THE RADIO-ELECTRIC EFFECT IN A STRONG MAGNETIC FIELD

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The stationary electric and magnetic fields induced in a metallic plate by an incident electromagnetic wave are calculated. The REE is of a resonant nature because of the excitation of standing magnetoplasma waves. An attempt is made in the paper to qualitatively explain the experimental results of M. S. Khaĭkin and coworkers[1,2].

INTRODUCTION

THE nonlinear relation between the current density and the electric field intensity leads to the appearance of a constant emf proportional to the high-frequency power and dependent, of course, on the nonlinearity mechanism (the radio-electric effect, REE).

The simplest nonlinearity mechanism is due to the dependence of the resistance of the magnetic field of the current. The resistance is particularly sensitive to the magnetic field at low temperatures. This means that the REE due to this dependence should be particularly noticeable at low temperatures. A constant magnetic field applied to a metal influences the magnitude of the REE significantly.

With an aim at discussing the experiments of Khaĭkin and Yakubovskiĭ[1] and of Kaĭkin and Semenchinskiĭ[2], we investigate the REE in the geometry shown in the figure; the static magnetic field H ($H_x = H_y = 0$, $H_z = H$) is parallel to the plane of the plate ($0 < y < d$), and the alternating electromagnetic field is polarized in the following manner[1)]:

$$\tilde{E}_x \neq 0,\quad \tilde{E}_y = \tilde{E}_z = 0,\quad \tilde{H}_x = \tilde{H}_y = 0,\quad \tilde{H}_z \neq 0. \tag{1}$$

After time averaging the relation $E_i = \rho_{ik} j_k$ and taking into account the dependence of the tensor ρ_{ik} on the magnetic field, we find that there are dc components of both the electric field $\bar{\mathbf{E}} = \langle \mathbf{E} \rangle$ and of the current density $\bar{\mathbf{j}} = \langle \mathbf{j} \rangle$ (the angle brackets denote averaging over the time):

$$\bar{E}_i = \langle \rho_{ik} j_k \rangle = \rho_{ik}^{(0)} \bar{j}_k + \frac{\partial \rho_{ik}^{(0)}}{\partial H_p} \langle \tilde{H}_p \tilde{j}_k \rangle \approx \rho_{ik}^{(0)} \bar{j}_k + \frac{\partial \rho_{ik}^{(0)}}{\partial H_p} \sigma_{km}^{(0)} \langle \tilde{H}_p \tilde{E}_m \rangle.$$

Here $\rho_{ik}^{(0)}$; $\sigma_{ik}^{(0)} = (\rho_{ik}^{(0)})^{-1}$ are the values of the resistivity and conductivity tensors at $\tilde{H} = 0$. We shall henceforth omit the index 0 throughout. The quantity

$$E_i^{\text{ext}} = \frac{\partial \rho_{ik}}{\partial H_p} \sigma_{km} \langle \tilde{H}_p \tilde{E}_m \rangle \tag{2}$$

represents the source of the extraneous dc emf, which produces in the sample, depending on the conditions, dc currents or potential differences or both (see below).

In their experiments, Khaĭkin and Yakubovskiĭ[1] investigated the REE in a pure Bi plate ≈1 mm thick. The electromagnetic wave frequency was $\omega \approx 1.3 \times 10^{11}\ \text{sec}^{-1}$ and the magnetic field reached $H \approx 10^5$ Oe. They measured the potential difference along the x axis (see the figure) from a direction opposite to that of the incidence of the electromagnetic wave. If it is assumed that the emf is due to the dependence of ρ_{ik} on the magnetic field then, according to (2),

$$E_x^{\text{ext}} = \frac{\partial \rho_{xk}}{\partial H_z} \sigma_{kx} \langle \tilde{E}_x \tilde{H}_z \rangle. \tag{3}$$

Even without allowance for the anisotropy of the crystal, the matrices ρ_{ik} and σ_{ik} have off-diagonal (Hall) elements:

$$\hat{\rho}_{\alpha\beta} = \begin{pmatrix} \rho_\perp & RH \\ -RH & \rho_\perp \end{pmatrix},\qquad \rho_{\alpha z} = 0,\qquad \rho_z = \rho_\parallel, \tag{4}$$
$$\hat{\sigma}_{\alpha\beta} = (\rho_\perp^2 + R^2H^2)^{-1} \begin{pmatrix} \rho_\perp & -RH \\ RH & \rho_\perp \end{pmatrix},\qquad \sigma_{\alpha z} = 0,\qquad \sigma_{zz} = \frac{1}{\rho_\parallel}.$$

