THE TUNNELING OF EXCITATIONS FROM A SUPERCONDUCTOR AND THE INCREASE OF T_c

A. G. ARONOV and V. L. GUREVICH

A. F. Ioffe Physico-technical Institute, USSR Academy of Sciences

Submitted May 6, 1972; resubmitted July 17, 1972

Zh. Eksp. Teor. Fiz. 63, 1809–1821 (November, 1972)

The nonequilibrium state of a superconductor, which arises because of tunneling transitions, is investigated. In this state both the total number of superconducting excitations and their energy distribution change (decrease). This should lead to an increase of T_c since the critical temperature depends on the number of excitations and on their distribution. A double tunnel structure is considered, consisting of a superconducting film with both of its surfaces covered by dielectric layers. Two semiconductors, one p-type and one n-type are in contact with the dielectric layers. In the presence of tunneling, holes from under the superconducting gap enter the n-type semiconductor, and quasiparticles from the region above the gap enter the p-type material. Their decrease is compensated by the production of excitation-hole pairs by phonons. In this connection the total concentration of the excitations decreases.

THE critical superconducting temperature T_c depends on both the number of excitations—that is, the number of quasiparticles above the gap and the number of holes below the gap—and on their distribution. Therefore, the establishment of a nonequilibrium distribution of the quasiparticles can lead to an increase in the value of T_c. In the present article we investigate the nonequilibrium state that arises in a superconductor because of tunneling of the excitations into a semiconductor that forms a tunnel junction with the superconductor.

The utilization of tunneling extraction of the excitations as a method for increasing the value of T_c was discussed in the article by Parmenter.[1] The tunneling from a superconducting film into another superconductor with a higher transition temperature, which in this case must be higher than the anticipated increase of T_c in the film,[1)] was investigated in Parmenter's work.

The increase of T_c due to the effect of the nonequilibrium distribution created by an uhf field was investigated in the articles by Éliashberg[2a] and by Ivlev and Éliashberg.[3] Then the authors of the present article considered[4] the question of increasing T_c due to the influence of the nonequilibrium distribution which arises as a consequence of the interband transitions associated with illumination of the superconductor by light of a definite spectral composition. The method of establishing the nonequilibrium distribution by tunneling, which is investigated in the present article, may turn out to be more convenient from an experimental point of view.

Let us consider the structure shown in the accompanying figure. Both surfaces of the superconducting film of thickness L_z are covered by thin dielectric layers. Tunneling of holes and electrons takes place through these dielectric layers into the semiconductors 1 and 2, which are n- and p-type, respectively. In order to be definite, we assume the semiconductors to be nondegenerate; for the sake of simplicity we neglect the distortion of the bands near their surfaces. The value of the voltages, which are applied to the dielectric layers, is assumed to be such that the top of the valence band in the n-type semiconductor is displaced from the lower edge of the gap by the amount $e\delta V_1 - \Delta$ (e is the electron charge), and the bottom of the conduction band is displaced from the upper edge of the gap by the amount $e\delta V_2 - \Delta$.

As a result of the tunneling, the quasiparticles[2)] will enter from semiconductor 1 into the region below the superconducting gap. In other words, an extraction of holes from below the gap will occur, that is, the injection of minority carriers into the n-type material. The concentration of quasiparticles below the gap will thereby increase. Then, by absorbing a phonon during the characteristic time τ_{ph} of the phonon collisions, the quasiparticles will undergo transitions into the region above the gap, from where they will be extracted into semiconductor 2, that is, an injection of minority carriers into the p-type semiconductor will occur. Here one can consider two limiting cases.

The first of these limiting cases corresponds to the regime of strong extraction, when the quasiparticle concentration above the gap and the hole concentration below the gap are appreciably smaller than the equilibrium concentration associated with a given temperature T. The virtue of this regime lies in the fact that the distribution of the quasiparticles and hence also the magnitude of the superconducting gap are here almost the same as in equilibrium at $T = 0$. The defect lies in the very large value of the tunneling current which is required in order to achieve strong extraction. In particular, this means that the transverse dimensions of the tunnel contact must be small, in order that the

[1)]The double tunnel structure (superconductor-insulator-semiconductor) which we consider in the present article is, in principle, free from such a limitation.

[2)]In the region below the gap the distribution functions of the quasiparticles, F_p, and of the holes, F_p^{holes}, in the superconductor are related by the equation $F_p^{holes} \equiv 1 - F_p$; the energies of the quasiparticles and of the holes are equal in magnitude and opposite in sign.

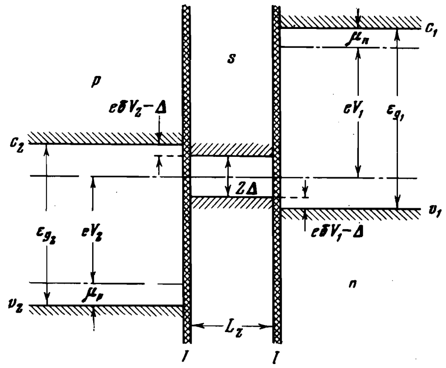

FIG. 1. The double tunnel structure consisting of a semiconductor-dielectric-superconductor sandwich. Here μ_n and μ_p denote the chemical potentials in the n- and p-type semiconductors, and eV_1 and eV_2 denote the potential differences which are applied between the semiconductors 1 and 2, respectively, and the superconductor s.

magnetic field arising from the tunneling current not destroy the superconductivity.

