# *PHENOMENOLOGICAL THEORY OF DEPOLARIZATION OF $\mu^+$ MESONS IN MAGNETIC MEDIA*

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Submitted May 31, 1972

Zh. Eksp. Teor. Fiz. **63**, 1873–1882 (November, 1972)

An equation set for depolarization of $\mu^+$ mesons in magnetic semicnoductors is formulated. It is shown that the presence of polarization of conductivity electrons results in the appearance of $\mu^+$-polarization along the direction of polarization of the medium electrons and also leads to an additional shift of the muon precession frequency in a magnetic field about the direction of the conductivity electron polarization and to a displacement of the precession stopping point. Polarization of captured electrons leads, in particular to a significant phase shift of the complex residual polarization. It is thus shown that $\mu^+$ mesons can be used effectively to study magnetic semiconductors. The results also show that the state of $\mu^+$ mesons in ferromagnetic metals (atomic or ionized) can be determined.

**1.** The Nosov-Yakovleva (NY) equations[1] were derived under the assumption that the polarization of the ensemble of electrons in a medium is equal to zero. This pertains both to electrons that captured by a meson and those which are subsequently scattered by the muonium. Perelomov[2] succeeded in formulating an equation for a magnetic medium in a particular case, namely when the magnetic field is longitudinal relative to the initial muon polarization and the polarization of the electrons of the medium, and assuming total polarization of the electrons of the medium.

Our present task is: a) to formulate in convenient form (in the NY operator formalism) equations for arbitrary polarization of the medium electrons and for arbitrary direction of the magnetic field, and b) to show that in fact the NY equations (as well as the Perelomov equations) do not presuppose the usual limitations (under which the Wangsness-Bloch equations are obtained) if the relaxing action of the medium is determined only by the scattering of the electrons by the muonium.

In fact, we consider in the present paper a situation wherein the muon is stopped in a magnetic semiconductor. Recently, many such magnetic substances were observed, in which doping produces an appreciable concentration of conduction electrons (up to $10^{19}$ cm$^{-3}$, for example, in europium chalcogenides doped with rare earths[3]). We assume that the ferromagnet in which the muon is stopped has a single-domain structure. Additional effects connected with the domain distribution will not be considered here.

We assume furthermore that the temperature of the medium is infinite, i.e., that kT is much larger than the level splitting of the ground state of muonium (but, of course, much less than the ionization potential).

**2.** In the most general case, the polarization matrix describing the spin state of the two electrons is given by[4]

$$\rho = {}^1/_4(1 + \mathbf{P}^1\sigma_1 + \mathbf{P}^2\sigma_2 + \Sigma Q_{ij}\sigma_{1i}\sigma_{2j}). \quad (1)$$

Here $\mathbf{P}^1$ and $\mathbf{P}^2$ are the polarization vectors of the incident and atomic electrons, $P_{ij}$ is the spin-correlation tensor. Inasmuch as in the cases considered by us the interaction between the electrons of the medium and the atomic electron takes place only during the time of the collision act, the correlation tensor $Q_{ij}$ is equal to zero. In this case, according to[5], the polarization of the atomic electron is given by

$$\mathbf{P}^{2\prime}(\theta) = \alpha(\theta)\mathbf{P}^1 + \beta(\theta)\mathbf{P}^2, \quad (2)$$

where $\alpha(\theta)$ and $\beta(\theta)$ are expressed in terms of the singlet and triplet amplitude of scattering to the angle $\theta$. If we average over all the scattering directions, we obtain

$$\overline{\mathbf{P}^{2\prime}} = \bar{\alpha}\mathbf{P}^1 + \bar{\beta}\mathbf{P}^2. \quad (3)$$

Thus, it can be assumed for simplicity that in exchange scattering there is a certain probability that in part of the ensemble the old polarization of the atomic electron is replaced by a new one, and a certain depolarization takes place in addition. In so far as the practical calculation of the coefficients $\bar{\alpha}$ and $\bar{\beta}$ for the medium is a very complicated matter, this does not decrease our ability of extracting information concerning the electrons of the medium.

