PHONON COMPONENT OF DYNAMIC DRAGGING OF DISLOCATIONS

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A unified analysis of phonon dissipation mechanisms is carried out which permits one to single out those processes which limit dynamic dragging of dislocations. By linear response theory the problem is reduced to calculation of the retarded two-particle Green's function. The short wave asymptotic of the function is found in the τ approximation. The Green's function expansion in the long-wave limit contains singular ladder diagrams whose summation is equivalent to solution of the phonon kinetic equation. It is shown that most of energy dissipation occurs near the dislocation within a range corresponding to the phonon mean free path ("phonon wind"). Only at high temperatures the "phonon wind" region contracts to such an extent that relaxation processes become predominant; however, these processes cannot be reduced to ordinary "phonon viscosity."

1. INTRODUCTION

ONE of the urgent problems in physics of real crystals is the identification of the principal mechanisms of dynamic dislocation dragging[1,2]. It has been reliably established by now that in real crystals there exist dynamic-mobility regions, in which the dislocation velocity is limited not by lattice defects but by outflow of energy to different branches of elementary excitations of the crystal. The dynamic energy losses also turned out to be significant for internal-friction processes connected with motion of dislocation between barriers, and for the overcoming of the barriers in the region of the thermal-fluctuation mobility of the dislocations, where the average dislocation velocity is limited by the waiting time in front of the barrier.

A decisive role in the dynamic dragging of dislocations is usually played by dissipative processes in the crystal phonon substances.[1)] It is usually assumed that the most important are two types of mechanisms, which are connected with the introduction of the concept of "phonon viscosity" and "phonon wind." We shall show that these concepts are restricted by the spatial dispersion of the dissipative processes in the phonon gas around the dislocations, so that it is necessary to review the theoretical estimates of the phonon dragging.

The deformation field of a straight-line dislocation that moves uniformly with velocity $v \ll c$ (c is the speed of sound) can be represented by means of the Fourier-integral expansion as a superposition of plane waves:

$$\varepsilon_{ij}(\mathbf{r}, t) = \varepsilon_{ij}(\mathbf{r} - \mathbf{v}t) = \int \frac{d\mathbf{q}}{(2\pi)^3} \varepsilon_{ij}{}^{\mathbf{q}} \exp[i(\mathbf{qr} - \Omega_{\mathbf{q}} t)], \qquad (1)$$

where $\epsilon_{ij}^{\mathbf{q}}$ is the Fourier transform of the static field of the dislocation, $\Omega_{\mathbf{q}} = \mathbf{q} \cdot \mathbf{v}$ and the integration is cut off on the upper limit at $q_m \sim r_0^{-1} \sim k_D$ (r_0 is the radius of the dislocation nucleus and k_D is the Debye boundary in the phonon spectrum). Expressed in these terms, the dissipation of the moving-dislocation energy reduces to a damping of plane elastic waves from the packet (1).

In his well-known series of papers[5,6], Mason proposed to calculate this damping in analogy with the theory developed by Akhiezer[7] for ultrasound absorption, using the concept of phonon-gas viscosity. However, Mason's estimate is incorrect, since the notion of a "phonon viscosity" that has no spatial dispersion is meaningful when applied to the damping of the dislocation packet (1) only for the long-wave part of the partial waves that have the adiabaticity property[8][2)] $\Omega_{\mathbf{q}} \gg \chi q^2$ ($\chi \approx cl$ is the temperature conductivity coefficient, $l = c\tau$ is the phonon mean free path, and τ is the phonon-gas relaxation time), i.e., for waves with wave vector $q \ll v/lc$. It is easy to verify, however, that the contribution to the energy dissipation of the adiabatic part of the packet (1) is negligibly small, and can be estimated by multiplying Mason's formula by the small parameter[3)] $(vr_0/cl)^2$.

A fundamentally different approach to the problem is used in the calculation of the so-called "phonon wind"[11-13], or elastic scattering of phonons by a moving dislocation, to which the transfer of dragging momentum to the dislocation is attributed. In this calculation, however, the phonon-phonon interaction was neglected, and this is valid, strictly speaking, only for the short-wave part of the packet (1), for which $\Omega_{\mathbf{q}}\tau \gg 1$, i.e., $q \gg c/lv$. To be sure, the situation in this case is not as hopeless as in the calculation of the "phonon viscosity," since the result is determined by large values of q, and one can count on such an estimate of the phonon dragging of the dislocation to be correct at sufficiently high velocities and low temperatures, when $v \gg c/k_D l$.