In fact, the number of quasiparticles above the gap in the superconducting film, which must be extracted during the time interval τ_{ph}, is equal to $n_0 S L_Z kT/\mu$ in order of magnitude, where n_0 is the total electron concentration in the superconductor, S is the area of the film, and μ is the chemical potential of the superconductor. It follows from here that the tunneling current density is given by

$$j \approx e n_0 \frac{L_z}{\tau_{ph}} \frac{kT}{\mu}. \tag{1}$$

We obtain $j \approx 3 \times 10^4$ A/cm^2 for $n_0 = 10^{22}$ cm^{-3}, $L_Z = 3 \times 10^{-5}$ cm, $\tau_{ph} = 10^{-10}$ sec, and $kT/\mu = 10^{-4}$. For a superconducting semiconductor film (such as $SrTiO_3$) with $n_0 = 10^{19}$ cm^{-3}, $kT/\mu = 10^{-2}$, $L_Z = 3 \times 10^{-5}$ cm, and $\tau_{ph} = 10^{-10}$ sec, we obtain a value for the tunneling current which is an order of magnitude smaller.

In the opposite case, namely, the regime of weak extraction, the distribution of the quasiparticles and holes differs very little from the equilibrium distribution. The qualitative picture which arises in this connection turns out to be different and resembles the one considered by the authors in[4b]. Briefly, the situation here reduces to the following. The phase transition into the superconducting state is a transition of the second kind. In this sense it is analogous, for example, to the transition in magnetically ordered systems. Meanwhile it is known that if a ferromagnetic substance is placed in an external magnetic field, the phase transition vanishes, and the ordering parameter (the magnetization) remains different from zero at all temperatures.

It is natural to pose the question whether anything like such an external field exists in the case of superconductors. It is found that a similar "quasifield" may exist in superconductors under nonequilibrium external conditions. Namely, let us imagine that the nonequilibrium distribution function of the quasiparticles undergoes a finite jump associated with a transition through the superconducting gap. In this connection let the magnitude of the discontinuity be determined only by the tunneling conditions and the magnitude itself does not depend on the magnitude of the gap. Then, as we shall see below, the width of the gap does not vanish as long as it turns out to be possible to maintain this discontinuity in the distribution function.

In the case of weak extraction which is considered below, in addition to a possible discontinuity of the distribution function, its change near the gap also occurs. In different cases different situations finally turn out to be possible: both the disappearance of the transition due to the discontinuity in the distribution function, and an increase of T_c because of its rearrangement.

Which factors will prevent the change in the distribution function which we need? In the first place, there are fluctuations of the electrostatic potential in a semiconductor, which are related to the random distribution of impurities inside and on the surface of the semiconductor. In order that the energy spread of the tunneling excitations should not be too large, these fluctuations must not too strongly smear the bottom of the conduction band and the top of the valence band. Another factor is the uncertainty in the energies of the states of the excitations, which is due to the scattering of the electrons by phonons. One can talk about a finite jump or about a change of the distribution function in general only by considering energy intervals which exceed this uncertainty. When the width of the gap is smaller than this uncertainty, our treatment becomes inapplicable.

Let us proceed to a quantitative solution of the problem. Let us choose the coefficients u_p and v_p in the Bogolyubov transformation in the following way:

$$u_p^2 = \frac{1}{2}\left[1 + \frac{|\xi_p|}{(\xi_p^2 + \Delta^2)^{1/2}}\right], \quad v_p^2 = \frac{1}{2}\left[1 - \frac{|\xi_p|}{(\xi_p^2 + \Delta^2)^{1/2}}\right]. \tag{2}$$

Here $\xi_p = (p^2/2m) - \mu$, p is the quasimomentum of the electrons, and m is their effective mass. Then the problem under consideration will be symmetric with respect to the extraction of excitations from the region above the gap and the extraction of holes from below the gap; in this case μ coincides with the Fermi level of the equilibrium metal. Such a choice of the quantities u_p and v_p differs from the usual choice for $\xi_p < 0$; however, it is somewhat more convenient for our problem. We then have

$$u_p v_p = \Delta / 2\zeta_p, \tag{3}$$

$$\zeta_p = \pm\sqrt{\xi_p^2 + \Delta^2}, \tag{4}$$

where the sign in front of the square root agrees with the sign of the quantity ξ_p. For $\xi_p < 0$ we shall assume $u_p > 0$ and $v_p < 0$. For $\xi_p > 0$ we regard both of these quantities as positive. For $|\xi_p| \gg \Delta$ one finds that $u_p \to 1$, $v_p \to 0$, and the quasiparticle creation and annihilation operators go over, respectively, into the operators for the creation and annihilation of noninteracting electrons. The quantity ζ_p represents the energy of the quasiparticles. In the representation we have chosen, the equation for the gap in the BCS theory[5] has the form

$$1 = gN(0) \int_0^{\hbar\omega_D} d\xi \, \frac{F(-\xi) - F(\xi)}{(\xi^2 + \Delta^2)^{1/2}}, \tag{5}$$

where g is the effective coupling constant of the electron-electron attraction, N(0) is the density of states near the Fermi surface, and ω_D is the limiting phonon frequency. Thus, it is necessary to find the distribution function of the excitations.

The operator describing electron-phonon collisions in a superconductor has the form (see the article by Éliashberg[2b])

$$\left[\frac{\partial F_p}{\partial t}\right]_{coll}=\frac{2\pi}{\hbar}\sum_q |\alpha_q|^2\{[F_{p-q}(1-F_p)N_q-F_p(1-F_{p-q})(N_q+1)] \times(u_{p-q}u_p-v_{p-q}v_p)^2\delta(\zeta_{p-q}+\hbar\omega_q-\zeta_p)+[F_{p+q}(1-F_p)(N_q+1) - F_p(1-F_{p+q})N_q](u_{p+q}u_p-v_{p+q}v_p)^2\delta(\zeta_{p+q}-\hbar\omega_q-\zeta_p)\} \quad (6)$$

Here $N_q = [\exp(\hbar\omega_q/kT) - 1]^{-1}$ and α_q is a quantity which is proportional to the matrix element of the electron-phonon interaction. For acoustic phonons one has

$$|\alpha_q|^2=\Lambda^2 q/2\mathscr{V}\rho w, \quad (7)$$

where Λ is the deformation potential constant, $\mathscr{V}$ is the volume, ρ is the density of the crystal, and w is the speed of sound. We consider temperatures such that the optical phonons (if such modes exist) are not excited.