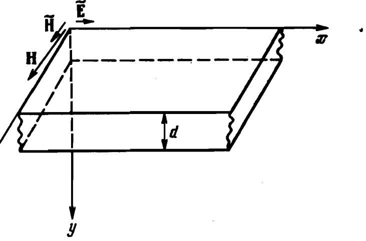
Here $\rho_\perp (\rho_\parallel)$ is the transverse (longitudinal) resistivity, which depends on the magnetic field H; R is the Hall constant (which depends little on H in the limiting cases). Substituting (4) in (3) we obtain

$$E_x^{\text{ext}} = (2\|\hat{\rho}\|)^{-1} (\partial \|\hat{\rho}\| / \partial H) \langle \tilde{H}_z \tilde{E}_x \rangle, \tag{5}$$

where

$$\|\hat{\rho}\| = \rho_\perp^2 + R^2H^2 \tag{6}$$

is a determinant made up of the matrix elements $\rho_{\alpha\beta}$ (see (4)).



[1)]The theories of the radio-electric or of the opto-electric effect are dealt with in many papers. In the two papers known to us, by Gurevich and Mezrin [3,4], an attempt is made to bring the region of theoretically investigated parameters closer to the region that is important from the point of view of understanding the experiments of [1,2]. Namely, in [3] they investigated the REE in a magnetic field H at low frequencies, and in [4] at high frequencies but at H = 0. In both cited papers, no account was taken of the specifics of the electronic energy structures of Bi and of the character of the propagation of electromagnetic waves in Bi in a strong magnetic field. As will be shown later, both circumstances are essential for the understanding of the experiments of [1,2]. Notice should also be taken of the work of Gurevich and Rumyantsev [5], which is close in its formulation to the second section of the present paper.

In metals with equal numbers of electrons and holes (such as Bi), $\rho_\perp \sim H^2$ and $\rho_\perp \gg RH$ in strong magnetic fields. Therefore

$$E_x^{ext} \approx 2\langle \tilde{E}_x \tilde{H}_z \rangle / H. \quad (7)$$

Substituting in the last formula the values from[1] ($H \approx 10^3$ Oe, $\tilde{E} \approx 1$ V/cm, $\tilde{H} \approx 1/300$ Oe), we obtain $E_x^{ext} \approx 3\ \mu$V/cm, which agrees in order of magnitude with the results of the cited paper.

Before we discuss in detail the dependence of E_x^{ext} on the magnetic field (according to[1], the REE in Bi at $H \lesssim 2.5$ kOe increased monotonically with increasing H, and a complicated oscillatory dependence, undoubtedly connected with excitation of magnetoplasma waves[6], was observed in strong fields), we make a few remarks.

1) According to (5), the coefficient of $\langle \tilde{H}_z \tilde{H}_x \rangle$ should have a maximum value at $H \sim H_l$ (H_l is the field at which the electron-orbit radius r_H is equal to the mean free path l). This statement follows from the fact that in order of magnitude we have

$$\|\hat{\rho}\| = (m/ne^2\tau)^2 f(H/H_l)$$

(see[7], Sec. 26). The notation is standard: $f(H/H_l)$ is a certain function, the asymptotic form of which depends on the electron spectrum (if the Fermi surface is closed, then in strong fields at $n_1 \neq n_2$ we have $f \sim H^2$, and at $n_1 = n_2$ we have $f \sim H^4$). Owing to the Hall term in[6], the electron spectrum does not affect strongly the coefficient $(2\|\rho\|)^{-1}(\partial\|\rho\|/\partial H)$.

2) In the absence of a constant magnetic field, there is no quadratic REE (in terms of the amplitude of the alternating field) due to this mechanism; according to (5) and (6), $(\|\hat{\rho}\|)^{-1}(\partial\|\hat{\rho}\|/\partial H) = 0$ at $H = 0$.

3) Strictly speaking, the foregoing formulas (particularly (5) and (7)) are valid only when the alternating field satisfies the quasistatic conditions

$$\omega \ll \omega_c, \nu; \quad l, r_H \ll \delta, \quad (8)$$

where ω_c is the cyclotron frequency, ν is the collision frequency ($\nu = v_f/l$, v_F is the Fermi velocity), and δ is the depth of the skin layer or the electromagnetic wavelength in the metal. If the conditions (8) are violated, then the static resistance cannot be explanded in powers of the alternating magnetic field, and the more detailed analysis presented below is necessary.