Nevertheless, there have been no investigations of the contribution made to the dissipation by the main part of the packet (1), for which $v/lc \lesssim q \lesssim c/lv$, let alone of the case of velocities that are not too high, $v \lesssim c/k_D l$, when $\Omega_{\mathbf{q}}\tau \lesssim 1$ for all the waves in the superposition (1) and allowance for phonon relaxation

1)In metals at low temperatures, when the phonon gas is frozen out, it is necessary to take into account the interaction between the moving dislocations and the conduction electrons [3,4].

2)For details see Sec. 3 below.

3)The same considerations pertain also to another relaxation mechanism of dislocation dragging, the so-called "thermoelastic losses" [9,10].

is essential. It is therefore relevant to consider from a unified point of view the damping of the entire packet of plane waves (1) with allowance for the phonon-phonon interaction. The present paper is devoted to the solution of this problem.

2. FORMULATION OF PROBLEM

We consider a straight-line dislocation with Burgers vector **b**, moving uniformly in a crystal with velocity v. We assume that velocity v is high enough to assume above-the-barrier dislocation motion, but still lower than the velocity of sound, so that "relativistic" effects can be neglected.

The behavior of the phonon subsystem of the crystal in the field of the moving dislocation can be described by the Hamiltonian

$$H = H_0 + H_A + V(t) \equiv \mathscr{H} + V(t), \qquad (2)$$

where H_0 and H_A are respectively the harmonic and anharmonic terms of the phonon Hamiltonian $\mathscr{H}$, and V(t) is the Hamiltonian of the interaction of the phonons with the dislocation. In the continual approximation, V(t) takes the form

$$V(t) = \int \frac{d\mathbf{q}}{(2\pi)^3} \sum_{\alpha\beta} \Gamma_{\alpha\beta}{}^{\mathbf{q}} \xi_\alpha \xi_\beta \exp(i\Omega_{\mathbf{q}} t). \qquad (3)$$

The Greek indices α and β (as well as the Greek letters γ, δ, ρ, which will be used later on) denote phonon states specified by the aggregate of the wave vector k and the polarization λ: $\alpha = (\mathbf{k}_1, \lambda_1)$, $\beta = (\mathbf{k}_2, \lambda_2)$, $\xi_\alpha = a_\alpha + a^+_{\bar{\alpha}}$, $\bar{\alpha} = (-\mathbf{k}_1, \lambda_1)$, a^+_α and a_α are the phonon creation and annihilation operators;

$$\Gamma_{\alpha\beta}{}^{\mathbf{q}} = A_{ij}{}^{\alpha\beta} \varepsilon_{ij}{}^{\mathbf{q}}, \quad A^{\alpha\beta} \sim {}^1\!/_4 \Lambda (\omega_\alpha \omega_\beta)^{1/2} / \mu, \qquad (4)$$

Λ is a certain mean value of the third-order moduli, μ is the shear modulus, and ω_α is the phonon frequency. Repeated Latin indices imply summation. To simplify the intermediate steps, we shall assume the crystal volume to be equal to unity and use a system of units in which Planck's constant is $\hbar = 1$.

If we denote by $\Delta\rho(t)$ the deviation of the density matrix of the system from the equilibrium value

$$\rho_0 = Z^{-1} e^{-\mathscr{H}/T}$$

($Z = \mathrm{Sp}\, e^{-\mathscr{H}/T}$, T is the temperature in energy units), then the average energy dissipation per unit time can be easily shown to be

$$D = -\mathrm{Sp}\left\{ \Delta\rho(t) \frac{\partial V(t)}{\partial t} \right\}. \qquad (5)$$

We substitute in (5) the matrix $\Delta\rho(t)$ calculated in the approximation linear in the perturbation[14]:

$$\Delta\rho(t) = \frac{1}{i} \int_{-\infty}^{t} e^{i\mathscr{H}(t'-t)} [V(t'), \rho_0]\, e^{-i\mathscr{H}(t'-t)} dt', \qquad (6)$$

where the square brackets denote, as usual, the commutator of the operators contained in them. This leads to the following expression for the dissipation:

$$D = -i \int \frac{d\mathbf{q}}{(2\pi)^3} \Omega_{\mathbf{q}} G^R(\mathbf{q}, \Omega_{\mathbf{q}}). \qquad (7)$$