We shall not explicitly take into account the elastic collisions of excitations with impurity centers. However, we shall assume that these collisions are frequent enough so that the distribution function F_p will only depend on the excitation energy ζ_p.

Let us begin with an investigation of the simpler complete extraction. In this case, in the zero-order approximation $F(\xi) = 0$ for $\xi > 0$ and $F(\xi) = 1$ for $\xi < 0$. Essentially the only quantity which is relevant to the calculation is the characteristic time τ_{ph} for the transitions of the excitations through the gap. Let us analyze the conservation laws associated with such transitions. We have

$$\zeta_p+\hbar\omega_{p'-p}=\zeta_{p'}. \quad (8)$$

Let us substitute the value of the function ζ_p here:

$$(\xi_p^2+\Delta^2)^{1/2}+(\xi_{p'}^2+\Delta^2)^{1/2}=2wp_F(1-x)^{1/2}; \quad (9)$$

x denotes the cosine of the angle between p and p′; in calculating the right-hand side of Eq. (9) we assumed that $p \approx p' \approx p_F \equiv \sqrt{2m\mu}$. From Eq. (9) we have

$$x=1-[(\xi_p^2+\Delta^2)^{1/2}+(\xi_{p'}^2+\Delta^2)^{1/2}]^2/4w^2p_F^2.$$

First of all let us remark that it is impossible to satisfy relation (9) for

$$[(\xi_p^2+\Delta^2)^{1/2}+(\xi_{p'}^2+\Delta^2)^{1/2}]^2\geqslant 4w^2p_F^2$$

that is, the single-phonon processes are incapable of transfering the excitation through the gap. If $\Delta > wp_F$, then single-phonon processes $1/\tau_{ph} = 0$ for any initial state. Electron-phonon collisions do not occur for

$$[(\xi_p^2+\Delta^2)^{1/2}+(\xi_{p'}^2+\Delta^2)^{1/2}]^2< 4w^2p_F^2$$

In this connection, if

$$wp_F\leqslant kT, \quad (10)$$

then the energy transferred in a collision is equal to wp_F in order of magnitude. The scattering, which is accompanied by the transfer of the excitations through the gap, occurs at large angles. However, if

$$wp_F\gg kT, \quad (11)$$

then phonons having an energy of the order of kT (phonons of larger energies are not excited) play the fundamental role in the collision processes). The scattering occurs at small angles of the order of kT/wp_F.

By using expression (6) we obtain the following expression for $1/\tau_{ph}$, the probability for the transition of excitations from the state with quasimomentum p below the gap into the region above the gap:

$$\frac{1}{\tau_{ph}}=\pi\sum_p|\alpha_{p'-p}|^2N_{p'-p}\delta((\xi_{p'}^2+\Delta^2)^{1/2}+(\xi_p^2+\Delta^2)^{1/2}-\hbar\omega_{p'-p}) \times\left[1-\frac{\xi_p\xi_{p'}-\Delta^2}{(\xi_p^2+\Delta^2)^{1/2}(\xi_{p'}^2+\Delta^2)^{1/2}}\right]. \quad (12)$$

It is convenient to express the answer in terms of the quantity l_a, which is the mean free path of the conduction electrons in a normal conductor for $wp_F \ll kT$. For this purpose we utilize the relation

$$|\alpha_q|^2=\frac{\pi}{2}\frac{\hbar^5\omega_q}{\mathscr{V}kTm^2l_a}.$$

We obtain the following result:

$$\frac{1}{\tau_{ph}(\varepsilon)}=\frac{1}{8mp_Fw^2kTl_a}\int_\Delta^{2mp_F-\varepsilon}\frac{\varepsilon' d\varepsilon'}{(\varepsilon'^2-\Delta^2)^{1/2}}\frac{(\varepsilon+\varepsilon')^2}{e^{(\varepsilon+\varepsilon')/kT}-1} \times\left[1-\frac{(\varepsilon'^2-\Delta^2)^{1/2}(\varepsilon^2-\Delta^2)^{1/2}+\Delta^2}{\varepsilon\varepsilon'}\right], \quad (13)$$

where $\epsilon = |\xi|$. It is clear from this formula that $1/\tau_{ph} \approx v_F/l_a$ in order of magnitude for $\Delta \ll \omega p_F \ll kT$. However, if $wp_F \gg kT$ and $\Delta \ll kT$, then for $\epsilon < 2wp_F$ we have

$$\frac{1}{\tau_{ph}}=\frac{u^2(\varepsilon)(kT)^2}{8mp_Fw^2l_a}\int_0^{(2wp-\varepsilon)/kT}dx\frac{(x+\varepsilon/kT)^2}{e^{x+\varepsilon/kT}-1}, \quad (13a)$$

and for $\Delta \gg kT$ the value of $1/\tau_{ph}$ is exponentially small for all values of ϵ.