Under the conditions of[1], the relations between the quantities written out above are:

$$\frac{\omega}{\nu} \approx 10^3, \quad \frac{\omega}{\omega_c} \approx 10^{-1}, \quad \frac{r_H}{l} \approx 10^{-4}, \quad \frac{l}{\delta} \approx 10^2.$$

The estimates correspond to the maximum magnetic field and the lowest temperature; ω_c depends strongly on the direction and on the type of carrier. We used in the estimate an effective mass $m^* \approx 10^{-1}$ m (m is the free-electron mass). Here δ denotes the depth of the skin layer at $H = 0$.

REE UPON EXCITATION OF MAGNETOHYDRODYNAMIC WAVES

As is well known[6], magnetoplasma waves are the consequence of temporal (and not spatial) dispersion of the electric conductivity of metals having equal numbers of electrons and holes. This makes it possible to use the hydrodynamic equations to calculate the REE in Bi. We start from the simplest model of Bi, that of a semimetal with equal numbers of electrons and holes ($n_1 = n_2 = n$), with isotropic dispersion laws ($m_{1,2}$ are the corresponding effective masses and $\tau_{1,2}$ the free path times). If we disregard the pressure of the electron gas, the hydrodynamic equations coincide with the equations of motion with fraction, which can be easily expressed in terms of the electron and hole current densities $\mathbf{j}_1$ and $\mathbf{j}_2$:

$$\frac{d\mathbf{j}_\alpha}{dt} + \frac{1}{\tau_\alpha}\mathbf{j}_\alpha = \frac{ne^2}{m_\alpha}\left(\mathbf{E} + \frac{(-1)^\alpha}{nec}[\mathbf{j}_\alpha \mathbf{H}]\right), \quad \alpha = 1, 2. \quad (9)^*$$

The sign of the charge or of the effective mass is accounted for by the factor $(-1)^\alpha$ ($m_\alpha > 0$, $e > 0$), The current density in the sample is the sum of the electron and hole densities:

$$\mathbf{j} = \mathbf{j}_1 + \mathbf{j}_2. \quad (10)$$

Here **E** and **H** are the true intensities of the electric and magnetic fields. All the quantities in (9) (**E**, **H**, and $\mathbf{j}_\alpha$) can be expressed in the form of sums:

$$\mathbf{E} = \overline{\mathbf{E}} + \tilde{\mathbf{E}}, \quad \mathbf{H} = \overline{\mathbf{H}} + \tilde{\mathbf{H}}, \quad \mathbf{j}_\alpha = \overline{\mathbf{j}}_\alpha + \tilde{\mathbf{j}}_\alpha, \quad (11)$$

where $\tilde{\mathbf{E}}$, $\tilde{\mathbf{H}}$, and $\mathbf{j}_\alpha$ are the electric and magnetic fields and the current density in the wave of frequency ω, $\overline{\mathbf{H}}$ is the constant magnetic field applied to the sample (it will be designated **H** as before), and $\overline{\mathbf{E}}$ and $\overline{\mathbf{j}}_\alpha$ are static values quadratic in the amplitude of the wave incident on the sample. Their appearance is due to nonlinear effects and their calculation is the subject of the present section[2)]:

In (9), the nonlinearities are contained in the Hall terms as well as in $d\mathbf{j}_\alpha/dt$, since it follows that

$$\frac{d\mathbf{j}_\alpha}{dt} = \frac{\partial \mathbf{j}_\alpha}{\partial t} + (\mathbf{v}_\alpha \nabla)\mathbf{j}_\alpha = \frac{\partial \mathbf{j}_\alpha}{\partial t} + \frac{(-1)^\alpha}{ne}(\mathbf{j}_\alpha \nabla)\mathbf{j}_\alpha.$$

It can be shown, however, that in the region where magnetoplasma waves exist the terms $((-1)^\alpha/ne)(\mathbf{j}_\alpha \cdot \nabla)\mathbf{j}$ are much smaller than the Hall terms, and will therefore be neglected henceforth.