**3.** The equations for the density matrix can be formally derived by using first the basis of muonium wave functions with a definite direction of the spin projection on a certain axis, and taking into account the following conditions: the normalization of the density matrix before and after the collision, the equality of the muon polarizations before and after collisions, change of the polarization of the electrons of part of the ensemble into polarization of the medium electron and partial depolarization of the atomic electron as a result of the scattering. It remains then to make a formal transition to operator variables and to use the NY equations for an arbitrary direction of the magnetic field (introducing at the same time a correction to the relaxation term). The derivation leads to rather obvious equations, so that the details are relegated to the Appendix. We note only that the coefficient 2 in front of the "spin-flip" frequency $\nu$, which appears first in the paper of Nasov and Yakovleva[1], is wrong; moreover, one cannot speak in general of a partial restoration of the polarization of the atomic electron after a second scattering by another electron of the medium, if the medium electrons are not polarized. On the other hand, the NY equations turn out to be perfectly correct (accurate to a renormalization of the parameter $\nu$).

**4.** The system of equations for the polarization density matrix $\rho$ consists of three parts: three equations for the time variation of the muon polarization, three equations for the time variation of the electron polarization of the atomic muonium, and nine equations for the time variation of the correlations of muon and electron polarizations. The first three equations are not altered at all, since they contain the medium only via $\omega_0$:

$$\frac{d\rho_{\varkappa 0}}{dt} = -\frac{\omega_0}{2} e_{\varkappa\lambda l}\rho_{\lambda l}(t) - \zeta e_{\varkappa\mu\lambda}\omega_\mu{}'\rho_{\lambda 0}(t). \tag{4}$$

Here $|\boldsymbol{\omega}'| = eH/m_e c$, where H is the resultant magnetic field at the point where the muonium is located, and $\zeta = |\mu_{\mu^+}/\mu_e|$ is the ratio of the moduli of the magnetic moments of the muonium and the electron.

In the NY equations, the density matrix components corresponding to electron polarization in muonium in a nonmagnetic medium contain a term $-2\nu\rho_{0k}$. Retaining the notation, we retain also the coefficient 2, but now $\rho_{0k}$ is replaced by the difference $\rho_{0k} - f\mathscr{P}_k/\nu$, where $\mathscr{P}_k$ is the medium-electron polarization component. In other words, there is a probability 2f that the polarization of the muonium electron will turn into the polarization of the medium electron. As seen from (3), all that actually happens in each scattering act is a rotation of the muonium-electron polarization towards the medium-electron polarization, but this does not matter, since we are dealing with an ensemble and writing the equations for the polarization directly. The difference between $\nu$ and f corresponds to partial depolarization of the muonium electron in exchange scattering. Thus, for a magnetic medium the indicated three equations take the form

$$\frac{d\rho_{0k}}{dt} = \frac{\omega_0}{2} e_{k\lambda l}\rho_{\lambda l}(t) + e_{kml}\omega_m{}'\rho_{0l}(t) - 2\nu\left[\rho_{0k}(t) - \frac{f}{\nu}\mathscr{P}_k\right]. \tag{5}$$

The equations for the polarization correlations $\rho_{\kappa k}$ contained previously the relaxation term $-2\nu\rho_{\kappa k}$. Following exchange scattering in the magnetic medium, however, a correlation will set in even if it did not exist before. Obviously, this correlation is proportional to the product of the muon-polarization component that exists at the scattering instant by the medium-electron polarization component, i.e., the correlation term is replaced by

$$-2\nu\left(\rho_{\varkappa k}(t) - \frac{f}{\nu}\rho_{\varkappa 0}(t)\mathscr{P}_k\right).$$

Thus, the equation for the density-matrix components corresponding to the correlations of the polarization of the magnetic medium take the form

$$\frac{d\rho_{\varkappa k}}{dt} = \frac{\omega_0}{2}[e_{\varkappa k\lambda}\rho_{\lambda 0}(t) - e_{\varkappa kl}\rho_{0l}(t) - \zeta e_{\varkappa\mu\lambda}\omega_\mu{}'\rho_{\lambda k} + e_{kml}\omega_m{}'\rho_{\varkappa l}(t)] \\ - 2\nu\left[\rho_{\varkappa k}(t) - \frac{f}{\nu}\rho_{\varkappa 0}(t)\mathscr{P}_k\right]. \tag{6}$$

Although these equations are obvious, it is tacitly assumed in the present derivation that $\rho_{\kappa k}$ corresponds exactly to the polarization correlations. This makes it all the more desirable to obtain a rigorous derivation of Eqs. (4)—(6); this will be done in the Appendix.