Let us find the distribution function $F^{(-)}(\epsilon)$ of the excitations under the gap in the energy range $\epsilon \ll \eta$ (where η is the characteristic energy transfer in collisions of electrons with phonons). In writing down the kinetic equation in this region one can neglect all transitions (both the absorption and the emission of a phonon) occurring between states below the energy band, since their probabilities are strongly suppressed because of the Pauli exclusion principle. Then the kinetic equation takes the following simple form:

$$-\frac{F^{(-)}(\varepsilon)}{\tau_{ph}(\varepsilon)}+\frac{1-F^{(-)}(\varepsilon)}{\tau_-(\varepsilon)}=0, \quad (14)$$

where $\tau_-(\epsilon)$ is the characteristic time governing the intensity of the tunneling into the region under the gap as a function of ϵ. Hence

$$F^{(-)}(\varepsilon)=(1+\tau_-/\tau_{ph})^{-1} \quad (15a)$$

or for $\tau_- \ll \tau_{ph}$ we have $F^{(-)}(\epsilon) = 1 - (\tau_-/\tau_{ph})$. In this case the function is slightly different from unity. This difference becomes smaller as the value of ϵ increases, when τ_{ph}^{-1} decreases. Expression (15a) is unsuitable for $\epsilon \gtrsim \eta$, since then it is no longer possible to neglect the transitions between the states under the gap. However, in this region it is still possible to simply neglect the difference of $F^{(-)}(\epsilon)$ from unity.

In similar fashion we obtain the following expression for the distribution function of the excitations above the gap for $\epsilon < wp_F$:

$$F^{(+)}(\varepsilon)=(1+\tau_{ph}'/\tau_+)^{-1}\approx\tau_+/\tau_{ph}', \quad (15b)$$

in the case $\tau_+ \ll \tau'_{ph}$, where $\tau_+(\epsilon)$ characterizes the rate of extraction of the quasiparticles from the region above the gap. We have $\tau_+(\epsilon) = \tau_-(\epsilon) \equiv \tau_S(\epsilon)$ in our approximation of symmetric extraction. If we neglect the difference of the function $F^{(-)}(\epsilon)$ from unity in evaluating the time τ'_{ph}, we find $\tau'_{ph} = \tau_{ph}$. We shall use this equality in what follows.

Let us go on to the calculation of the function $\tau_S(\epsilon)$ which determines the number of quasiparticles with energy passing from the region above the gap into semiconductor 2 per unit time. The current of quasiparticles with a given energy flowing from the superconductor into the semiconductor is given by

$$\frac{1}{L_z\Sigma}\sum_{\mathbf{p}} v_z D\delta(\varepsilon-\zeta_{\mathbf{p}})u_{\mathbf{p}}^2F(\zeta_{\mathbf{p}}),\quad \Sigma\equiv\sum_{\mathbf{p}}\delta(\varepsilon-\zeta_{\mathbf{p}}). \tag{16}$$

Here $\zeta_p = (\xi_p^2 + \Delta^2)^{1/2}$ denotes the energy of the quasiparticles, D is the transmission coefficient of the barrier between the metal and the semiconductor (for the case of the normal metal, L_z is the thickness of the superconducting film, and the z axis is perpendicular to the film boundary. The summation is taken over all values $|\mathbf{p}| > p_F$ corresponding to a positive component $v_z = \partial\zeta/\partial p_z$ of the velocity on the z axis. We assume that the conduction band of the semiconductor is empty.

The quantity (16) is none other than $F(\epsilon)/\tau_S(\epsilon)$. The total current J can be expressed in the following way:

$$J = 2e\int_0^\infty d\varepsilon\frac{1}{L_z}\sum_{\mathbf{p}} v_z D\delta(\varepsilon-\zeta_{\mathbf{p}})u_{\mathbf{p}}^2F(\zeta_{\mathbf{p}}) \tag{17}$$

(the factor of 2 arises from the summation over the spins). Integrating the numerator of (16) with respect to p_z, we find

$$\frac{1}{\tau_s(\varepsilon)} = \frac{u^2(\varepsilon)}{2\pi\Sigma}\sum_{\mathbf{p}_\perp} D(\varepsilon, \mathbf{p}_\perp). \tag{18}$$

Later it will be necessary for us to know the explicit form of the function D which generally depends on p'_z (the z-component of the electron's quasimomentum in the semiconductor), p_z, and $p_\perp$. We shall neglect the dependence of D on the last two factors since the quasimomentum of the excitations in a metal is large, and the characteristic (for our problem) interval of its variation is small. However, the dependence on p'_z should be taken into consideration. We consider two limiting cases.

a) The case of an abrupt barrier. For characteristic values of p'_z, the characteristic distance over which the height of the potential barrier changes substantially, is much smaller than $\hbar/p'_z$. The coefficient D is proportional to p'_z[6] and by introducing the dimensionless coefficient D_a one can write down

$$D = D_a p'_z / p_F. \tag{19}$$

b) The case of a smooth (flat) barrier when the opposite inequality holds. The quasiclassical approximation can be used to calculate the coefficient D. The coefficient D is small; to the first approximation it does not depend on p'_z and is equal to a certain constant value D_b.

