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PHASE TRANSITION IN NUCLEAR MATTER AND NON-PAIR NUCLEAR FORCES

A. B. MIGDAL

L. D. Landau Institute of Theoretical Physics of the USSR Academy of Sciences

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The π-meson spectrum in nuclear matter is found, with nucleon correlations taken into account. It is shown that two branches of the meson spectrum exist. When the density is less than the nuclear density an instability of the meson field arises, leading to a phase transition; this changes the ground-state energy of nuclei and should lead to additional scattering of mesons by nuclei. The distortion of the meson Green function in nuclear matter results in important changes in the forces between the nucleons. These changes are lost when the usual assumption of pair interaction is made.

1. INTRODUCTION

IN a sufficiently deep potential well, the energy levels of a particle may descend to a value at which formation of particles from the vacuum is possible. Instability of the vacuum arises and particles are created until their interaction ensures stability. In the case of Fermi particles, this process leads to a fairly unimportant change of structure of the vacuum—by virtue of the Pauli principle, particles cannot accumulate in the "dangerous" levels, and the process ceases after occupation of the critical levels.

In the case of Bose particles, the process stops when repulsion between the particles makes further formation of particles unfavorable. If there is a critical field in a large volume ($R \gg \hbar/\mu c$), then a Bose condensate arises in the system. These questions are considered in detail in[1]. We

We must take special note of one possibility, which stems from the formation of a meson condensate in a sufficiently deep electric well. As is shown in[1], a bound state arises not only for mesons with charge of the same sign, but also for mesons with opposite charge, for which in the non-relativistic problem there is only repulsion (sic). Because of this phenomenon, in a sufficiently deep well a meson condensate appears and, perhaps, this will result in stability of nuclei with charge Z such that formation of meson pairs is possible: $Ze^2/R \gtrsim 2\mu c^2$. This corresponds to $Z \gtrsim 137^{3/2}$. For such Z, the gain of energy from formation of the condensate will, perhaps, exceed the Coulomb repulsion energy. Unfortunately, only a rough estimate is possible, and the question of the stability of such nuclei remains open.

As will be shown below, in nuclear matter of density $n_c \approx n_0/2$ where n_0 is the ordinary nuclear density, instability of the meson field arises and a static meson-field-condensate appears, which can be manifested in different experiments. The appearance of the condensate decreases the energy of nuclear matter, i.e., increases the binding energy of the nucleus. A change in the equation of state of nuclear matter occurs, which must be taken into account in calculations of the evolution of neutron stars. As will be shown, the mean square of the condensate field $\langle \varphi_0^2 \rangle$ is a periodic function of the spatial coordinates, and this should be manifested in the scattering of π mesons and electrons by nuclei.

Mesons in nuclear matter interact closely with excitations that may be called spin sound. Two branches of the excitations, which make a contribution to the effective interaction between the nucleons, arise. Exchange of these distorted mesons leads to a non-pair component in the interaction forces between the particles. This alteration of the interaction forces is lost in the ordinary approach in which the pair interaction of free nucleons is inserted in the Hamiltonian of the nucleus.

2. MESONS IN NUCLEAR MATTER[1)]

A. The Meson Polarization Operator

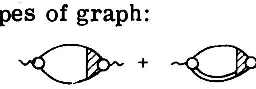
The energy of a π meson in nuclear matter can be expressed in terms of the meson polarization operator $\Pi(k, \omega)$:

$$\omega^2(k) = 1 + k^2 + \Pi(k, \omega) \tag{1}$$

($\hbar = \mu = c = 1$). By virtue of isotopic invariance, all three components φ_1, φ_2 and φ_3 of the meson field will have the same spectra. Therefore, all the calculations can be performed for the interaction with matter of neutral mesons; we recall that

$$\varphi_{\pi^+} = (\varphi_1 + i\varphi_2)/\sqrt{2}, \quad \varphi_{\pi^-} = (\varphi_1 - i\varphi_2)/\sqrt{2}, \quad \varphi_{\pi^0} = \varphi_3.$$

One can convince oneself that, for the values of ω and k of interest ($\omega \lesssim 1$, $k < m \approx 7$ where m is the mass of a nucleon), the meson polarization operator is determined by two types of graph:

$\Pi(k,\omega) =$  $= \Pi_p + \Pi_\pi$ (2)

The first graph corresponds to the transformation of a meson into a particle and a hole in the nucleon Fermi sea. The second graph corresponds to the absorption of a meson with the transformation of a nucleon of the nucleus into the resonance $\Delta_{3/2\,3/2}$.