**5.** Now a few words concerning the initial conditions. Nosov and Yakovleva used as the initial conditions the vanishing the density-matrix components corresponding to the electron polarization and to the polarization components, and also equality of the polarization of the muon in the muonium to its initial polarization when the muon is produced in the $\pi$ decay. The situation now is somewhat more complicated, since nonzero conditions can also arise for the components $\rho_{0k}$ and $\rho_{\kappa k}$. When the muon slows down it captures those electrons whose relative velocity with respect to the muon is of the order of the Bohr velocity. Consequently, electrons from different shells will be captured. However, the muonium will have a very large kinetic energy and will therefore easily lose those electrons which it acquired from other shells. It is therefore difficult to predict reliably the polarization of those muonium electrons that remain the muonium after the slowing down. In any case this polarization can differ from the polarization of the conduction electrons that experience exchange scattering by the muonium.

The polarization of the electrons stopped in the muonium at the start of the depolarization process (i.e., after the slowing down of the muonium) will be designated S. Then the initial conditions for the polarization of the electrons in the muonium are

$$\rho_{0k}(0) = S_k. \tag{7}$$

The correlation of the polarizations at the initial instant of time can be connected only with the product of the initial polarizations of the electron and muon, since the hyperfine interaction has "not yet been turned on." Thus, for $\rho_{\kappa k}$ we have the initial conditions

$$\rho_{\varkappa k}(0) = P_\varkappa(0)S_k, \tag{8}$$

where $\mathbf{P}(0)$ is the polarization of the muon at the instant of its production. For $\rho_{\kappa 0}$ there remains the old initial conditions:

$$\rho_{\varkappa 0}(0) = P_\varkappa(0). \tag{9}$$

**6.** To understand the structure of the equations, it is very useful to write down the system following the Laplace transformation. For concreteness, we direct the magnetic field in the medium along the axis 1. If the direction of the magnetic field does not coincide with the direction of the conduction-electron magnetization, then the system (4)—(6) cannot be divided into subsystems and it is necessary to consider a system of 15 equations. In the present paper we confine ourselves to the following situations: the magnetic field is perpendicular to the direction of the initial polarization of the muon and is so strong that the magnetization of the conduction electrons coincides with the direction of the external field. In addition, we assume that the magnetization of the electrons that can be captured when the muon and the muonium slow down is directed along the field.

Following the preceding paper[6], we use complex variables that are plane vectors (in the plane 2, 3):

$$\rho_\mu = \rho_{30} + i\rho_{20}, \quad \rho_e = \rho_{03} + i\rho_{02};$$
$$\mathrm{G} = \rho_{13} + i\rho_{12}, \quad \mathrm{E} = \rho_{31} + i\rho_{21}. \tag{10}$$

In the situation indicated, the system breaks up into eight, six, and one equation. After the transformation

(10), the system of eight equations goes over into a system of four equations for the complex combinations. These last four equations are

$$
\begin{aligned}
\frac{d\rho_\mu}{dt} &= i\zeta\frac{eH}{m_e c}\rho_\mu + \frac{i\omega_0}{2}\mathbf{G} - \frac{i\omega_0}{2}\mathbf{E}, \\
\frac{d\mathbf{P}_e}{dt} &= -i\frac{eH}{m_e c}\mathbf{P}_e - \frac{i\omega_0}{2}\mathbf{G} + \frac{i\omega_0}{2}\mathbf{E} - 2\nu\mathbf{P}_e, \\
\frac{d\mathbf{G}}{dt} &= \frac{i\omega_0}{2}\rho_\mu - \frac{i\omega_0}{2}\mathbf{P}_e - \frac{ieH}{m_e c}\mathbf{G} - 2\nu\mathbf{G}, \\
\frac{d\mathbf{E}}{dt} &= -\frac{i\omega_0}{2}\rho_\mu + 2f\mathscr{P}_1\mathbf{P}_\mu + \frac{i\omega_0}{2}\mathbf{P}_e + \frac{i\zeta eH}{m_e c}\mathbf{E} - 2\nu\mathbf{E}.
\end{aligned}
\tag{11}
$$

Here and below, $\mathscr{P}_i$ are the projections of the polarization vector of the electrons of the medium.

