/l}K(\xi')\Psi(\xi')d\xi',\\ K(\xi)=-j_{00}\left[\frac{RS}{l}+\int_0^{L/l}\frac{d\xi'}{\sigma(W_0(\xi'))}\right]^{-1}\frac{\partial}{\partial W_0}\left[\frac{1}{\sigma(W_0(\xi))}\right].\end{gathered} \tag{5.10}$$

We expand the functions $\Psi(\xi)$ and $\Phi(\xi)$ in terms of the eigenfunctions of the operator $\hat{\mathscr{H}}$:

$$\Psi(\xi)=\sum c_m\psi_m(\xi),\quad \Phi(\xi)=\sum b_n\psi_n(\xi). \tag{5.11}$$

We can then determine from (5.9) the coefficients c_m in terms of b_m:

$$\begin{gathered}c_m=\frac{\delta j_0 b_m}{(\gamma_m-\gamma)P(\xi)}\\ b_m=\left(\int P(\xi')\psi_m^2(\xi')d\xi'\right)^{-1}\int\Phi(\xi')P(\xi')\psi_m(\xi')d\xi',\end{gathered} \tag{5.12}$$

where $\gamma_m P(\xi)$ are the eigenvalues of the operator $\hat{\mathscr{H}}$. Substituting the values of the function $\Psi(\xi)$ in the expressions for δj_0 (5.10), we obtain an equation for b_m:

$$\sum\frac{b_m}{\gamma_m-\gamma}\int_0^{L/l}d\xi'\frac{K(\xi')\psi_m(\xi')}{P(\xi')}=1. \tag{5.13}$$

Let $\gamma_M = 0$. It can be directly verified that $\psi_M(\xi) = W_{0\xi}(\xi)$. For one soliton, $W_{0\xi}$ vanishes over the length of the sample once, i.e., $M = 1$, and the lowest level γ_0 corresponds to the eigenfunction $\psi_0(\xi)$. We write down the sum (5.13):

$$1=\frac{b_0}{\gamma_0-\gamma}\int_0^{L/l}d\xi'\frac{K(\xi')\psi_0(\xi')}{P(\xi')}-\frac{b_1}{\gamma_1}\int_0^{L/l}d\xi'\frac{K(\xi')\psi_1(\xi')}{P(\xi')}+\ldots \tag{5.14}$$

The individual terms in the sum are small, since they are proportional to $(\gamma_m)^{-2}$. The functions $P(\xi)$ and $K(\xi)$ are monotonic, and we can therefore put approximately

$$\int_0^{L/l}d\xi'\frac{K(\xi')\psi_1(\xi')}{P(\xi')}\approx\left\langle\frac{K(\xi)}{P(\xi)}\right\rangle\int_0^{L/l}d\xi' W_{0\xi'}(\xi')\approx 0. \tag{5.15}$$

Then

$$\gamma_0-\gamma\approx b_0\int_0^{L/l}d\xi'\frac{K(\xi')\psi_0(\xi')}{P(\xi')} \tag{5.16}$$

and the stability criterion takes the form

$$\gamma_0<b_0\int_0^{L/l}d\xi'\frac{K(\xi')\psi_0(\xi')}{P(\xi')}. \tag{5.17}$$

Stationary waves in the form of two or more domains are unstable regardless of the external load. (We note that this stationary state sets in for phase trajectories that pass near the saddle points.) Indeed, in analogy with[5], we can verify that among the perturbations of the stationary state there are always some that do not change the total voltage on the sample, and therefore are not stabilized by the external load.

In conclusion, let us dwell briefly on the opposite limiting case, when the Maxwellian relaxation time is large and an inequality opposite to (2.9) is satisfied. Taking into account the weak space-time dependence of the variables in the transport equation (2.3), we can transform the system (2.1)–(2.3) into

$$\frac{\partial E}{\partial t}+V(E)\frac{\partial E}{\partial y}-D(E)\frac{\partial^2 E}{\partial y^2}=4\pi e[G(t)-N_0V(E)]\varepsilon_0^{-1}. \quad (5.18)$$

where

$$V(E)=\frac{\sigma_0 E}{eN_0}\left(\frac{\tau_{00}\sigma_0 E_0^2}{N_0 T}\right)^{-q/(r+q)},$$

$$D(E)=\frac{q^2-q+r}{r+q}\frac{V(E)}{e}E^{(2-r-q)/(r+q)}$$

G(t) is the integration constant of the continuity equation.

Expression (5.18) is similar to Eq. (5) of [9]. By investigating this expression on the phase plane, we can obtain all the considered types of stationary waves of the electric field and electron density and electron temperature.

[1)]The conditions $r + q > 0$ and $r - q < 0$ are realized, for example, in momentum scattering by charged particles ($q = 3/2$) and energy scattering by piezoacoustic phonons ($r = 1/2$). These scattering mechanisms can appear in InSb at helium temperatues [5].

[2)]We note that in the absence of an external magnetic field the j(W) plot has the shape of the letter S.

[3)]The singular points correspond to equilibrium states of the dynamic system on the phase plane, i.e., to points at which $W_\xi = W_{\xi\xi} = 0$.

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