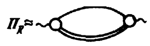
The principal idea of the results given below is that, right up to very high densities of nuclear matter ($n \gg n_0$), we can assume that the meson-nucleon vertex, like the vertex of the transition to the resonance $\Delta_{3/2\,3/2}$, does not vary in nuclear matter. In fact, both vertices correspond to the exchange of several mesons or nucleon pairs. The radius of the corresponding form

[1)]In this section, we confine ourselves to a schematic account of the results. A more detailed discussion will be given in another paper.

factor $r_n \sim 1/m$ and consequently, there will be a noticeable effect in nuclear matter at density $n \sim r_n^{-3} \sim 100\, n_0$.

The shaded vertex which appears in the first graph of formula (2) is determined by the methods of the theory of Fermi liquids, with interaction constants found from a comparison of the theoretical and experimental values of the magnetic moments of nuclei (spin-spin interaction of nucleons)[2].

As regards the second graph in (2), this has been estimated by Ericson and Hüfner[3] with neglect of the interaction of $\Delta_{3/2\,3/2}$ and a nucleon, i.e., they used the expression



(2′)

with vacuum vertices and without allowance for the difference in the mass shifts of the nucleon and the Δ-particle in nuclear matter. Satisfactory agreement with experiments on the scattering of π mesons in C^{12} was obtained. Therefore, we shall use the expression (2′) as a reasonable estimate of Π_R.

We give the final expression for the two terms of the polarization operator:

$$\Pi_R = -0.6\frac{n}{n_0}k^2\frac{2\omega_R}{\omega_R^2-\omega^2}, \quad (3)$$

$$\Pi_P = -2.3\left(\frac{n}{n_0}\right)^{1/3}k^2\frac{\Phi(k,\omega)}{1+g^-(k)\Phi(k,\omega)}. \quad (4)$$

In formula (3), we have neglected the damping constant $\gamma(k)$ of the Δ-resonance

$$\gamma = \gamma_0 k^3,$$

where γ_0 is determined from the observed width of the resonance

$$\gamma_R = \gamma_0 k_R^3 \approx 60\ \text{MeV} \approx 0{,}45.$$

This neglect is permissible for $k^2 < 15$. For large k^2, the quantity Π_R falls off sharply. The position of the resonance $\omega_R = 2.4$. The general factor in (3) has been chosen from the requirement that, on the mass shell ($\omega^2 = 1 + k^2$), the condition (cf., e.g.,[3])

$$\Pi_R \to -4\Pi n F_R,$$

be fulfilled, where F_R is the amplitude of the resonance scattering ($\pi^0 n$) at zero angle.

The quantity $\Phi(k, \omega)$ appearing in (4) is given below (see (7)). The unperturbed vertex in the first term in (2) is equal to $\Gamma = (\boldsymbol{\sigma}\cdot\mathbf{k}/2m)g$, with $g^2/4\pi = 14$. The denominator in (4) has arisen from the summation of the graphs of the spin-spin interaction of the nucleons in nuclear matter[2]. The spin-spin interaction is characterized by the quantity $g^-(k)$. For $k \ll p_F$, $g^-(k) \approx 1$. The expression (4) is a sum of neutron and proton terms. We have confined ourselves to the case N = Z ($p_F^{(n)} = p_F^{(p)}$). For Z = 0, the expression (4) must be halved.

B. The Meson Energy Spectrum. The Two Branches of the Spectrum

Collecting the expressions given above, we obtain the following equation for $\omega^2(k)$:

$$\omega^2(k) = 1 + \alpha k^2 - \beta k^2\frac{\Phi(k,\omega)}{1+g^-(k)\Phi(k,\omega)}, \quad (5)$$

where

$$\alpha = 1 - 0.6\frac{n}{n_0}\frac{2\omega_R}{\omega_R^2-\omega^2}, \quad \beta = 2.3\left(\frac{n}{n_0}\right)^{1/3}. \quad (5')$$

The quantity $g^-(k)$ has been found approximately by Osadchiev and Troitskiĭ[4] (in their notation, $g^-(k) = g^{nn} - g^{np}$):

$$g^-(k) \approx [1 - 0.5(k^2/p_F)](n/n_0)^{1/3}. \quad (6)$$

The last factor has arisen by virtue of the fact that $g^-(k)$, by definition, contains the factor $p_F \sim (n/n_0)^{1/3}$.