The system of six equations has a right-hand side corresponding to a gradual onset of polarization in the direction of the medium-electron polarization, i.e., in our case, in the direction of the field. In other words, the medium operates like a thermostat. This system of equations is given by

$$
\begin{aligned}
\frac{d\rho_{10}}{dt} &= -\frac{\omega_0}{2}(\rho_{23} - \rho_{32}), \\
\frac{d\rho_{01}}{dt} &= \frac{\omega_0}{2}(\rho_{23} - \rho_{32}) - 2\nu\rho_{01} + 2f\mathscr{P}_1, \\
\frac{d\rho_{23}}{dt} &= \frac{\omega_0}{2}(\rho_{10} - \rho_{01}) - 2\nu\rho_{23} + \frac{eH}{m_e c}\rho_{22} + \frac{\zeta eH}{m_e c}\rho_{33}, \\
\frac{d\rho_{32}}{dt} &= -\frac{\omega_0}{2}(\rho_{10} - \rho_{01}) - 2\nu\rho_{32} - \frac{\zeta eH}{m_e c}\rho_{22} - \frac{eH}{m_e c}\rho_{33}, \\
\frac{d\rho_{22}}{dt} &= -H\rho_{33} + \zeta H\rho_{32} - 2\nu\rho_{22}, \quad \frac{d\rho_{33}}{dt} = -\zeta H\rho_{23} + H\rho_{32} - 2\nu\rho_{33}.
\end{aligned}
\tag{12}
$$

The initial conditions for the systems (11) and (12) are

$$\rho_{01} = S_1, \quad \rho_{31} = S_1, \quad S_{30} = 1 \ \text{ at } \ t = 0;$$

The remaining components of the density matrix at the initial instant of time are equal to zero.

7. Although we are considering a single-domain sample, the equations for the density matrix are suitable in fact for a much larger class of cases. In order for them to be valid it suffices that the muonium remain within a single domain during the time of its deceleration, when intense charge exchange not accompanied by depolarization takes place. To obtain the entire picture of the observed process of the change of polarization, it is sufficient to average the solutions of (4)–(6) over many domains.

8. Returning to our situation, we solve first the system of equations (12). The solution reduces to a solution of a system of algebraic equations obtained following the Laplace transformation. For the Laplace transform of the polarization component in the direction of the axis 1 we obtain

$$P_1(\lambda) = \frac{(\lambda S_1 + 2f\mathscr{P}_1)\omega_0^2}{2\lambda\{\lambda^3 + 4\nu\lambda^2 + \lambda[4\nu^2 + \omega_0^2(1 + x_+^2) + \omega_0^2\nu]\}}. \tag{13}$$

We have used here the standard notation

$$x_+ = (1 + \zeta)x, \quad x = eH / m_e c\omega_0.$$

From (13) we obtain for the polarization component along the direction 1 (the direction of the medium-electron polarization)

$$P_{\infty 1} = \frac{1}{\tau}P_1\left(\frac{1}{\tau}\right) = \frac{1}{2}(\omega_0\tau)^2\frac{(S_1 + 2f\tau\mathscr{P}_1)}{(1 + 2\nu\tau)^2 + (\omega_0\tau)^2(1 + 2\nu\tau + x_+^2)}. \tag{14}$$

In the analysis of the time dependence of the polarization, we confine ourselves to cases when only changes occurring within times much longer than $1/\omega_0$ can be observed. Taking the inverse Laplace transform of (13) and taking into account only the terms indicated above[1], we obtain

$$\rho_{10} = \frac{f}{\nu}\mathscr{P}_1 + \left(\frac{S_1}{2y} - \frac{f}{\nu}\mathscr{P}_1\right)\exp\left\{-\frac{\nu t}{y}\right\}. \tag{15}$$

Here $y = 1 + x_+^2 + 4\nu^2/\omega_0^2$.

Proceeding from (15) to a real case, when the muonium either loses an electron to the conduction band or enters in a chemical reaction, we can easily obtain for the time dependence of the polarization

$$P_1(t) = \frac{1}{\tau}\int_0 \rho_{10}(t')e^{-t'/\tau}dt' + e^{-t/\tau}\rho_{10}(t), \tag{16}$$

or in explicit form

$$P_1(t) = \frac{f}{\nu}\mathscr{P}_1 + \frac{{}^1\!/_2 S_1 - f\mathscr{P}_1 y/\nu}{y + \nu\tau}\left[1 - \exp\left\{-t\left(\frac{1}{\tau} + \frac{\nu}{y}\right)\right\}\right]. \tag{17}$$