Here $G^R(\mathbf{q}, \omega_{\mathbf{q}})$ is the Fourier transform of the retarded two-particle Green's function

$$G^R(\mathbf{q}, \omega) = \int_{-\infty}^{\infty} dx\, e^{i\omega x} \Big\{ -i\theta(x) \sum_{\alpha\beta\gamma\delta} \Gamma_{\alpha\beta}{}^{\mathbf{q}} \Gamma_{\gamma\delta}{}^{\mathbf{q}} \times \mathrm{Sp}\, (\rho_0 [e^{i\mathscr{H}x} \xi_\alpha \xi_\beta e^{-i\mathscr{H}x}, \quad \xi_\gamma{}^+ \xi_\delta{}^+]) \Big\}. \qquad (8)$$

Neglecting in (7) terms of order $(v/c)^4$ and higher[4)] in comparison with the first non-vanishing term, which is of order $(v/c)^2$, we obtain an expression for the viscous component of the dissipation

$$D = -i \int \frac{d\mathbf{q}}{(2\pi)^3} \Omega_{\mathbf{q}}{}^2 \frac{\partial G^R(\mathbf{q}, \omega)}{\partial \omega} \Big|_{\omega=0}. \qquad (9)$$

As is well known[15], the function $G^R(\mathbf{q}, \omega)$ coincides with the Fourier transform of the corresponding causal Green's function

$$G(\mathbf{q}, i\omega_n) = \frac{1}{2} \int_{-1/T}^{1/T} dx\, e^{i\omega_n x} \Big\{ -\sum_{\alpha\beta\gamma\delta} \Gamma^{\mathbf{q}}_{\alpha\beta} \Gamma^{\mathbf{q}}{}_{\gamma\delta} \times \mathrm{Sp}\, (\rho_0 T_x e^{i\mathscr{H}x} \xi_\alpha \xi_\beta e^{-i\mathscr{H}x} \xi_\gamma{}^+ \xi_\delta{}^+) \Big\} \qquad (10)$$

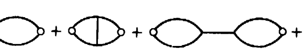
(T_x is the ordering operator[15]) on the discrete set of points $\omega = i\omega_n = 2\pi i n T$ (n = 1, 2, ...). Thus, the problem reduces to a calculation of the causal Green's function (10) and to its analytic continuation with respect to frequency into the upper half-plane: $i\omega_n \to \omega + i\eta$. This method is convenient in that it makes it possible to use a diagram technique.

3. CALCULATION OF THE GREEN'S FUNCTION $G^R(\mathbf{q}, \Omega_{\mathbf{q}})$

In the analysis of the function G^R we confine ourselves to allowance, in the Hamiltonian H_A, of the first anharmonic term corresponding to the three-phonon processes:

$$H_A = \frac{1}{3!} \sum_{\alpha\beta\gamma} \Gamma_{\alpha\beta\gamma} \xi_\alpha \xi_\beta \xi_\gamma. \qquad (11)$$

The diagram expansion of the Green's function (10) in powers of the anharmonicity is then given by

$G(\mathbf{q}, i\omega_n) =$  (12)

Each line corresponds here to a complete single-particle Green's function $G_\beta(i\omega_n)$, i.e., the graphic summation has already been carried out and only the irreducible diagrams are left. The circles at the beginning and end of each diagram denote the incomplete vertex

$$\prec\; = \sum_{\alpha\beta} \Gamma^{\mathbf{q}}_{\alpha\beta} \ldots \qquad (12a)$$

In diagram language, the "phonon wind" corresponds to inclusion of only the first diagram in the expansion (12). This diagram is calculated in the harmonic approximation, a procedure that is valid, as already mentioned, only in the region $q \gg c/lv$. Mason's "phonon viscosity," to the contrary, is obtained by summing all the diagrams of (12) under the assumption that $q \ll v/lc$, followed by extrapolation of the results to the region $q \sim k_D$.