Averaging Eqs. (9) over the time and denoting by $\hat{\sigma}_\alpha$ the tensor of the static electric conductivity of the electrons ($\alpha = 1$) and of the holes ($\alpha = 2$), we obtain (recognizing that $\langle \tilde{\mathbf{j}}_\alpha \rangle = 0$ and $\langle \tilde{\mathbf{E}} \rangle = 0$)

$$\overline{\mathbf{j}}_\alpha = \hat{\sigma}_\alpha\{\overline{\mathbf{E}} - \mathbf{E}_\alpha^{ext}\}, \quad \alpha = 1, 2, \quad (12)$$

where

$$\mathbf{E}_\alpha^{ext} = -\frac{(-1)^\alpha}{nec}\langle[\tilde{\mathbf{j}}_\alpha \tilde{\mathbf{H}}]\rangle \quad (13)$$

is the source of the extraneous dc emf acting on the electrons ($\alpha = 1$) and the holes ($\alpha = 2$).

Adding the equations in (12), we obtain

$$\overline{\mathbf{j}} = \hat{\sigma}\{\overline{\mathbf{E}} - \mathbf{E}^{ext}\}, \quad \mathbf{E}^{ст} = -\hat{\rho}\sum_\alpha \frac{(-1)^\alpha}{nec}\hat{\sigma}_\alpha\langle[\tilde{\mathbf{j}}_\alpha \tilde{\mathbf{H}}]\rangle, \quad (14)$$

$\hat{\sigma}$ is the static electric conductivity tensor of Bi and

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*$[\mathbf{jH}] \equiv \mathbf{j} \times \mathbf{H}$.

2)Of course, Eqs. (9) cannot be used to analyze all the quadratic effects; for example, it is impossible to take into account the nonlinearity contained in the collision integral (heating of the electron gas etc.). It seems to us that under the conditions of the discussed experiments [1,2] the principal role in the REE was played by nonlinearities contained in the equations of motion.

$\hat{\rho}$ is the static resistivity tensor ($\hat{\rho} = \hat{\sigma}^{-1}$); $\hat{\sigma}$ and $\hat{\rho}$ were used in the Introduction (see (4)).

To make (14) concrete, it is necessary to express the partial currents $\tilde{\mathbf{j}}_\alpha$ in terms of the electric field $\mathbf{E}$. According to (9),

$$\tilde{\mathbf{j}}_\alpha = \sigma_\alpha \mathbf{E}, \tag{15}$$

where the matrix elements $\tilde{\sigma}_\alpha$ are:

$$\tilde{\sigma}_{xx}^{\alpha} = \tilde{\sigma}_{yy}^{\alpha} = \frac{\sigma_\alpha(\omega)}{1+(\sigma_\alpha(\omega)H/nec)^2},$$

$$\tilde{\sigma}_{xy}^{\alpha} = -\tilde{\sigma}_{yx}^{\alpha} = (-1)^\alpha \frac{H}{nec}\frac{\sigma_\alpha^2(\omega)}{1+(\sigma_\alpha(\omega)H/nec)^2}, \tag{16}$$

$$\tilde{\sigma}_{zz}^{\alpha} = \sigma_\alpha(\omega), \qquad \tilde{\sigma}_{xz}^{\alpha} = \tilde{\sigma}_{yz}^{\alpha} = \tilde{\sigma}_{zx}^{\alpha} = \tilde{\sigma}_{zy}^{\alpha} = 0,$$

$$\sigma_\alpha(\omega) = \frac{ne^2\tau_\alpha}{m_\alpha}\frac{1}{1-i\omega\tau_\alpha}.$$

Recognizing that $\hat{\tilde{\sigma}}_\alpha \equiv \tilde{\sigma}_\alpha$ ($\omega = 0$) and that the static magnetic field is the very largest parameter in the region of existence of magnetoplasma waves (i.e., $\omega_c \gg \omega, \nu$), we find that the largest component of the extraneous current

$$\mathbf{j}^{\text{ext}} = \sum_\alpha \frac{(-1)\hat{\sigma}_\alpha}{nec}\langle[\tilde{\mathbf{j}}_\alpha \tilde{\mathbf{H}}]\rangle \tag{17}$$

is the x-component (relative to the inverse magnetic field) and

$$j_x^{\text{ext}} \approx -\frac{1}{4H}\left(\frac{nec}{H}\right)^2\left\{\frac{1}{\sigma_1}+\frac{1}{\sigma_1(\omega)}+\frac{1}{\sigma_2}+\frac{1}{\sigma_2(\omega)}\right\}\langle \tilde{E}_x\tilde{H}_z^*\rangle + \text{c.c.}, \tag{18}$$