We consider now other polarization components. Solving the system (11), we obtain for the Laplace transform

$$\rho_{\mu+}(\mu) = Q / D, \tag{18}$$

where

$$
\begin{aligned}
\mu &= \lambda - i\zeta eH / m_e c, \\
Q(\lambda) &= [(\mu + 2\nu)(\mu + 2\nu + ix_+\omega_0)^2 + {}^1\!/_4\omega_0^2(2\mu + 4\nu + ix_+\omega_0) \\
&\quad - {}^1\!/_2 i\omega_0 S_1(\mu + 2\nu + ix_+\omega_0)^2]P_{\mu 3}(0), \\
D(\mu) &= \mu(\mu + 2\nu + ix_+\omega_0)^2(\mu + 2\nu) \\
&\quad + {}^1\!/_4\omega_0^2(2\mu + 2\nu + ix_+\omega_0)(2\mu + 4\nu + i\omega_0 x_+) \\
&\quad + if\mathscr{P}_1\omega_0(2\mu + 2\nu + ix_+\omega_0)(\mu + 2\nu + ix_+\omega_0).
\end{aligned}
$$

For the residual polarization, as always, we have

$$
\begin{aligned}
P_{\perp\infty} &= \frac{1}{\tau}\rho_{\mu+}\left(\frac{1}{\tau}\right) = [q(q + \delta)^2 + {}^1\!/_4 b^2(2q + \delta) - iS_1 b(q + \delta)^2/2] \\
&\times [q(q + \delta)^2 + {}^1\!/_4 b^2(1 + \delta + q)(2q + \delta) + i\mathscr{P}_1(f\tau)b(1 + q + \delta)(q + \delta)]^{-1}.
\end{aligned}
\tag{19}
$$

Here $q = 1 + 2\nu\tau$, $\delta = ix_+\omega_0\tau$, $b = \tau\omega_0$.

Solving the equations $D = 0$ for the case of "large exchanges" ($\nu \gg \omega_0$) we obtain for the value of the small root

$$\mu = -\frac{\omega_0^2}{4\nu} - \frac{if}{2\nu}\mathscr{P}_1\omega_0. \tag{20}$$

Thus, we have found that scattering by polarized electrons of the medium produces, besides the trivial damping, also a shift in the precession frequency (we note that in accordance with (18) a quantity equal to the precession frequency of the free muon has already been subtracted from $\mu$). The physical meaning of this frequency shift is extremely simple, namely, owing to the frequent scattering, the muon is located at all times in the magnetic field of an electron, and this field is perpendicular to the muon polarization while the electron spin, the direction of which determines the field direction, does not have time to flip during the time between the scattering acts. We note that this frequency shift remains also when the external field is turned off. Since f and $\nu$ are of the same order and the frequency

[1]To take the large roots ($\sim\omega_0$) into account it is necessary to solve a cubic equation or to use a computer; at the present time, the experiment still does not require the study of such strong damping.

of the hyperfine splitting is very large, one can observe a very small polarization $\mathscr{P}_1$ of the conduction electrons. Unfortunately, the polarization $\mathscr{P}_1$ always enters multiplied by f, so that measurements yield only the product $f\mathscr{P}_1$. In other words, one can determine only the order of magnitude of $\mathscr{P}_1$, but with very high sensitivity. We note in this connection that if muonium (and consequently also hydrogen) were to be present in the metal in a bound state, then this would inevitably lead to large observable precession-frequency shifts in ferromagnets. On the other hand, the absence of large shifts should indicate that the muon (and the proton) in the metal is in the free state.

Finally, notice should be taken of one more feature in the behavior of the polarization of $\mu^+$ mesons in magnetic semiconductors. As seen from (19), the polarization of the captured electrons in a direction transverse to the initial polarization of the $\mu^+$ mesons leads to the appearance of a phase shift in the residual polarization, provided only that $\omega_0\tau \sim 1$. (As is well known, the tangent of the phase angle is defined as the ratio of the imaginary and real parts of the complex residual polarization, and the phase itself corresponds to the initial angle with which the precession with muon frequency begins.)

In the case of ultrastrong fields, it can be easily seen from the equation D = 0 that the condition for the stopping of the precession is now

$$x_+ = 1/2\zeta + f\mathscr{P}_1/\omega_0\zeta.$$

The second term in the last expression corresponds to an additional shift of the points where the precession stops; we note that this shift depends only on $f\mathscr{P}_1$ but not on $\nu$.

In conclusion, I am deeply grateful to Yu. G. Abov, B. T. Geĭlikman, V. G. Nosov, O. B. Firsov, and V. A. Khodel' for discussions.

