NEUTRINO THERMAL CONDUCTIVITY IN COLLAPSING STARS

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Gasdynamic calculations of gravitational collapse of stars show that 1) the collapse dynamics is determined to a large extent by neutrino emission processes, the main role being played by the Urca-process; 2) at a certain stage of collapse the central core of the star becomes opaque with respect to its own neutrino radiation. Neutrino (and antineutrino) energy and momentum transfer processes are considered and incorporated into the gasdynamics. A system of neutrino gasdynamics equations is derived in the thermal conductivity approximation. This system generalizes the photon radiative thermal conductivity familiar from radiative gasdynamics. An additional diffusion equation for lepton charges is obtained and follows from an analysis of β-interaction kinetics in a medium of arbitrary nuclear composition. All transfer coefficients are calculated for the case of high temperatures when the baryon component of matter consists of free nucleons. A spherically symmetric boundary problem for the central core of a star is formulated.

If a star has exhausted its reserve of nuclear fuel during its evolution and if its mass (more accurately, the mass of its central dense core) is sufficiently large, then the catastrophic process of gravitational collapse inevitably occurs[1]. Gasdynamic calculations of the collapse were performed for stars with different masses exceeding the Chandrasekhar limit[2-4]. They have shown that the star becomes opaque to neutrinos during a definite stage. It must be emphasized that the character of the neutrino radiation and of the neutrino energy transfer determines in many respects the dynamics of the gravitational collapse[1], so that a physically correct description of neutrino mechanisms of radiation and energy (and momentum) transfer become very important for the problem of gravitational collapse of a star as a whole. The onset of neutrino opacity can be one of the causes of the stopping of the catastrophic contraction of the collapsing core of the star and the formation, as a result of the stopping, of a powerful shock wave propagating through the shell of the star, which was partially involved in the collapse process prior to this occurrence. The singularities of the neutrino energy transfer can strongly influence the parameters of the shock wave and the fraction of the star-shell mass ejected into the interstellar space.

In this paper[1)] we derive the neutrino thermal-conductivity approximation on the basis of general kinetic transport equations for neutrinos and antineutrinos. The main difference between the neutrino and radiant thermal conductivities, corresponding to the fermion nature of the neutrino, lies in the additional transfer of leptonic charge (besides the energy transfer). As a result we obtain an additional diffusion-type equation for the chemical potential of the neutrino. All these effects are included in the system of gasdynamics equations, so that we arrive at a neutrino gasdynamics analogous to radiative gasdynamics (the initial transport equations take into account, in turn, the effects of the velocity of the matter). The concrete conditions inside the collapsing stars are such that the principal role among the weak interactions are played by β processes (Urca processes) on free nucleons. We write out for this case the final expressions for all the coefficients of the neutrino thermal conductivity, and the equations themselves are supplemented with the suitable boundary conditions. The entire analysis is given in a spherical coordinate system for the spherically-symmetrical case, which is of greatest interest in neutrino transport (this is important for an optical thickness on the order of unity).

1. NEUTRINO TRANSPORT EQUATIONS WITH ALLOWANCE FOR THE MOTION OF THE MEDIUM

The intensities of the neutrino (ν) and antineutrino ($\tilde{\nu}$) radiation $I_\nu(r, \epsilon_\nu, \mu, t)$ and $I_{\tilde{\nu}}(r, \epsilon_{\tilde{\nu}}, \mu, t)$ satisfy transport equations that can be written in a relativistically covariant form in analogy with the case of photons, to take into account the motion of matter. The equations for radiation transfer in a moving medium were first derived by Thomas[5] and were subsequently considered in[6,7]. If we disregard scattering and confine ourselves to pure absorption[2)], then the neutrino transport equations[3)] in the spherically symmetrical case takes in the chosen reference frame the form[10]

$$\frac{1}{c}\frac{\partial I_\nu}{\partial t}+\mu\frac{\partial I_\nu}{\partial r}+\frac{1-\mu^2}{r}\frac{\partial I_\nu}{\partial \mu}=-\frac{I_\nu}{l_\nu}+q_\nu. \quad (1.1)$$

The mean free path l_ν, the bulk emissivity q_ν, and

[1)]A preliminary report of these results is given in a preprint of the Institute of Applied Mathematics of the USSR Academy of Sciences (No. 18, 1971). where additional details dealing mainly the mathematical calculations can also be found.