where $\sigma_\alpha = \sigma_\alpha$ ($\omega = 0$) (see (16)). We took into account the fact that $\langle \tilde{E}_x^* \tilde{H}_z^* \rangle = \langle \tilde{E}_x \tilde{H}_z \rangle = 0$. For metals with $n_1 = n_2$ we have $\rho_{xx} > \rho_{xy}$, so that the x-component of the extraneous field will also be maximal:

$$E_x^{\text{ext}} \approx \frac{1}{4}\frac{\rho_{xx}}{H}\left(\frac{nec}{H}\right)^2\left\{\frac{1}{\sigma_1}+\frac{1}{\sigma_1(\omega)}+\frac{1}{\sigma_2}+\frac{1}{\sigma_2(\omega)}\right\}\langle \tilde{E}_x\tilde{H}_z^*\rangle + \text{c.c.} \tag{19}$$

Since we have in these terms

$$\rho_{xx} \approx \left(\frac{H}{nec}\right)^2 \frac{1}{1/\sigma_1 + 1/\sigma_2}, \tag{20}$$

it follows that

$$E_x^{\text{ext}} \approx \frac{1}{(1/\sigma_1+1/\sigma_2)H}\left\{\frac{1}{\sigma_1}+\frac{1}{\sigma_2}+\frac{1}{\sigma_1(\omega)}+\frac{1}{\sigma_2(\omega)}\right\}\frac{1}{4}\langle \tilde{E}_x\tilde{H}_z^*\rangle + \text{c.c.} \tag{21}$$

As $\omega \to 0$ we arrive at formula (7) ($\frac{1}{4}\langle \tilde{E}_x \tilde{H}_z^* \rangle + \frac{1}{4}\langle \tilde{E}_x^* \tilde{H}_z \rangle = \langle \operatorname{Re} \tilde{E}_x \operatorname{Re} \tilde{H}_z \rangle$). At $\omega \neq 0$ we have

$$E_x^{\text{ext}} \approx \frac{1}{H}\operatorname{Re}\langle \tilde{E}_x\tilde{H}_z^*\rangle + \frac{\omega\tau_{\text{eff}}}{2H}\operatorname{Im}\langle \tilde{E}_x\tilde{H}_z^*\rangle, \tag{22}$$

where

$$\frac{1}{\tau_{\text{eff}}} = \frac{1}{m_1+m_2}\left(\frac{m_1}{\tau_1}+\frac{m_2}{\tau_2}\right). \tag{23}$$

We note that in the derivation of (22) we did not assume that $\omega\tau_{\text{eff}} \gg 1$, but the magnetoplasma waves exist precisely when this condition is satisfied.

It is convenient to characterize the metal in this region of magnetic fields and frequencies by a dielectric constant, and if we retain only the principal terms in the expansion in powers of $1/H$, then the dielectric constant in the plane perpendicular to H can be regarded as diagonal and isotropic

$$\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon = \varepsilon' + i\varepsilon'',$$

$$\varepsilon = \frac{4\pi n(m_1+m_2)c^2}{H^2}\left(1+\frac{i}{\omega\tau_{\text{eff}}}\right). \tag{24}$$

The value of E_x^{ext} depends very strongly on the concrete configurations of the alternating fields in a plate through which magnetoplasma waves pass. To determine the REE it is therefore necessary to calculate $\langle \tilde{E}_x \tilde{H}_z^* \rangle$ with allowance for the magnetoplasma-wave excitation conditions. It is most important here to specify the amplitude E_0 of the wave incident on the plate, or the energy flux Q from the source ($Q = (c/8\pi)E_0^2$).

The intensities of the electric and magnetic fields in the plate are equal to

$$\tilde{E}_x = \frac{2E_0(\sqrt{\varepsilon}+1)}{(\sqrt{\varepsilon}+1)^2-(\sqrt{\varepsilon}-1)^2e^{2ikd}}\left(e^{iky}+e^{2ikd}\frac{\sqrt{\varepsilon}-1}{\sqrt{\varepsilon}+1}e^{-iky}\right) \tag{25}$$

$$\tilde{H}_z = -\sqrt{\varepsilon}\frac{2E_0(\sqrt{\varepsilon}+1)}{(\sqrt{\varepsilon}+1)^2-(\sqrt{\varepsilon}-1)^2e^{2ikd}}\left(e^{iky}-e^{2ikd}\frac{\sqrt{\varepsilon}-1}{\sqrt{\varepsilon}+1}e^{-iky}\right),$$

where

$$k = k' + ik'' = \frac{\omega}{c}\sqrt{\varepsilon} = \frac{\sqrt{4\pi n(m_1+m_2)}\,\omega}{H}\left(1+\frac{i}{2\omega\tau_{\text{eff}}}\right). \tag{26}$$