## APPENDIX

### FORMAL DERIVATION OF EQUATIONS FOR THE POLARIZATION DENSITY MATRIX OF MUONIUM IN A MAGNETIC MEDIUM

We consider an auxiliary problem, the case when the medium electrons are described by a wave function. Let the spin wave function of the electron by

$$\psi_{em} = k\psi_\uparrow + f\psi_\downarrow, \quad kk^* + ff^* = 1. \tag{A.1}$$

The arrows denote here the proper wave functions with respect to the quantization axis. Assume, further, that at a certain instant of time the spin states of the atomic muonium are described by the wave function

$$\psi_{\text{Mu}}^{(a)} = a\psi_{\uparrow\uparrow} + c\psi_{\downarrow\uparrow} + d\psi_{\uparrow\downarrow} + b\psi_{\downarrow\downarrow} \tag{A.2}$$

(the first arrow pertains to the muon and the second to the electron). In (A.2), the wave function of the muonium has been expanded in terms of the direct product of the proper wave functions of the muon and electron in the muonium. The coefficients a, b, c, and d satisfy the normalization condition

$$|a|^2 + |b|^2 + |c|^2 + |d|^2 = 1. \tag{A.3}$$

We consider first the model corresponding to a simple exchange of polarizations between the medium electron and the atomic electron. This does not correspond to reality, but the result will not change in the subsequent transition to the density matrix, if the result is taken to mean the form of the equations rather than the connection between the phenomenological and microscopic parameters.

Following the scattering act, the spin wave function of the muonium will take the same form (A.2), but now with other coefficients:

$$\psi_{\text{Mu}}^{(f)} = A\psi_{\uparrow\uparrow} + C\psi_{\downarrow\uparrow} + D\psi_{\uparrow\downarrow} + B\psi_{\downarrow\downarrow}. \tag{A.4}$$

We have seven equations with which to determine the new coefficients A, C, D, and B. There is no eighth equation, since these four coefficients can have a common phase factor which does not affect the major quantities.

1) The normalization equation

$$|A|^2 + |C|^2 + |D|^2 + |B|^2 = 1. \tag{A.5}$$

2) The equality of the polarization of the electron in muonium to the value obtained as a result of the scattering act (in our model, simply the equality of the medium-electron polarization prior to the scattering act). In explicit form, this condition yields for the z-projection

$$|A|^2 - |D|^2 + |C|^2 - |B|^2 = |k|^2 - |f|^2, \tag{A.6}$$

and for the x- and y-projections

$$A^*D + C^*B = k^*f. \tag{A.7}$$

3) The polarization of the muon should not be altered by the scattering act; in explicit form, this condition yields for the z-projection

$$|a|^2 + |d|^2 - |c|^2 - |b|^2 = |A|^2 + |D|^2 - |C|^2 - |B|^2, \tag{A.8}$$

and for the x- and y-projections

$$a^*c + d^*b = A^*C + D^*B. \tag{A.9}$$

The system of equations (A.5)—(A.9) is nonlinear. It can be easily reduced to a linear form, however, if it is noted that formally it does not contain the interaction between the muon and the electron at all. We therefore seek solutions satisfying the conditions

$$A/D = k/f = C/B. \tag{A.10}$$

We verify first that these conditions do not contradict (A.5) and (A.7). If the condition (A.1) for the normalization of the wave function of the medium electron are satisfied, these equations reduce to the following:

$$|A|^2 + |C|^2 = |k|^2. \tag{A.11}$$

Thus, it remains to solve the system (A.8)—(A.11). For the moduli of the coefficients A, B, C, and D we get

$$\begin{aligned} |A| &= |k|\sqrt{(1+R)/2}, \quad |C| = |k|\sqrt{(1-R)/2}, \\ |D| &= |f|\sqrt{(1+R)/2}, \quad |B| = |f|\sqrt{(1-R)/2}, \end{aligned} \tag{A.12}$$

where $R \equiv |a|^2 + |d|^2 - |c|^2 - |b|^2$ corresponds to the projection of the muon polarization prior to the scattering act on the direction of the z axis (the quantization axis). For the phase differences we obtain

$$e^{i(\varphi_C-\varphi_A)} = 2Q/\sqrt{1-R^2}, \quad e^{i(\varphi_B-\varphi_A)} = 2Q|k|f/k|f|\sqrt{1-R^2}. \tag{A.13}$$
$$e^{i(\varphi_D-\varphi_A)} = f|k|/|f|k.$$

Here $Q \equiv a^*c + d^*b$ corresponds to complex polarization of the muon (prior to the scattering act) on the xy plane.