[2)]Scattering effects in the relativistically covariant transport equation are discussed in [7,8]. On going over to the thermal-conductivity approximation, with which we shall deal later on, it is preferable to use the approximate procedure employed for photons [9], namely combine the absorption coefficient with the scattering coefficient in the final expressions.

[3)]The antineutrino case can be analyzed in perfect analogy. At the end of our analysis we simply add the contribution of the antineutrinos.

the intensity I_ν are connected here with the corresponding quantities $l_{\nu 0}$, $q_{\nu 0}$, and $I_{\nu 0}$, in the proper reference frame in which the matter is at rest, by the relations[5,6]

$$l_{\nu 0} = L l_\nu, \qquad q_{\nu 0} = L^2 q_\nu, \qquad \frac{I_{\nu 0}}{\varepsilon_{\nu 0}^3} = \frac{I_\nu}{\varepsilon_\nu^3} = \frac{N_\nu}{c^2 h^3}, \tag{1.2}$$

$$L = \Theta\left(1 - \frac{v\mu}{c}\right) = \Theta^{-1}\left(1 + \frac{v\mu_0}{c}\right)^{-1}, \qquad \Theta = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}. \tag{1.3}$$

These relations ensure invariance of (1.1) under Lorentz transformations. The arguments of the functions $l_{\nu 0}$, $q_{\nu 0}$, and $I_{\nu 0}$ are the neutrino energy $\epsilon_{\nu 0}$ and the direction cosine μ_0 in the proper reference frame. They are connected with ϵ_ν and μ by the relations

$$\varepsilon_{\nu 0} = L\varepsilon_\nu, \quad \mu_0 = L^{-1}\Theta(\mu - v/c). \tag{1.4}$$

According to (1.2), the quantity N_ν is invariant. For the subsequent calculations we shall need the differential connection between the neutrino energy and the solid angle in both reference frames; this connection is obtained from (1.4) (μ is assumed fixed when the first relation is differentiated):

$$d\varepsilon_{\nu 0} = L d\varepsilon_\nu, \quad d\omega_0 = L^{-2} d\omega, \tag{1.5}$$

where $d\omega = 2\pi d\mu$ and $d\omega_0 = 2\pi d\mu_0$. In the nonrelativistic approximation ($v/c \ll 1$), neglecting terms of order $(v/c)^2$ in (1.3), we obtain

$$L = 1 - v\mu/c = 1 - v\mu_0/c. \tag{1.6}$$

Neutrinos are subject to Fermi-Dirac statistics. Therefore, in accordance with the Pauli principle, the emissivity $q_{\nu 0}$ of the matter should be lower the larger the occupancy of the phase space. In other words, instead of stimulated emission we obtain here in the case of photons "stimulated absorption." The emissivity is obviously given by

$$q_{\nu 0} = (1 - N_\nu) B_{\nu 0}, \tag{1.7}$$

where $B_{\nu 0}$ no longer depends on the intensity of the neutrino emission and is determined by the properties of the medium. To find $B_{\nu 0}$, we consider the equilibrium case, when the emission intensity is determined by Fermi-Dirac statistics[11]:

$$I_{\nu 0} = I_{e0} = \frac{\varepsilon_{\nu 0}^3}{c^2 h^3}[1 + \mathscr{E}(\varepsilon_{\nu 0} - \psi_\nu)]^{-1} = \frac{\varepsilon_{\nu 0}^3}{c^2 h^3} N_{e0}, \tag{1.8}$$

$$\mathscr{E}(x) = \exp(x/kT), \tag{1.8'}$$

where ψ_ν is the chemical potential of the neutrino. Using now (1.1) with zero left-hand side, (1.7), and (1.8), we get

$$B_{\nu 0} l_{\nu 0} = I_{e0}\left(1 - \frac{c^2 h^3}{\varepsilon_{\nu 0}^3} I_{e0}\right)^{-1} = \frac{\varepsilon_{\nu 0}^3}{c^2 h^3}\mathscr{E}(\psi_\nu - \varepsilon_{\nu 0}). \tag{1.9}$$