These expressions become much simpler in limiting cases. If $k''d \gg 1$ (thick plate), we can disregard the reflected wave. We do not write out the values of E_x^{ext} in this case, since it is of no interest when it comes to explaining the results of[1] and[2], because no standing magnetoplasma waves are excited in thick plates.

We consider the case of thin plates ($k''d \lesssim 1$). Since $\epsilon' \gg \epsilon''$, we retain the imaginary part of ϵ only in the exponentials. Then

$$E_x^{\text{ext}} \approx -\frac{4E_0^2\sqrt{\varepsilon}}{H(\varepsilon-1)}\frac{1}{\chi-\cos 2k'd}\left\{\frac{1}{\varepsilon-1}[2\sqrt{\varepsilon}\operatorname{ch}2k''(d-y) + (\varepsilon+1)\operatorname{sh}2k''(d-y)] + \frac{\omega\tau_{\text{eff}}}{2}e^{-2k''y}\sin 2k'(d-y)\right\} \tag{27}$$

where

$$\chi \equiv \chi(\varepsilon, k''d) = \frac{(\sqrt{\varepsilon}+1)^4e^{2k''d}+(\sqrt{\varepsilon}-1)^4e^{-2k''d}}{2(\varepsilon-1)^2}, \quad \varepsilon \equiv \varepsilon'. \tag{28}$$

Formulas (27) and (28), together with (23), (24), and (26), solve our problem of determining E_x^{ext}. We note that owing to the large effective dielectric constant ϵ, the extraneous field $\mathbf{E}^{\text{ext}}$ is quite sensitive to the value of the coordinate y (especially near $y = d$).

STATIONARY FIELD AND CURRENT IN THE REE

Knowing the extraneous emf $\mathbf{E}^{\text{ext}}$, we can calculate the static field $\overline{\mathbf{E}}$ and the current density $\overline{\mathbf{j}}$ induced in the sample, which satisfy, naturally, the equations of electrostatics:

$$\operatorname{rot}\overline{\mathbf{E}} = 0, \quad \operatorname{div}\overline{\mathbf{j}} = 0, \quad \overline{\mathbf{j}} = \hat{\sigma}(\overline{\mathbf{E}} - \mathbf{E}^{\text{ext}}). \tag{29}$$

The solution of these equations depends essentially on the geometry of the problem: if the plate is very thin and the electromagnetic wave is uniformly incident on the entire surface of the plate, then $\overline{\mathbf{j}} = 0$, and a constant emf is produced between the edges of the plate. If the plate is thick or if the electromagnetic wave is incident only on part of the plate surface, then closed currents and the associated electric field are excited in the plate (we are speaking here of an "open-circuited" plate).

We assume that $\mathbf{E}^{\text{ext}}$ depends only on the coordinate y, although in the experiments[1,2] this was not strictly fulfilled, since the electromagnetic wave was incident only on the central part of the plate. In addition we assume that $E_y^{ext} = e_z^{ext} = 0$ and E_x^{ext} is given by (27). Recognizing that the components $\sigma_{z\beta}$ of the tensor $\hat{\sigma}$ vanish ($\beta = x, y$), we obtain from (29)

$$\bar{E}_z = 0, \quad \bar{E}_x = \text{const}, \quad \bar{j}_z = \bar{j}_y = 0,$$
$$\bar{E}_y = -\frac{\sigma_{yx}}{\sigma_{yy}}(\bar{E}_x - E_x^{ext}), \bar{j}_x = \left(\sigma_{xx} - \frac{\sigma_{xy}\sigma_{yx}}{\sigma_{yy}}\right)(\bar{E}_x - E_x^{ext}). \tag{30}$$