We give the expression for $\Phi(k, \omega)$[2]:

$$\Phi(k,\omega) = \frac{2\pi^2}{mp_F}\int\frac{d^3p}{(2\pi)^3}\frac{n(\mathbf{p})-n(\mathbf{p}-\mathbf{k})}{E(\mathbf{p}-\mathbf{k})-E(\mathbf{p})+\omega}. \quad (7)$$

Integration leads to a somewhat cumbersome expression. In the limiting cases, we find

$$\Phi(k,\omega) = 1 - \frac{\omega}{2kv_F}\ln\left|\frac{\omega+kv_F}{\omega-kv_F}\right| + \frac{i|\omega|\pi}{2kv_F}\theta(kv_F-\omega), \quad k \ll 2p_F; \quad (8)$$

$$\Phi\left(\frac{k}{2p_F}=x,\ \omega=0\right) = \frac{1}{2} + \frac{1}{4x}(1-x^2)\ln\left|\frac{1+x}{1-x}\right|,$$

$$\Phi\left(\frac{k}{2p_F}=x,\ \omega\right) = \frac{1}{3}\left(x^2 - \frac{\omega^2}{k^2v_F^2}\right)^{-1}, \quad x \gg 1.$$

Analysis of these expressions leads to the curves depicted in the Figure ($n = n_0$; the straight line $\omega^2 = k^2v_F^2$ is shown as a dashed line).

The existence of two branches of the spectrum for $k^2 < k_1^2$ is physically perfectly clear. It is natural to call the upper branch the meson branch—on decrease of the nucleon density, ω^2 goes over into its value for free mesons. The lower branch may be called spin sound. On switching off the interaction of the π mesons with the nucleons ($\beta \to 0$), this branch is obtained from the condition

$$1 + g^-(k)\Phi(k,\ \omega) \to 0,$$

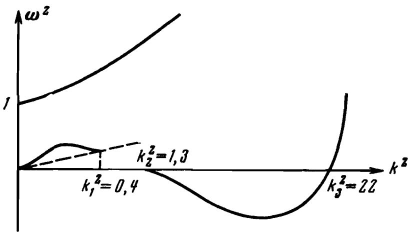
which corresponds to the pole of the shaded vertex of the first graph of (2). This condition is analogous to the well known equation for the frequency of "zero sound." There is a break in the spin-sound branch at the point k_1^2.

In the interval $k_2^2 < k^2 < k_3^2$, a solution with $\omega^2 < 0$ arises. Such a solution implies instability of the meson field in the medium: the meson field increases until interaction between the mesons come into play.

We have not taken into account meson absorption associated with the creation of more than one particle and hole. Such absorption does not make a very large contribution to $\omega^2(k)$, even for $\omega \sim 1$. For $\omega \ll 1$, the damping associated with the formation of two particles and two holes falls off $\sim (\omega/4)^2$.

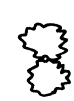
3. PHASE TRANSITION IN NUCLEAR MATTER

The meson-field instability noted above leads to the formation of a meson condensate and to a phase transition. From Eq. (5) for ω^2, we can find that the region of negative values of ω^2 first appears at densities $n = n_c \approx \frac{1}{2} n_0$.



In order to trace the appearance of the condensate, we shall study as our model a meson field with interaction $H' = \frac{1}{4}\lambda\varphi^4$, $0 < \lambda \ll 1$. Finding the energy density of the ground state of the system in the presence of the static field φ_0, we obtain

$$E^{(\pi)}(\varphi_0) = \sum_k (1 + k^2)\frac{\varphi_{0k}\varphi_{0,-k}}{2}$$

+  + $\frac{1}{4}$ 

$$= \sum_k \{1 + k^2 + \Pi(k, 0)\}\frac{\varphi_{0k}\varphi_{0,-k}}{2} + \frac{\lambda}{4V}\int \varphi_0^4 dV. \tag{9}$$

We choose for φ_0 the expression

$$\varphi_0 = a \sin k_0 x. \tag{10}$$

(It can be shown that more complicated expressions, e.g.,

$$\varphi_0 = a(\sin k_0 x + \sin k_0 y + \sin k_0 z),$$

give a higher ground-state energy.) Substituting (10) into (9) and finding the minimum, we obtain

$$a^2 = -\frac{4\omega_0^2}{3\lambda}\theta(n - n_c), \quad E^{\pi}_{min} = -\frac{\omega_0^4}{6\lambda}\theta(n - n_c), \tag{11}$$

where $\omega_0^2 = \omega^2(k_0^2) < 0$ is determined by the minimum of the expression in the curly brackets of formula (9), i.e., by the minimum of the right-hand side of (5) with $\omega = 0$. One can convince oneself that the appearance of the condensate makes the vacuum stable. In fact, to first order in λ, the correction to the meson polarization operator will be

$$\delta\Pi = 3\lambda\overline{\varphi_0^2} = {}^3/_2\lambda a^2 = -2\omega_0^2\theta(n - n_c), \quad k \neq 2k_0. \tag{12}$$

Since ω_0^2 is the minimum value of the right-hand side of (5) with $\omega = 0$, after addition of $\delta\Pi$, ω^2 nowhere goes to zero and, consequently, the vacuum becomes stable against the formation of the meson field.