Let us carry out the integration in Eq. (18), taking the following relation between the energies of the excitations in the metal and in the semiconductor into account:

$$\varepsilon_{\mathbf{p}'} = \varepsilon_{\mathbf{p}} - e\delta V,\quad \mathbf{p}'_\perp = \mathbf{p}_\perp,$$

where $e\delta V$ is the difference between the energy level at the bottom of the conduction band and the middle of the superconducting gap. For case (a) we obtain the following result:

$$\text{FO}\quad \frac{1}{\tau_s(\varepsilon)} = \frac{1}{3\sqrt{2}}\frac{D_a}{L_z}\left(\frac{m_0}{m}\right)^{3/2}\frac{(\varepsilon-e\delta V)^{3/2}}{\mu m^{1/2}}u^2(\varepsilon)\frac{(\varepsilon^2-\Delta^2)^{1/2}}{\varepsilon}, \tag{20a}$$
$$\text{for}\ \varepsilon > e\delta V;$$
$$\frac{1}{\tau_s(\varepsilon)} = 0\ \text{for}\ \varepsilon < e\delta V.$$

In case (b) we find

$$\frac{1}{\tau_s(\varepsilon)} = \frac{1}{2}u^2(\varepsilon)\frac{(\varepsilon^2-\Delta^2)^{1/2}}{\varepsilon}D_b\frac{m_0}{m}\frac{1}{L_z p_F}(\varepsilon - e\delta V) \tag{20b}$$

under the previous condition $\epsilon > e\delta V$. Here m_0 denotes the effective mass in the semiconductor.

It is convenient to express the quantities in (20) in terms of an observable quantity, namely, the total current density J_0 passing through the tunneling contact under certain characteristic conditions. We choose as such conditions $\Delta/kT \ll 1$ and $e\delta V/kT \ll 1$. We then have

$$J_0 = \frac{\sqrt{2}}{3\pi^2}\frac{eD_a m_0^{3/2}(kT)^{5/2}}{\hbar^3 p_F}F_{3/2}(0), \tag{21a}$$

where

$$F_\nu(0) = \int_0^\infty \frac{x^\nu dx}{e^x+1}$$

is the Fermi integral; hence

$$J_0 = \frac{1}{2\pi^2\hbar^3}eD_b m_0(kT)^2F_1(0). \tag{21b}$$

Let us introduce the characteristic time

$$\frac{1}{\tau_J} = \frac{2}{3}\frac{J_0}{en_0L_z}\frac{\mu}{kT} \tag{22}$$

Then in case (a) we have

$$\frac{1}{\tau_s(\varepsilon)} = \frac{1}{F_{3/2}(0)}\frac{1}{\tau_J}\left(\frac{\varepsilon-e\delta V}{kT}\right)^{3/2}u^2(\varepsilon)\frac{(\varepsilon^2-\Delta^2)^{1/2}}{\varepsilon}, \tag{23a}$$

and in case (b) we have

$$\frac{1}{\tau_s(\varepsilon)} = \frac{1}{F_1(0)}\frac{1}{\tau_J}\frac{\varepsilon-e\delta V}{kT}u^2(\varepsilon)\frac{(\varepsilon^2-\Delta^2)^{1/2}}{\varepsilon}. \tag{23b}$$

It is clear from the form of these expressions that one can discuss a regime of strong extraction only if $e\delta V < \Delta$. Further, in order to be definite, let us consider strong extraction in case (a). By substituting (15a) and (15b) in Eq. (5) with Eqs. (13a) and (23a) taken into consideration, we obtain

$$\ln\frac{\Delta_0}{\Delta} = 2\frac{\tau_J}{\tau_0}F_{3/2}(0)(kT)^{3/2}\int_\Delta^\infty\frac{\varepsilon d\varepsilon}{(\varepsilon^2-\Delta^2)^{1/2}}\chi\left(\frac{\varepsilon}{kT}\right) \tag{24}$$
$$\times\left[(\varepsilon-e\delta V)^{3/2}(\varepsilon^2-\Delta^2)^{1/2}+\frac{\tau_J}{\tau_0}F_{3/2}(0)\chi\left(\frac{\varepsilon}{kT}\right)\varepsilon(kT)^{3/2}\right]^{-1},$$

where Δ_0 denotes the half-width of the gap in equilibrium at T = 0, and

$$\chi(x) = \int_x^{2wp_F/kT}\frac{y^2dy}{e^y-1},\quad \tau_0^{-1} = \frac{(kT)^2}{8mp_Fw^2l_a}. \tag{25}$$

Let us solve Eq. (24) by the method of successive approximations, using the smallness of the parameter

$$\gamma = \frac{\tau_J}{\tau_0}\chi(0)F_{3/2}(0)\left(\frac{kT}{\Delta_0}\right)^{3/2}.$$

In the zero-order approximation $\Delta = \Delta_0$, and in the next approximation for $\gamma \ll 1 - (e\delta V/\Delta_0)$ we find

$$\Delta=\Delta_0\left[1+\frac{\gamma\ln\gamma}{(1-e\delta V/\Delta_0)^{3/2}}\right]. \qquad (26)$$

Now let us consider the regime of weak extraction. In this case one can represent the distribution function as the sum of the equilibrium Fermi function $F_0(\zeta_p)$ and a small nonequilibrium correction:

$$F(\zeta_p)=F_0(\zeta_p)-kT\frac{\partial F_0}{\partial\zeta_p}\varphi(\zeta_p). \qquad (27)$$

Linearizing the collision operator (6) with respect to this correction term and integrating over the angle between p and p′, we obtain the following result for the region below the gap:

$$\left[\frac{\partial F_p^{(-)}}{\partial t}\right]_{\rm coll}=\frac{1}{\tau_0}\frac{1}{(kT)^3}\left\{\int\limits_\Delta^\varepsilon\frac{\varepsilon' d\varepsilon'}{(\varepsilon'^2-\Delta^2)^{1/2}}\frac{(\varepsilon'-\varepsilon)^2}{e^{(\varepsilon-\varepsilon')/kT}-1}\cdot\right.$$
$$\times U_-[\varphi^{(-)}(\varepsilon')-\varphi^{(-)}(\varepsilon)]F_0(\varepsilon')[1-F_0(\varepsilon)]- \qquad (28)$$
$$-\int\limits_\varepsilon^\infty\frac{\varepsilon' d\varepsilon'}{(\varepsilon'^2-\Delta^2)^{1/2}}\frac{(\varepsilon'-\varepsilon)^2}{e^{(\varepsilon-\varepsilon')/kT}-1}U_-F_0(\varepsilon')[1-F_0(\varepsilon)][\varphi^{(-)}(\varepsilon')-\varphi^{(-)}(\varepsilon)]$$
$$\left.+\int\limits_\Delta^\infty\frac{\varepsilon' d\varepsilon'}{(\varepsilon'^2-\Delta^2)^{1/2}}\frac{(\varepsilon+\varepsilon')^2}{e^{(\varepsilon+\varepsilon')/kT}-1}U_+[1-F_0(\varepsilon)][1-F_0(\varepsilon')][\varphi^{(+)}(\varepsilon')-\varphi^{(-)}(\varepsilon)]\right\}$$