This universal connection between $B_{\nu 0}$ and $l_{\nu 0}$ is the neutrino analog of Kirchoff's law. With the aid of (1.7–1.9) we can write the right-hand side of (1.1) for a medium at rest in the form

$$-\frac{I_{\nu 0}}{l_{\nu 0}} + q_{\nu 0} = -\frac{I_{\nu 0}}{l_{\nu 0}^*} + B_{\nu 0} = -\frac{I_{\nu 0} - I_{e0}}{l_{\nu 0}^*}, \tag{1.10}$$

where $l_{\nu 0}^*$ is the mean free path with allowance for the stimulated absorption:

$$l_{\nu 0}^* = l_{\nu 0}[1 + \mathscr{E}(\psi_\nu - \varepsilon_{\nu 0})]^{-1}. \tag{1.11}$$

In the chosen reference frame, in which the matter moves, we obtain with the aid of (1.2), (1.7–(1.9), (1.4), and (1.11), respectively,

$$-\frac{I_\nu}{l_\nu} + q_\nu = -\frac{I_\nu L}{l_{\nu 0}} + \frac{q_{\nu 0}}{L^2} = -\frac{1}{l_{\nu 0}^*}\left(I_\nu L - \frac{I_{e0}}{L^2}\right). \tag{1.12}$$

From this, in accordance with (1.2), we get the equilibrium intensity

$$I_e = L^{-3} I_{e0} = \frac{\varepsilon_\nu^3}{c^2 h^3}[1 + \mathscr{E}(L\varepsilon_\nu - \psi_\nu)]^{-1}. \tag{1.13}$$

We see therefore that I_e is an anisotropic function.

In the nonrelativistic case, using (1.6), we obtain from (1.13) the moments of the neutrino intensity $U_{\nu e}$, $K_{\nu e}$, and $S_{\nu e}$:

$$\begin{aligned} U_{\nu e} &= \frac{2\pi}{c}\int_{-1}^{+1} d\mu \int_0^\infty I_e d\varepsilon_\nu = \frac{2\pi}{c}\int_{-1}^{+1}\frac{d\mu}{L}\int_0^\infty I_e\left(\frac{\varepsilon_{\nu 0}}{L}\right) d\varepsilon_{\nu 0} \\ &= \frac{2\pi}{c}\int_{-1}^{+1}\frac{d\mu}{L^4}\int_0^\infty I_{e0}\, d\varepsilon_{\nu 0} = \frac{4\pi}{c}\int_0^\infty I_{e0}\, d\varepsilon_{\nu 0} \\ &= \frac{4\pi (kT)^4}{c^2 h^3}\int_0^\infty \frac{x^3\, dx}{1 + \exp(x - \psi_\nu/kT)} = 4\pi\left(\frac{kT}{ch}\right)^3 kT F_3\left(\frac{\psi_\nu}{kT}\right), \end{aligned} \tag{1.14}$$

where F_3 is a Fermi-Dirac function of third order. Analogous calculations yield

$$K_{\nu e} = \frac{2\pi}{c}\int_{-1}^{+1}\mu^2 d\mu \int_0^\infty I_e\, d\varepsilon_\nu = \frac{4\pi}{3}\left(\frac{kT}{ch}\right)^3 kT F_3\left(\frac{\psi_\nu}{kT}\right) = \frac{1}{3}U_{\nu e}, \tag{1.15}$$

$$S_{\nu e} = 2\pi\int_{-1}^{+1}\mu\, d\mu \int_0^\infty I_e\, d\varepsilon_\nu = {}^4/_3\, v U_{\nu e}. \tag{1.16}$$

As a result of the anisotropy, I_e is the nonzero density of the energy flux $S_{\nu e}$, whereas $U_{\nu e}$ and $K_{\nu e}$ are the same as in a medium at rest in the nonrelativistic approximation. At $\psi_\nu = 0$ we have[11] $U_{\nu e} = (7/16)aT^4$, where $a = (8/15)\pi^5 k^4/(ch)^3$ is the radiation-density constant.