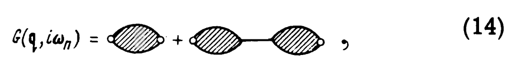
We shall calculate the Green's function in two limiting cases, $q \gg l^{-1}$ and $q \ll l^{-1}$. The fact that we do not know the function G^R in the region $q \sim l^{-1}$ is immaterial, since it will be shown that in the region $l^{-1} < q < k_D$ the integral (7) is determined by the upper limit, and in the interval $0 < q < l^{-1}$ the value of the integral is limited by the behavior of the function G^R in the vicinity of the point $q_0 \sim v/l_c$.

4) The expansion of D in the parameter v/c contains only even powers, since the energy dissipation in this problem cannot depend on the direction of the dislocation motion.

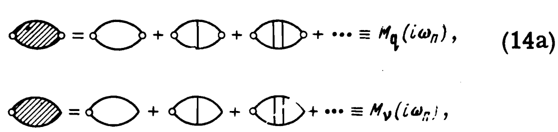
When $q \gg l^{-1}$, the diagrams in expansion (11) contain no singularities, so that in this region the Green's function can be obtained in the usual relaxation-time approximation. In the τ-approximation, the function G^R coincides formally with the harmonic function G_0^R, provided that we replace in the latter the infinitesimally small damping by the finite quantity i/τ:

$$G^R(\mathbf{q},\Omega_{\mathbf{q}})=\sum_{\alpha\beta}|\Gamma_{\alpha\beta}{}^{\mathbf{q}}|^2\left\{\frac{2(\omega_\beta-\omega_\alpha)(n_\alpha-n_\beta)}{(\omega_\alpha-\omega_\beta)^2-(\Omega_{\mathbf{q}}+i/\tau)^2}+\frac{2(\omega_\alpha+\omega_\beta)(n_\alpha+n_\beta+1)}{(\omega_\alpha+\omega_\beta)^2-(\Omega_{\mathbf{q}}+i/\tau)^2}\right\}. \quad (13)$$

In the region of small q ($q \ll l^{-1}$), the ladder diagrams in the expansion (12) contain divergences[16], and this leads to the need for summing such diagrams in all orders of perturbation theory. The result of the summation can be represented schematically in the form



where



$$> \; = \sum_{\alpha\beta}\Gamma_{\bar{\alpha}\bar{\beta}\nu\ldots}, \qquad \nu=(\mathbf{q},\lambda).$$

The graphic formula (14) corresponds to the analytic expression

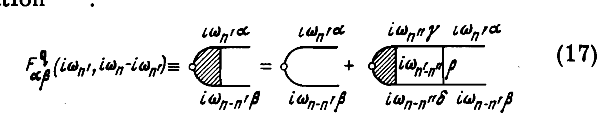
$$G(\mathbf{q},i\omega_n)=M_{\mathbf{q}}(i\omega_n)+\sum_\lambda M_\nu(i\omega_n)G_{\bar{\nu}}(i\omega_n)M_{\bar{\nu}}(i\omega_n). \quad (15)$$

It is convenient to express $M_{\mathbf{q}}$ and M_ν in terms of the same unknown function $F^{\mathbf{q}}_{\alpha\beta}(\omega,\omega')$

$$M_{\mathbf{q}}(i\omega_n)=T\sum_{\alpha\beta}\sum_{n'}\Gamma_{\alpha\beta}{}^{\mathbf{q}}F_{\alpha\beta}{}^{\mathbf{q}}(i\omega_{n'},i\omega_n-i\omega_{n'}), \quad (16)$$

$$M_\nu(i\omega_n)=T\sum_{\alpha\beta}\sum_{n'}\Gamma_{\bar{\alpha}\bar{\beta}\nu}F_{\alpha\beta}{}^{\mathbf{q}}(i\omega_{n'},i\omega_n-i\omega_{n'}),$$

which is determined by the Bethe-Salpeter integral equation[16]:



An analysis of (17) can be carried out in analogy with the corresponding investigation performed by a number of authors[15,16].