If we now introduce the muon density matrices $\rho$ and $\tilde{\rho}$, constructed on the basis of the wave functions, with definite muon and electron spin projections: $\rho_{nm} = \overline{n^*m}$ and $\tilde{\rho}_{NM} = \overline{N^*M}$ (the average is taken over the ensemble), and also introduce the medium-electron density matrix $T_{lq} = \overline{l^*q}$ (in our model; more rigorously it would be necessary to take the density matrix of the electrons in the muonium after the scattering act), then we obtain from (A.12) and (A.13) the relations between the density matrices before and after the scattering.

Before we proceed to the operator representation, it is useful to call attention to the following circumstance. According to quantum mechanics, a particle can have a definite spin projection only on a single axis. But this does not mean that the choice of the quantization axis determines the polarization direction; one can choose as the quantization axis, for example, the x axis, and the particle can be polarized along the y axis. The electrons of the medium are not polarized. This can mean, for example, that each electron of the ensemble has a certain direction, along which its spin has a definite value, but when the entire ensemble is considered the picture becomes isotropic, and therefore even single scattering with change of the spin direction of the atomic electron and electron of the medium leads to depolarization. Therefore the idea that the second scattering act restores the polarization partially (as assumed by Nosov and Yakovleva[1]) is in error. The picture can be even more complicated: the electron can be in such a state that its spin states are not determined by a wave function at all; then depolarization occurs even without averaging over the ensemble. However, to understand the muon depolarization picture this is no longer very important, since an ensemble of muons is observed all the same.

As shown convincingly by Nosov and Yakovleva, it is more convenient to use for muonium the operator representation of the density matrix, namely, following Fano's ideas[7], Yakovleva[8] proposed to use the following operator representation:

$$\rho = {}^1\!/_2 \rho_{\varkappa k} U_\varkappa U_k, \tag{A.14}$$

where $U_\varkappa$ and $U_k$ are two-row matrix operators corresponding to muon and the electron, namely, the zeroth components are unit operators $U_0 = \chi/\sqrt{2}$ and the vector components are $\mathbf{U} = \boldsymbol{\sigma}/\sqrt{2}$. Formula (A.14) was corrected as against those in[1,7] by adding the coefficient 1/2. This coefficient is necessary to make the maximum polarization equal to unity.

Expanding in (A.14), we can obtain relations between the density-matrix components in the operator representation and in the representation corresponding to a basis of wave functions having definite muon and electron spin projections, and also formulas for the inverse transformation. Using these formulas, as well as the NY equations for the polarization density matrix of muonium in an arbitrary magnetic field, we obtain (5) and (6). In writing down the formulas, we already took into account the fact that what actually occurs is not exchange of polarization of the atomic electron and the medium electron, but the more complicated process described by formula (3).

At first glance we are faced with a paradox consisting in the following. We found that the polarization correlations that occur after scattering are expressed directly in terms of the product of the corresponding components of the muon and medium-electron polarizations prior to scattering. On the one hand this is understandable, since the medium electrons are independent of the muon and the only correlation that they can bring about is precisely the one found in the polarization of the muon and the electron. On the other hand, one can visualize a muonium density matrix corresponding to zero polarizations (at a given instant of time) of the muon and the electron, but with correlation of the polarizations still present. But then, were we to replace the atomic electron by a medium electron whose state differs little from the state of the atomic electron, the correlations would clearly remain. The reason is that we have obtained on the basis of (A.13) expressions for ten quantities, whereas we had only seven equations. We could obtain these ten quantities only because we have actually assumed that each muonium atom, taken separately, exists at a given instant of time in some pure state, i.e., it can be described by a wave function. Since collisions with electrons of the medium are quite rare (the interaction with the medium is actually small), the muonium density matrix is obtained by averaging over all the particles of the ensemble. Each of the particles of this ensemble is then described by a wave function. The meaning of smallness of interaction consists therefore in the fact that the scattering time multiplied by the number of scattering acts is much smaller than unity; in other words, the muon becomes depolarized at those instants of time when the muonium can be described by a wave function. We note that in addition to the fact that such an approach to concrete situations is realistic, any other description would call for concrete information concerning the interaction between the muonium and the electrons of the medium.

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Translated by J. G. Adashko
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