All the preceding formulas remain valid for antineutrinos, with the only substitution

$$\psi^{\sim} = -\psi_\nu, \tag{1.17}$$

which follows from the thermodynamic condition for annihilation and pair production of particles (relation (1.17) will be justified in Sec. 3 below from the point of view of kinetics).

2. NEUTRINO THERMAL CONDUCTIVITY APPROXIMATION

This approximation is based on the fact that there exists a small parameter, namely the ratio of the neutrino mean free path to the characteristic scale of the problem. It is the approximation calculation of the small differential terms in the left-hand side of (1.1) which leads to the sought approximation[12]. In a moving medium there is also another small parameter, v/c. Later on, confining ourselves to the nonrelativistic approximation, we shall retain only terms of first order in $\beta = v/c$. The product of the velocity ratio β by the differential terms (1.1) increases the order of their smallness. We shall assume this product to be a quantity of second order of smallness if it contains derivatives with respect to the radius. We write down the transport equation (1.1), using (1.12) in the form

$$DI_\nu = -\frac{1}{l_{\nu 0}^*}\left(I_\nu L - \frac{I_{e0}}{L^2}\right), \tag{2.1}$$

where the linear differential operator is given by

$$D=\frac{1}{c}\frac{\partial}{\partial t}+D_{r\mu}=\frac{1}{c}\frac{\partial}{\partial t}+\mu\frac{\partial}{\partial r}+\frac{1-\mu^2}{r}\frac{\partial}{\partial\mu}. \tag{2.2}$$

In the zeroth approximation (at $DI_\nu = 0$) we obtain for I_ν the equilibrium solution (1.13). Substituting I_e in the left-hand side of (2.1), we obtain the first approximation:

$$I_\nu^{(1)}=I_e-\frac{l_{\nu0}^*}{L}DI_e. \tag{2.3}$$

We shall subsequently need also the second approximation:

$$I_\nu^{(2)}=I_e-\frac{l_{\nu0}^*}{L}DI_\nu^{(1)}=I_\nu^{(1)}+\frac{l_{\nu0}^*}{L}D\left(\frac{l_{\nu0}^*}{L}DI_e\right). \tag{2.4}$$

Since the difference $I_\nu^{(2)} - I_\nu^{(1)}$ is, by definition, of second order of smallness, it is necessary to neglect the effects of motion in the second term of (2.4), i.e., to put $L = 1$ and $I_e = I_{e0}$, and to omit from the operator D the derivative $c^{-1}\partial/\partial t$, which generally speaking can be regarded as a small quantity of second order. We then obtain in place of (2.4)

$$I_\nu^{(2)}=I_\nu^{(1)}+l_{\nu0}^*D_{r\mu}(l_{\nu0}^*D_{r\mu}I_{e0}). \tag{2.5}$$

We calculate $I_\nu^{(1)}$, taking into account the small correction due to the motion of the medium and retaining the derivative with respect to time. Applying the operator D to I_e, we must bear in mind that all the differentiation operators should be carried out at a constant value of ϵ_ν. Therefore

$$DI_e=-I_e[1+\mathscr{E}(\psi_\nu-L\varepsilon_\nu)]^{-1}D\left(\frac{L\varepsilon_\nu-\psi_\nu}{kT}\right). \tag{2.6}$$

Taking into account (1.6) and the fact that the derivative $c^{-1}\partial/\partial t$ is of second order of smallness, we obtain

$$D\left(\frac{L\varepsilon_\nu-\psi_\nu}{kT}\right)=L\varepsilon_\nu D\left(\frac{1}{kT}\right)-D\left(\frac{\psi_\nu}{kT}\right)-\frac{\varepsilon_\nu}{kT}D_{r\mu}\left(\frac{v}{c}\mu\right). \tag{2.7}$$