Here

$$U_\pm(\varepsilon,\ \varepsilon',\ \Delta)=1+[(\varepsilon'^2-\Delta^2)^{1/2}(\varepsilon^2-\Delta^2)^{1/2}\pm\Delta^2]/\varepsilon\varepsilon'. \qquad (29)$$

The collision operator for the region above the gap differs from expression (28) by the change $\varphi^{(-)}(\epsilon)\rightleftarrows\varphi^{(+)}(\epsilon)$.

In the case of the "symmetric extraction" of quasiparticles and holes (which is the case under consideration), the rate of quasiparticle tunneling from semiconductor 1 into the state with energy $\zeta=-\epsilon$ is given by

$$[1-F_0(-\varepsilon)]/\tau_s(\varepsilon)=F_0(\varepsilon)/\tau_s(\varepsilon), \qquad (30)$$

and the rate of quasiparticle tunneling from the region above the gap into semiconductor 2 is given by

$$F_0(\varepsilon)/\tau_s(\varepsilon).$$

In accordance with the adopted approximation, in the expressions for the tunneling rates we have replaced the quasiparticle distribution function by its equilibrium value.

Furthermore, in our case

$$\varphi^{(+)}(\varepsilon)=-\varphi^{(-)}(\varepsilon)\equiv\varphi(\varepsilon). \qquad (31)$$

as one can easily verify. Setting the sum of expressions (28) and (30) equal to zero and using (31), we obtain the following integral equation concerning the function $\varphi(\epsilon)$:

$$\frac{1}{\tau_0(kT)^3}\left\{\int\limits_\Delta^\varepsilon\frac{\varepsilon' d\varepsilon'}{(\varepsilon'^2-\Delta^2)^{1/2}}\frac{U_-(\varepsilon'-\varepsilon)^2[\varphi(\varepsilon')-\varphi(\varepsilon)]}{(e^{(\varepsilon'-\varepsilon)/kT}-1)(e^{-\varepsilon'/kT}+1)}\right. \qquad (32)$$
$$-\int\limits_\varepsilon^\infty\frac{\varepsilon' d\varepsilon'}{(\varepsilon'^2-\Delta^2)^{1/2}}\frac{U_-(\varepsilon'-\varepsilon)^2[\varphi(\varepsilon')-\varphi(\varepsilon)]}{(e^{(\varepsilon'-\varepsilon)/kT}-1)(e^{-\varepsilon'/kT}+1)}$$
$$\left.-\int\limits_\Delta^\infty\frac{\varepsilon' d\varepsilon'}{(\varepsilon'^2-\Delta^2)^{1/2}}\frac{U_+(\varepsilon+\varepsilon')^2[\varphi(\varepsilon')+\varphi(\varepsilon)]}{[e^{-(\varepsilon+\varepsilon')/kT}-1](e^{\varepsilon'/kT}+1)}\right\}=-\frac{1}{\tau_s(\varepsilon)}.$$

Let us investigate the solution of this equation in the most interesting case $\Delta\ll kT$, and in order to do so we apply a procedure similar to the one used in[3]. Let us start with an analysis of the solution in the energy range $\epsilon\ll kT$. As we shall verify below, values of ϵ' of the order of kT are important in the integrals over ϵ'. Therefore, the first integral inside the curly brackets is small in comparison with the other two, and we shall discard it. In the other two integrals we neglect Δ in comparison with ϵ' and ϵ in comparison with kT. In addition, to within the same approximation we replace the lower limits of these two integrals by zero. Finally the functions $\varphi(\epsilon')$ appearing in the integrand mutually cancel each other, and the following expression is obtained for the function $\varphi(\epsilon)$ in the region of small energies:

$$\varphi(\varepsilon)=-\frac{\tau_1}{\tau_s(\varepsilon)}\frac{1}{u^2(\varepsilon)}, \qquad (33)$$

where $1/\tau_1=7\zeta(3)/\tau_0$ and $\zeta(x)$ is the Riemann zeta function. The expression in the denominator of (33) comes from the coherence factors of the type $(u_{p+q}u_p-v_{p+q}v_p)^2$ which appear in the probabilities for electron-phonon collisions; it differs substantially from unity only for values of ϵ of the order of Δ.

The mutual cancellation of the contributions from the functions $\varphi(\epsilon')$ inside the sign of the integral over ϵ' is due to the following reason. As has already been noted, energy values ϵ' of the order of kT are important in the integral over ϵ'. The integral terms describe the arrival of the quasiparticles in a state with a small (in comparison with kT) energy ϵ due to the nonequilibrium corrections to the distribution function. But the regions with $\epsilon'>0$ and $\epsilon'<0$ are almost symmetrically distributed with respect to this state, and the functions $\varphi^{(-)}(\epsilon)$ and $\varphi^{(+)}(\epsilon)$ are equal in magnitude and opposite in sign. Therefore, their total contribution vanishes to within the accuracy of our approximation.