In order to make a realistic estimate of the energy $E^{(\pi)}$, we can use the expressions for the meson-field energy given in the current algebra[5].

We note that φ_0 is not a classical quantity, but a parity-changing operator. The mean value $\langle\varphi_0\rangle$ can be expressed in terms of a quantity which changes sign on reflection. If the spin density in the ground state of the system is equal to zero, such a quantity cannot be constructed from anything, and $\langle\varphi_0\rangle = 0$. In any case, $\langle\varphi_0\rangle$ is defined in such a way that the term

$$H' = \frac{g}{2m}\int \psi^+\sigma\tau_\alpha\psi\nabla\varphi_{0\alpha}\,dV$$

does not violate the parity.

The φ_0^2 that we have found is now a classical quantity and the algebraic operations given above are valid.

It should be noted that yet another phase transition is logically possible. For density $n > 2n_0$, α becomes negative and the meson branch is lowered. If ω^2 then becomes a negative quantity greater in magnitude than ω_0^2, the meson condensate changes discontinuously—the wave vector k_0 will correspond to the minimum of the meson branch. Unfortunately, it is difficult to obtain an estimate proving the inevitability or absence of such a phase transition.

4. NON-PAIR MECHANISM OF NUCLEAR FORCES

Since mesons moving in nuclear matter differ markedly from free mesons, the forces between the nucleons in a nucleus cannot be obtained exactly from the pair interaction of nucleons in the vacuum.

If two nucleons exchange several mesons, then large meson 4-momenta ($k^2 \sim m^2$) play a role in the integration over the intermediate states, and the distortion of the meson branch in the nucleus has little effect. For such graphs, the vacuum interaction of the nucleons can be used. However, graphs with exchange of one or two mesons, which are responsible for the forces over greater distances, should be calculated with allowance for the distortions of the meson Green functions in the medium.

As an illustration, we shall compare the interaction which is given by the graph of one-meson exchange in the vacuum and in the medium. The vacuum interaction has the form

$$V(\mathbf{r}_1, \sigma_1; \mathbf{r}_2, \sigma_2) = -g^2\frac{(\sigma_1\nabla_1)(\sigma_2\nabla_2)}{4m^2}G(\mathbf{r}_1 - \mathbf{r}_2, \omega); \tag{13}$$

$G(\mathbf{r}, \omega)$ is the meson Green function in the mixed representation, and ω is the transferred energy. It is not difficult to verify that for $r \gtrsim 1$ the transferred energy can be neglected. In fact,

$$G(\mathbf{r}, \omega) = \int \frac{e^{i\mathbf{kr}}}{\omega^2 - (1 + k^2)}\frac{d^3k}{(2\pi)^3},$$

with

$$\omega = \frac{(\mathbf{p} + \mathbf{k})^2}{2m} - \frac{\mathbf{p}^2}{2m} \approx \frac{k^2}{2m}.$$

Since $r \gtrsim 1$ implies $k \lesssim 1$, we have $\omega^2 \ll 1 + k^2$. In the limit we have

$$G(\mathbf{r}, \omega) = -\frac{e^{-r}}{4\pi r}, \quad \omega \to 0.$$

The corrected interaction $\tilde{V}$ is obtained from (13) by replacing G by $\tilde{G}$, where

$$\tilde{G}_{\omega\to 0} = -\int \frac{e^{i\mathbf{kr}}}{1 + k^2 + \Pi_R(k, 0)}\frac{d^3k}{(2\pi)^3} \approx -2\frac{e^{-1,4r}}{4\pi r}.$$

In the denominator of the integrand, only the component Π_R of the meson polarization operator is present. The graphs corresponding to Π_P are obtained by repeated application of the interaction $\tilde{V}$. Thus, the interaction $\tilde{V}$ differs substantially from the vacuum interaction V.

It is clear that this component of the nuclear forces cannot be obtained from the usual approach. To take these phenomena into account, it is possible to take the following approach to the "microscopic theory of the nucleus," i.e., to the determination of nuclear properties from the interaction of nucleons and mesons in the vacuum. All graphs associated with the exchange of several particles (large momenta—short distances) are found in the gas approximation, whereas the interaction at distances $r \gtrsim 1$ should be determined with the inclusion of exchange of distorted mesons.

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