From the condition that no macroscopic current flows through the plate

$$\int_0^d j_x(y)\,dy = 0,$$

we get

$$\bar{E}_x = \frac{1}{d}\int_0^d E_x^{ext}(y)\,dy, \tag{31}$$

and from the last equation of (30) we obtain the density distribution of the stationary current in the plate in the presence of REE. According to (4) we have $\sigma_{xx} - \sigma_{xy}\sigma_{yx}/\sigma_{yy} = \rho_\perp^{-1}$: Therefore

$$j_x(y) = (\bar{E}_x - E_x^{ext})/\rho_\perp. \tag{32}$$

The flow of closed currents in the plate (the "closing" is at infinity) produces in the plate an induced magnetic field H defined (in this case) by

$$\bar{H}_x = \bar{H}_y = 0, \quad \bar{H}_z(y) = \frac{4\pi}{c}\int_0^y \bar{j}_x(y')\,dy'. \tag{33}$$

In the considered geometry, the induced magnetic field $\bar{H}$ does not go outside the plate, but under real conditions (a plate of finite length, inhomogeneous irradiation), the static magnetic field produced by the closed currents should surround the sample. Recently Khaĭkin and Semenchinskiĭ[2] observed a magnetic flux produced by a stationary closed current excited by microwave radiation in single-crystal Bi. To estimate the induced magnetic field, we can apparently use the value of $H_z(y)$ at $0 < y < d/2$. Since, however, $H_z(y)$ is sensitive to the value of the coordinate y, we do not write out the corresponding formulas. In addition, we do not write out the result of integration in general form in (31), which is too cumbersome. A simplification is obtained if it is recognized that $\epsilon \gg 1$ and $k''d \ll 1$. Then

$$\bar{E}_x \approx -\frac{E_0^2}{\sqrt{\pi n(m_1+m_2)c^2}}\,\frac{\Gamma + {}^1/_2\omega\tau_{\text{eff}}\sin^2 k'd - {}^1/_4(1-\sin(2k'd)/2k'd)}{\Gamma^2 + \sin^2 k'd} \tag{34}$$

where

$$\Gamma = 2\varepsilon^{-1/2} + k''d \ll 1, \tag{35}$$

and in the derivation of (34) and (35) we have used expressions (23), (24), and (26).

The formulas presented show that the stationary field $\bar{E}_x$ (as well as $\bar{H}_z$) excited in the plate has a resonant dependence on the magnetic field. The resonant values of the external magnetic field are determined from the condition $k'd = N\pi$ (N = 0, 1, 2, ... are integers), corresponding to the condition for the excitation of standing magnetoplasma waves in a plate. According to (26),

$$H_N = \frac{d\omega}{N}\sqrt{\frac{4n(m_1+m_2)}{\pi}}, \tag{36}$$

and in the determination of the "line" width Γ it is necessary to distinguish between very thin plates ($k''d \ll 1/\sqrt{\epsilon}$) and thicker ones $1 \gg k''d \gg 1/\sqrt{\epsilon}$. In the former case the line width Γ is determined by the escape of electromagnetic energy from the plate, and in the second by the damping of the magnetoplasma waves. The total line width is determined by (35).

Owing to the second (high-frequency) term in (22), the dependence of $\bar{E}_x$ on the magnetic field is very complicated. The oscillation amplitude has the following order of magnitude:

$$\bar{E}_x|_{k'd=N\pi} - \bar{E}_x|_{k'd=(N+1/2)\pi} = \frac{E_0^2}{4[\pi n(m_1+m_2)c^2]^{1/2}}\left(\frac{1}{\Gamma^2} + 2\omega\tau_{\text{eff}}\right) \tag{37}$$

with E_x having opposite signs at $k'd = N\pi$ and $k'd = (N + 1/2)\pi$.

Formula (34) is valid at field values for which $\Gamma \ll 1$, i.e., when

$$\frac{H}{H_d} + \frac{H_d}{H} \ll \sqrt{\frac{cl}{v_F d}}, \quad H_d = \sqrt{\frac{\pi n(m_1+m_2)v_F c d}{l}}$$

Since $\sqrt{cl/v_F d} \gg 1$, the latter condition means that

$$(d/l)\sqrt{\pi n(m_1+m_2)v_F^2} \ll H \ll \sqrt{\pi n(m_1+m_2)c^2}. \tag{38}$$

In this magnetic-field interval, formula (34) seems to describe correctly (in any case, qualitatively), the results obtained in[1].

As to the results of[2] (observation of the magnetic field of currents circulating in the plate), they obviously depend strongly on the concrete geometry of the experiment and cannot be obtained from an idealized analysis.

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