The final expression for $I_\nu^{(1)}$ as a function of $\epsilon_{\nu0}$, in accordance with (2.3), (2.6), (2.7), and (1.4), is

$$I_\nu^{(1)}=\frac{I_{e0}}{L^3}+\frac{I_{e0}\,l_{\nu0}^*}{L^4[1+\mathscr{E}(\psi_\nu-\varepsilon_{\nu0})]}\left[\varepsilon_{\nu0}D\left(\frac{1}{kT}\right)-D\left(\frac{\psi_\nu}{kT}\right)-\frac{\varepsilon_{\nu0}}{kT}D_{r\mu}\left(\frac{v}{c}\mu\right)\right] \tag{2.8}$$

In analogy with (1.14)–(1.16), we find the moments of the intensity $I^{(1)}$ by integrating subsequently with respect to $\epsilon_{\nu0}$:

$$U_\nu^{(1)}=\frac{2\pi}{c}\int_{-1}^{+1}d\mu\int_0^\infty I_\nu^{(1)}\,d\varepsilon_\nu=\frac{2\pi}{c}\int_0^\infty d\varepsilon_{\nu0}\int_{-1}^{+1}I_\nu^{(1)}\frac{d\mu}{L},$$

$$K_\nu^{(1)}=\frac{2\pi}{c}\int_0^\infty d\varepsilon_{\nu0}\int_{-1}^{+1}\frac{\mu^2}{L}I_\nu^{(1)}\,d\mu,$$

$$S_\nu^{(1)}=2\pi\int_0^\infty d\varepsilon_{\nu0}\int_{-1}^{+1}\frac{\mu}{L}I_\nu^{(1)}\,d\mu. \tag{2.9}$$

The first integration with respect to the angle in (2.9), with L taken from (1.6), is carried through to conclusion. It remains to integrate with respect to the energy $\epsilon_{\nu0}$, which can be done only if $l_{\nu0}^*$ is known as a function of $\epsilon_{\nu0}$. However, the term in excess of the equilibrium term is important only for the energy flux $S_\nu^{(1)}$, whereas in the moments $U_\nu^{(1)}$ and $K_\nu^{(1)}$ the integrands are small quantities of second order ($\sim l_{\nu0}^*/(ct_{\text{char}})$ $\sim l_0^* r_{\text{char}}^{-1}$, there t_{char} is the characteristic time). If they are substituted in the gasdynamics equations, they yield terms of the viscosity type. The ratio of the viscosity terms to the pressure is thus of second order of smallness. We shall therefore retain in the gasdynamics equations the equilibrium values (1.14) and (1.15) in place of $U_\nu^{(1)}$ and $K_\nu^{(1)}$. Thus, in first-order approximation we have from (2.9)

$$U_\nu^{(1)}=U_{\nu e},\quad K_\nu^{(1)}={}^1/_3\,U_{\nu e},$$

$$S_\nu^{(1)}=\frac{4}{3}vU_{\nu e}-\frac{4\pi}{3}\int_0^\infty l_{\nu0}^*\left(\frac{\partial I_{e0}}{\partial r}\right)_{\varepsilon_{\nu0}}d\varepsilon_{\nu0}=\frac{4}{3}vU_{\nu e}+H_\nu. \tag{2.10}$$

To abbreviate the notation, we use in the expression for the energy flux H_ν the derivative $(\partial I_{e0}/\partial r)_{\epsilon_{\nu0}}$ at a constant value of $\epsilon_{\nu0}$.

It is easy to write down the neutrino gasdynamics equations in the general relativistic case by using the energy-momentum conservation law for matter with neutrino (and antineutrino) radiation and by introducing the energy-momentum tensors T_{ik} and W_{ik} for the matter and for the radiation[13]. In the case of an ideal gas and using the known expressions for the components W_{ik} in terms of the radiation intensity (at zero rest mass of ν and $\bar{\nu}$ we have here an analogy with the photon), we can transform to a convenient system of equations that includes the equations of motion and the entropy equation[14]. Finally, in the spherically-symmetrical nonrelativistic case of interest to us, by transforming the derivatives of the tensor (see Kochin's book[15]) we reduce these equations to the form

$$\rho\frac{dv}{dt}=-\frac{\partial P}{\partial r}+\left[-\frac{\partial K}{\partial r}-\frac{1}{r}(3K-U)-\frac{1}{c^2}\frac{\partial S}{\partial t}\right]-\frac{Gm}{r^2}\rho, \tag{2.11}$$