Following an analytic continuation of $F^{\mathbf{q}}_{\alpha\beta}$ with respect to frequency and separation of the singular terms[5)] in (17), the substitution

$$F_{\alpha\beta}{}^{\mathbf{q}}(\omega'+i\eta_1,\ \omega-\omega'+i\eta_2)=4\pi\delta_{\mathbf{q},\mathbf{k}_1+\mathbf{k}_2}\delta_{\lambda_1\lambda_2}f_\beta{}^{\mathbf{q}}[\delta(\omega'-\omega_\beta)+\delta(\omega'+\omega_\beta)] \quad (18)$$

($\delta_{\lambda_1\lambda_2}$ is the Kronecker symbol) leads to an inhomogeneous kinetic equation for $f^{\mathbf{q}}_\beta$:

$$i(\mathbf{q}\mathbf{v}_\beta-\omega)n_\beta(n_\beta+1)f_\beta{}^{\mathbf{q}}(\omega)=\Gamma_{\beta\beta'}{}^{\mathbf{q}}n_\beta(n_\beta+1)+J(f_\beta{}^{\mathbf{q}}(\omega)). \quad (19)$$

Here $\mathbf{v}_\beta = \partial\omega_\beta/\partial\mathbf{k}$, $\beta = (\mathbf{k}, \lambda)$, $\beta' = (\mathbf{q}-\mathbf{k}, \lambda)$, $J(f^{\mathbf{q}}_\beta(\omega))$ is the collision integral in the usual form:

$$J(f_\beta{}^{\mathbf{q}}(\omega))=2\pi\sum_{\alpha\gamma}\{|\Gamma_{\bar{\alpha}\gamma\beta}|^2n_\alpha(n_\gamma+1)(n_\beta+1)\times(f_\alpha{}^{\mathbf{q}}-f_\gamma{}^{\mathbf{q}}-f_\beta{}^{\mathbf{q}})\delta(\omega_\alpha-\omega_\gamma-\omega_\beta)+{}^1/_2|\Gamma_{\bar{\alpha}\bar{\gamma}\beta}|^2n_\alpha n_\gamma(n_\beta+1)(f_\alpha{}^{\mathbf{q}}+f_\gamma{}^{\mathbf{q}}-f_\beta{}^{\mathbf{q}})\delta(\omega_\alpha+\omega_\gamma-\omega_\beta)\}. \quad (20)$$

Following the analytic continuation $i\omega_n \to \omega + i\eta$ we can express $M_{\mathbf{q}}$ and M_ν in terms of the function $f^{\mathbf{q}}_\beta(\omega)$:

$$M_{\mathbf{q}}(\omega+i\eta)=\frac{4i\omega}{T}\sum_\beta n_\beta(n_\beta+1)f_\beta{}^{\mathbf{q}}(\omega)\Gamma^{\mathbf{q}}_{\beta\beta'}, \quad (21)$$

$$M_\nu(\omega+i\eta)=\frac{4i\omega}{T}\sum_\beta n_\beta(n_\beta+1)f_\beta{}^{\mathbf{q}}(\omega)\Gamma_{\bar{\beta}\bar{\beta}'\nu}.$$

The solution of (19) for temperature low enough that Umklapp processes can be neglected, according to[8,16], leads to $f^{\mathbf{q}}_\beta(\omega) \sim \omega$, which corresponds, with allowance for (21) and (15) and after substitution in (9), to a zero contribution to the viscous dissipation[6)]. At not too low temperatures, when the Umklapp processes are not small, Eq. (19) has the solution[8],

$$f_\beta{}^{\mathbf{q}}(\omega)=\frac{\omega_\beta}{T^2C}\,\frac{\sum_\alpha\Gamma^{\mathbf{q}}_{\alpha\alpha'}\omega_\alpha n_\alpha(n_\alpha+1)}{-i\omega+\chi_{ij}q_iq_j}. \quad (22)$$

Here $C = T^{-2}\sum_\beta \omega_\beta^2 n_\beta(n_\beta + 1)$ is the specific heat of the crystal and χ_{ij} is the temperature-conductivity tensor. Without going beyond the framework of the accuracy with which the expression for $G^R(\mathbf{q}, \Omega_{\mathbf{q}})$ is written at $q \gg l^{-1}$, we assume henceforth that $\chi_{ij} = \chi\delta_{ij}$, where $\chi = cl$. It is easy to verify that the second term in (15) makes a contribution of the order of $(v/c)^4$ to the energy dissipation, and this term will therefore henceforth be omitted. Thus, in the region $q \ll l^{-1}$, the singular component of the Green's function $G^R(\mathbf{q}, \Omega_{\mathbf{q}})$ is given by

$$G^R(\mathbf{q},\Omega_{\mathbf{q}})=\frac{4i\Omega_{\mathbf{q}}}{T^2C}\,\frac{\left|\sum_\beta\Gamma^{\mathbf{q}}_{\beta\beta'}\omega_\beta n_\beta(n_\beta+1)\right|^2}{-i\Omega_{\mathbf{q}}+\chi q^2}. \quad (23)$$