Substituting (23) into (33), for $\epsilon\ll kT$ we obtain the following results for cases a) and b), respectively:

$$\varphi(\varepsilon)=-\frac{1}{F_{3/2}(0)}\frac{\tau_1}{\tau_J}\left(\frac{\varepsilon-e\delta V}{kT}\right)^{3/2}\frac{(\varepsilon^2-\Delta^2)^{1/2}}{\varepsilon}, \qquad (34a)$$

$$\varphi(\varepsilon)=-\frac{1}{F_1(0)}\frac{\tau_1}{\tau_J}\frac{\varepsilon-e\delta V}{kT}\frac{(\varepsilon^2-\Delta^2)^{1/2}}{\varepsilon}. \qquad (34b)$$

If $\epsilon\gtrsim kT$ one can neglect the small terms proportional to Δ/ϵ in the kinetic equation for the function φ, that is, one can use the kinetic equation for the normal metal. In this connection the solution of the kinetic equation will have the following form (in order to be definite, we present the result for case b)):

$$\varphi(\varepsilon)=-\frac{1}{F_1(0)}\frac{\tau_1}{\tau_J}\frac{\varepsilon}{kT}\psi\left(\frac{\varepsilon}{kT}\right), \qquad (35)$$

where $\psi(x)$ is the solution of the following equation:

$$\int\limits_0^x\frac{dx'(x-x')^2[x'\psi(x')-x\psi(x)]}{(e^{x'-x}-1)(e^{-x'}+1)}-\int\limits_x^\infty\frac{dx'(x-x')^2[x'\psi(x')-x\psi(x)]}{(e^{x'-x}-1)(e^{-x'}+1)}$$
$$-\int\limits_0^\infty\frac{dx'(x+x')^2[x\psi(x)+x'\psi(x')]}{(e^{-x'-x}-1)(e^{x'}+1)}=\frac{7\zeta(3)}{2}x. \qquad (36)$$

This equation does not contain any parameters and can only be solved numerically. However, for our purposes the exact form of the function $\psi(\epsilon/kT)$ does not play a special role. It is sufficient to know that the solution of Eq. (32) can be represented in the form

$$\varphi(\varepsilon)=-\frac{8}{F_1(0)}\frac{\tau_1}{\tau_J}\frac{\varepsilon-e\delta V}{kT}\frac{(\varepsilon^2-\Delta^2)^{1/2}}{\varepsilon}\psi\left(\frac{\varepsilon}{kT}\right)\left[1+O\left(\frac{\Delta}{kT}\right)\right]. \qquad (37)$$

Here $\psi(0) = 1$, and for energies $\epsilon \gg kT$ the nonequilibrium correction $\varphi kT\partial F_0/\partial\epsilon$ to the distribution function falls off exponentially.

Substituting expression (37) into (27) and then into Eq. (5), we obtain the following equation for the quantity Δ:

$$\ln\frac{2\hbar\omega_D}{\Delta} = \int_\Delta^{\hbar\omega_D} \operatorname{th}\frac{\varepsilon}{2kT}\frac{d\varepsilon}{(\varepsilon^2-\Delta^2)^{1/2}} - \frac{1}{2}\int_{\varepsilon_0}^{\infty}\frac{d\varepsilon}{(\varepsilon^2-\Delta^2)^{1/2}}\frac{\varphi(\varepsilon)}{\operatorname{ch}^2(\varepsilon/2kT)}. \quad (38)$$

Here ϵ_0 is the larger of the two quantities Δ and $e\delta V$; we have extended the range of integration in the second integral to infinity in view of the rapid convergence of the integrand for $\epsilon \sim kT \ll \hbar\omega_D$. By integrating we obtain

$$\ln\frac{\Delta_0}{\Delta} = 2I\left(\frac{\Delta}{kT}\right) + \frac{1}{2}K, \quad (39)$$

where

$$I\left(\frac{\Delta}{kT}\right) = \int_\Delta^\infty \frac{d\varepsilon}{(\varepsilon^2-\Delta^2)^{1/2}}\frac{1}{e^{\varepsilon/kT}+1};$$

$$K = \int_{\varepsilon_0}^\infty \frac{d\varepsilon}{(\varepsilon^2-\Delta^2)^{1/2}}\frac{\varphi(\varepsilon)}{\operatorname{ch}^2(\varepsilon/2kT)}.$$

Since $\varphi < 0$, it follows that $K < 0$. This means that at any temperature the nonequilibrium value of Δ exceeds its equilibrium value.

Now let us concentrate our attention on case b) since here somewhat simpler quantitative expressions are obtained, even though the qualitative picture is similar in both cases. For $\Delta \ll kT$ we have

$$I\left(\frac{\Delta}{kT}\right) = \frac{1}{2}\ln\frac{\pi}{\gamma}\frac{kT}{\Delta} + O\left(\left(\frac{\Delta}{kT}\right)^2\right).$$

Then, by introducing the transition temperature under equilibrium conditions,

$$kT_{c0} = \gamma\Delta_0/\pi, \quad \ln\gamma = C = 0.577,$$

we can represent Eq. (38) in the form

$$\ln\frac{T_{c0}}{T} = -\frac{1}{2}\frac{1}{F_1(0)}\frac{\tau_1}{\tau_J}\int_{\varepsilon_0}^\infty \frac{\varepsilon - e\delta V}{kT}\psi\left(\frac{\varepsilon}{kT}\right)\frac{d\varepsilon}{\varepsilon\operatorname{ch}^2(\varepsilon/2kT)}. \quad (40)$$