$$\rho\left[\frac{d}{dt}\left(\frac{E}{\rho}\right)+P\frac{d}{dt}\left(\frac{1}{\rho}\right)\right]=-v\left[-\frac{\partial K}{\partial r}-\frac{1}{r}(3K-U)-\frac{1}{c^2}\frac{\partial S}{\partial t}\right]+\left[-\frac{1}{r^2}\frac{\partial}{\partial r}(r^2S)-\frac{\partial U}{\partial t}\right], \tag{2.12}$$

where P, E, and ρ are the pressure, internal energy, and the density of the ideal gas, $d/dt = \partial/\partial t + v\partial/\partial r$, and the integral moments of the intensities of the neutrino (and antineutrino) radiation are

$$U=\frac{2\pi}{c}\int_{-1}^{+1}d\mu\int_0^\infty I_\nu\,d\varepsilon_\nu,\quad K=\frac{2\pi}{c}\int_{-1}^{+1}\mu^2\,d\mu\int_0^\infty I_\nu\,d\varepsilon_\nu,$$

$$S=2\pi\int_{-1}^{+1}\mu\,d\mu\int_0^\infty I_\nu\,d\varepsilon_\nu. \tag{2.13}$$

The equation of motion (2.11) includes the Newtonian force, and m is the mass inside a sphere of radius r.

After substituting (2.10). (1.14), and (1.15) in (2.11) and (2.12), or more accurately, after substituting the values for ν and $\bar{\nu}$ with allowance for (17), neglecting the term $c^{-2}\partial S/\partial t$, which is of second order of smallness relative to the term $\partial K/\partial r$, and changing over to the Lagrangian coordinates t and m, we obtain the system of equations[4)]

$$\frac{\partial r}{\partial t}=v,\quad \frac{\partial r^3}{\partial m}=\frac{3}{4\pi\rho},$$

[4)]It is of interest to note the high accuracy with which neutrino radiation is described in these equations. Second-order corrections have been discarded in all the terms that take radiation into account. It can be shown that calculation of the neutrino-radiation intensity (2.5) to second order does not change this conclusion concerning the accuracy.

$$\frac{\partial v}{\partial t}=-4\pi r^2\frac{\partial}{\partial m}(P+K)-\frac{Gm}{r^2},$$
$$\frac{\partial}{\partial t}\left(\frac{E+U}{\rho}\right)+(P+K)\frac{\partial}{\partial t}\left(\frac{1}{\rho}\right)=-4\pi\frac{\partial}{\partial m}(r^2H), \quad (2.14)$$

where the moments of the radiation of ν and $\tilde{\nu}$ are

$$H=-\frac{7}{8}\frac{4acT^3}{3}4\pi r^2\rho\left(l_T\frac{\partial T}{\partial m}+Tl_\psi\frac{\partial \psi_\nu'}{\partial m}\right),$$
$$K=\frac{1}{3}U=\frac{15}{24\pi^4}aT^4\left[(\psi_\nu')^4+2\pi^2(\psi_\nu')^2+\frac{7\pi^4}{15}\right],$$
$$l_T=l_{T\nu}+l_{T\tilde{\nu}},\ l_\psi=l_{\psi\nu}-l_{\psi\tilde{\nu}},$$
$$l_{T\nu}=\frac{15}{7\pi^4}\int_0^\infty l_{\nu 0}\frac{x^4\exp\{2(x-\psi_\nu')\}}{(1+\exp\{x-\psi_\nu'\})^3}dx,$$
$$l_{\psi\nu}=\frac{15}{7\pi^4}\int_0^\infty l_{\nu 0}\frac{x^3\exp\{2(x-\psi_\nu')\}}{(1+\exp(x-\psi_\nu'))^3}dx, \qquad \psi_\nu'=\psi_\nu/kT. \quad (2.15)$$

The expressions for $l_{T\tilde{\nu}}$ and for $l_{\psi\tilde{\nu}}$, which correspond to the contribution of the antineutrino to the energy transport, are obtained by replacing $l_{\nu 0}$ by $l_{\tilde{\nu}0}$ and ψ_ν' by $-\psi_{\tilde{\nu}}'$ in the integrals. The integrals in (2.15) are analogs of the Rosseland averaging of the mean free path. The system of neutrino gasdynamics equations (2.14) and (2.15) would be complete were it not for the presence of the unknown function ψ_ν.