We note that the ultrasound absorption problem reduces[8] to an analysis of the formula of the type (23). The expressions for the thermoelastic damping and for the "phonon viscosity" are obtained as the first terms of the expansion of this formula in the parameter $\chi q^2/\omega_{\mathbf{q}}$ ($\omega_{\mathbf{q}} = cq$ is the frequency of the ultrasound). It is easy to verify here that the condition that the expansion parameter be small (the adiabaticity condition) coincides with the criterion for the validity of formula (23) itself ($ql \ll 1$). The situation is different with the analysis of the wave damping from the dislocation packet (1). In this case, the region in which such an expansion is valid is much narrower ($ql \ll v/c$), and the concepts of "phonon viscosity" and thermoelastic losses become meaningless when $q \gtrsim v/lc$.

4. ENERGY DISSIPATION AND DISLOCATION DAMPING CONSTANT

Expressions (13) and (23) for the function $C^R(\mathbf{q}, \Omega_{\mathbf{q}})$ together with formulas (7) and (9) solve in principle our

5)The function $F^{\mathbf{q}}_{\alpha\beta}(\omega, \omega')$ also has a non-singular component, alowance for which would lead in the final result to small corrections of the order of $(k_D l)^{-2}$.

6)More accurately, a contribution of order $(k_D l)^{-2}$ (see footnote 5).

problem of phonon dragging of dislocations[7)]. It is easy to verify that the main contribution to the integrals (7) and (9) are made by the regions $q \gg l^{-1}$ and $q \ll l^{-1}$. To calculate the dissipation it is therefore sufficient to know the asymptotic expressions obtained above for G^R at large and small q. Neglecting in (13) the second term, which makes a small contribution to the dissipation, we obtain

$$D=\frac{4}{\tau}\int\limits_{q>l^{-1}}\frac{d\mathbf{q}}{(2\pi)^3}\Omega_\mathbf{q}^2\sum_{\alpha\beta}|\Gamma_{\alpha\beta}^\mathbf{q}|^2\frac{(n_\beta-n_\alpha)(\omega_\alpha-\omega_\beta)}{[(\omega_\alpha-\omega_\beta)^2+\tau^{-2}]} + \frac{4}{T^3C}\int\limits_{q<l^{-1}}\frac{d\mathbf{q}}{(2\pi)^3}\Omega_\mathbf{q}^2\frac{\left|\sum_\beta\Gamma_{\beta\beta'}^\mathbf{q}\omega_\beta n_\beta(n_\beta+1)\right|^2}{-i\Omega_\mathbf{q}+\chi q^2}. \tag{24}$$

A more convenient measure of the effective dislocation dragging than the dissipation D is the damping constant B, defined as the coefficient of proportionality between the velocity v and the force F (per unit dislocation length) needed to maintain this velocity:

$$F=Bv. \tag{25}$$

The connection between the energy dissipation per unit time and the damping constant is obvious:

$$D=Bv^2L, \tag{26}$$

where L is the dislocation length. The value of B can be estimated from formulas (26) and (24) in order of magnitude, if it is recognized that

$$\sum_\beta\omega_\beta n_\beta(n_\beta+1)\Gamma_{\beta\beta'}^\mathbf{q}\approx\frac{\Lambda}{4\mu}T^2C\varepsilon_{ll}^\mathbf{q},\qquad|\varepsilon_{ij}^\mathbf{q}|^2\approx2\pi b^2L\frac{\delta(\mathbf{qn})}{q^2}\varphi_{ij}\left(\frac{\mathbf{q}}{q}\right).$$

Here $\mathbf{n}$ is a unit vector in the dislocation direction and φ_{ij} is a dimensionless function of the directions and is of the order of unity, with $\varphi_{ll} = 0$ for a screw dislocation.