First let us consider the case $e\delta V > \Delta$. Then the right-hand side of Eq. (40) does not depend on Δ, and this equation determines the transition temperature in the normal state, T_c, as a function of T_{c0}. Calculating the right-hand side to lowest order in $e\delta V/kT$, we obtain

$$\ln\frac{T_{c0}}{T_c} = -\frac{1}{2F_1(0)}\frac{\tau_1}{\tau_J}\left\{A + \frac{e\delta V}{kT_c}\ln\frac{e\delta V}{kT_c}\right\}, \quad (41)$$

where A is a constant of the order of unity:

$$A = \int_0^\infty dx\frac{\psi(x)}{\operatorname{ch}^2(x/2)}. \quad (42)$$

If $e\delta V < \Delta$, then $\epsilon_0 = \Delta$, and the equation for Δ has the form

$$\ln\frac{T_{c0}}{T} = -\frac{1}{2F_1(0)}\frac{\tau_1}{\tau_J}\left\{A - \frac{\Delta}{kT} + \frac{e\delta V}{kT}\ln c\frac{\Delta}{kT}\right\}. \quad (43)$$

Here

$$\ln c = \int_0^\infty dx \ln x\left(\frac{\psi(x)}{\operatorname{ch}^2(x/2)}\right)'.$$

Equation (40), which gives relations (41) and (43) in the appropriate limiting cases, enables us to analyze qualitatively the dependence of Δ on T and $e\delta V$ and the dependence of T_c on $e\delta V$. For $e\delta V \gtrsim kT$ the tunnel effect has little influence on the nature of the temperature dependence of the gap. As the value of $e\delta V$ decreases, the curves representing the temperature dependence of Δ always move higher and intersect the axis of abscissas at temperatures T_c, which increase as $e\delta V$ decreases. The transition temperature T_{c1} for $e\delta V = 0$ is determined by the expression

$$\frac{T_{c1} - T_{c0}}{T_{c0}} = \frac{A}{2F_1(0)}\frac{\tau_1}{\tau_J}. \quad (44)$$

The phase transition vanishes for $e\delta V < 0$. This is the case of the "quasifield" which was mentioned at the beginning of this article (also see article[4b] by the authors). In this case the width of the gap is damped out exponentially as the temperature increases.

What is the fundamental difference between the cases $e\delta V > 0$ and $e\delta V < 0$? In the first case the tunneling transitions perturb the distribution function in such a way that even though its discontinuity on going through the gap is increased in comparison with the equilibrium value, it still tends to zero together with the width of the gap. At the same time, for $e\delta V < 0$ the tunneling transitions create a discontinuity which does not depend on Δ, and the phase transition vanishes within the framework of the BCS theory. It is necessary however, to bear in mind that direct tunneling of the electrons from one semiconductor through the superconductor to the other semiconductor can occur for $e\delta V < 0$.

The behavior of the phase transition at $e\delta V = 0$ is determined by the nature of the perturbation of the distribution function of the excitations for small energies. In the case considered above, the probability of tunneling transitions vanishes when $\epsilon = 0$. This leads to the emergence of a finite transition temperature T_{c1}.

The vanishing of the probability for tunneling transitions at $\epsilon = 0$ is related to the following property. We have considered the tunneling transitions which occur with conservation of energy and conservation of the transverse component of the quasimomentum. On the other hand, this transverse component must be small since the electron's energy in the semiconductor is small (measured from the bottom of the conduction band), i.e., the energy in that state where the tunneling transition originates. But this implies that the tunneling can originate only from states of the superconductor occupying a small fraction of the total area of the Fermi surface. And as $\epsilon \to 0$ the tunneling probability vanishes due to the smallness of the corresponding phase volume. However, if the tunneling transitions take place without conservation of the transverse component of the quasimomentum (this might happen in the presence of sufficiently strong scattering by impurities or if the surfaces of the film are sufficiently rough), then the tunneling probability does not vanish in the case of a sufficiently smooth barrier as $\epsilon \to 0$. In this case $T_c \to \infty$ as $e\delta V \to 0$.

In other respects the qualitative picture of the effect is not very sensitive to the specific form of the expression for the tunneling probability. Quantitatively, however, it is clear that it should be more favorable to consider the case when the tunneling transitions occur without conservation of the transverse component of the quasimomentum, because in this case the tunneling probability may turn out to be substantially larger.

The authors thank G. E. Pikus for a helpful discussion.

Note added in proof. Through an error the authors have left out of Eq. (6) terms describing the production and annihilation of quasiparticle pairs from the condensate. Allowance for these terms makes it necessary to replace the last factor in (12) by

$$1 + \Delta^2 / \sqrt{(\xi_p^2 + \Delta^2)(\xi_{p'}^2 + \Delta^2)},$$

and the last term of (13) by $\lambda + \Delta^2/\epsilon\epsilon'$. Expression (13a) should be divided, and expressions (33), (34a), (34b), (37), and (40) as well as the first term in the denominator of (24) should be multiplied by

$$u^2(\varepsilon) = \tfrac{1}{2}(1 + \sqrt{\varepsilon^2 - \Delta^2}/\varepsilon).$$

Formula (29) should take the form

$$U_\pm(\varepsilon, \varepsilon', \Delta) = 1 \pm \Delta^2 / \varepsilon\varepsilon'.$$

In addition, it is necessary to multiply by 2 the expression for γ after formula (25) and the right-hand sides of (13a) and (32) by the formula for τ_0^{-1} after formula (25).

As a result, the final expression (41) remains unchanged. In (43) the coefficient of Δ/kT is $1/2 + \pi/4$ in place of unity, and in the expression for ln C it is necessary to add the term $(1 - \ln 2)/2$.

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Translated by H. H. Nickle
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