Changing over in (24) in the usual manner from summation to integration, we can easily obtain in the Debye approximation the following expression for the damping constant:

$$B=g\left|\frac{\Lambda}{\mu}\right|^2\frac{\hbar}{b^3}\left(\frac{k_Db}{2\pi}\right)^5\left[f_1\left(\frac{T}{\Theta}\right)+\alpha f_2\left(\frac{T}{\Theta}\right)\frac{\ln[k_Dl(c/v)^\gamma]}{k_Dl}\right], \tag{27}$$

where g, α, and γ are numerical coefficients of the order of unity (for a screw dislocation, in the isotropic approximation assumed by us, we have $\gamma = 0$, and for an edge dislocation γ differs from zero only at not too low temperatures, when the Umklapp processes are not small), Θ is the Debye temperature,

$$f_1(x)=x^5\int\limits_0^{1/x}\frac{t^5e^t\,dt}{(e^t-1)^2},\qquad f_2(x)=x^4\int\limits_0^{1/x}\frac{t^4e^t\,dt}{(e^t-1)^2}. \tag{28}$$

5. DISCUSSION

In formula (27), the first term coincides with the estimate of the "phonon wind," obtained earlier in the essential assumption of high dislocation velocities $v \gg c/k_Dl$ (see paper[12] by one of the authors, where an explicit expression for the coefficient $g|\Lambda/\mu|^2$ in terms of the Murnaghan moduli can be found). The second term in this formula is an estimate of the role of the phonon relaxation. We note that the relaxation component B is proportional to τ^{-1} and not to τ as in Mason's incorrect formula. At not too high temperatures, this component is much less than the first term, since $k_Dl \gg 1$ in most crystals even at a temperature on the order of the Debye temperature, and when the temperature decreases the mean free path increases exponentially (at low temperatures f_1 and f_2 depend on T in power-law fashion: $f_1 \sim T^5$ and $f_2 \sim T^4$). With increasing temperature, the mean free path decreases and the phonon-phonon interaction becomes manifest more and more in dislocation dragging. At sufficiently high temperatures the relaxation component may become decisive in the effect. At high temperatures, however, formula (27) is less reliable, since the entire calculation was performed assuming the dislocation perturbation of the phonon energy to be small, which is true strictly speaking only so long as the average thermal wavelength of the phonon is much larger than the dimension of the dislocation nucleus.

Thus, the main contribution to the dynamic dragging of dislocations is made by phonon scattering by the short-wave part of the packet (1): $ql > 1$, i.e., the greater part of the energy dissipation occurs in the immediate vicinity of the dislocation, within a radius $R \lesssim l$. Only at high temperatures does the region of the "phonon wind" contract enough to bring to the forefront the relaxation processes which, however, do not reduce to "phonon viscosity." The notion that phonon relaxation plays an important role in dynamic dislocation dragging, which is based on erroneous calculations in a number of papers[5,6], is an exaggeration, at least for temperatures below and of the order of the Debye temperature.

The developed approach can be easily compared with the phenomenological theory of Kosevich and Natsik[17], in which dislocation dragging is connected with the dispersion of the elastic moduli, if the coefficients of $\Omega_\mathbf{q}\epsilon_{ij}^\mathbf{q}\epsilon$ in (7) are interpreted as the imaginary parts of the dynamic elastic moduli $c_{ijkl}(\mathbf{q}, \Omega_\mathbf{q})$, calculated with allowance for the temporal and spatial dispersions. Deriving phenomenological formulas for dislocating dragging, Kosevich and Natsik investigated them under the assumption that there is no spatial dispersion of c_{ijkl}. The estimates presented above show, however, that it is not sufficient to take into account only the temporal dispersion of the elastic moduli in the analysis of the phonon dislocation draggings. It is seen from (23) that the spatial dispersion is negligible only in the longest-wavelength part of the partial waves of the packet (1), where $ql \ll v/c$; on the other hand, as shown above, the main contribution to the effect is made not by the long waves but by the short ones corresponding to the asymptotic forms of the elastic moduli at large values of q.

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[7)]Expression (23) cannot be substituted in (9), for this leads to a divergence at small q. The reason is that the contribution made by small q to the dislocation dragging force is not linear but quasilinear in the velocity v, the proportionality coefficient being logarithmically dependent on v (see below).

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