3. KINETICS OF β PROCESSES FOR EQUILIBRIUM DISTRIBUTION OF COMPLEX NUCLEI

For an arbitrary mixture of complex nuclei and nucleons, it would be necessary to write down a system of kinetic equation, the number of which would be equal to the number of types of particles. But the tremendous difference between the reaction rates due to strong and weak interactions[5)] enables us to describe the process by simpler relations. The concentration of all the complex nuclei (A, Z) whose distribution is determined by strong interactions depends in equilibrium fashion on the concentrations of the free nucleons[16,17]:

$$n_{A,Z}=\left(\frac{2\pi Am_pkT}{h^2}\right)^{3/2}\left(\frac{h^2}{2\pi m_pkT}\right)^{3A/2}\frac{n_n^{A-Z}n_p^Z}{2^A}\exp\frac{Q_{A,Z}}{kT}, \quad (3.1)$$

where the binding energies are given by

$$Q_{A,Z}=(A-Z)m_nc^2+Zm_pc^2-m_{A,Z}c^2. \quad (3.2)$$

At a specified number of baryons ρ/m_p, all that is left unknown in the system is then the ratio of the n and p concentrations, including their fraction in the composition of the nuclei

$$\vartheta=\frac{N_n}{N_p}=\frac{n_n+\Sigma(A-Z)n_{A,Z}}{n_p+\Sigma Zn_{A,Z}}. \quad (3.3)$$

The quantity ϑ can change only as a result of β processes. In a moving medium, these processes must be considered in the proper coordinate frame.

It is easily understood that the kinetic equation with allowance for all the possible β processes takes the form[6)]

$$-\rho\frac{d}{dt}\left(\frac{N_n}{\rho}\right)=\rho\frac{d}{dt}\left(\frac{N_p}{\rho}\right)=-\frac{\rho}{m_p}\frac{1}{(1+\vartheta)^2}\frac{d\vartheta}{dt}$$
$$=-n_p(W_{\text{а}}{}^{11}+W_{\text{б}'}{}^{11}+W_{\text{в}'}{}^{11})+n_n(W_{\text{б}}{}^{01}+W_{\text{в}}{}^{01}+W_{\text{а}'}{}^{01})$$
$$+\sum n_{A,Z}(-W_{\text{а}}{}^{ZA}+W_{\text{б}}{}^{ZA}+W_{\text{в}}{}^{ZA}-W_{\text{г}}{}^{ZA}+W_{\text{а}'}{}^{ZA}-W_{\text{б}'}{}^{ZA}-W_{\text{в}'}{}^{ZA}+W_{\text{г}'}{}^{ZA}), \quad (3.4)$$

where the summation extends over all possible complex nuclei, including their excited and spin states. The total probabilities W_i^{ZA} of the β processes pertain to the nucleus (A, Z). The letter subscripts denote the type of the β process, in the primed processes the leptons ν and $\tilde{\nu}$ are absorbed by the nucleus (A, Z), and in the unprimed ones they are emitted. According to classical theory of allowed β processes[9], we write down the corresponding differential probabilities $w_i^{ZA}(W_i^{AZ}=\int w_i^{ZA}d\epsilon d\omega)$:

a) $e^-+(A,Z)\to(A,Z-1)+\nu,\ w_{\text{а}}^{ZA}=G_{Z,Z-1}\Omega_eN_e\Omega_\nu(1-N_\nu),$
$\varepsilon_e=\varepsilon_\nu-\varepsilon_{Z,Z-1};$

b) $e^++(A,Z)\to(A,Z+1)+\tilde{\nu},\ w_{\text{б}}^{ZA}=G_{Z,Z+1}\Omega_{\tilde{e}}N_{\tilde{e}}\Omega_{\tilde{\nu}}(1-N_{\tilde{\nu}}),$
$\varepsilon_{\tilde{e}}=\varepsilon_{\tilde{\nu}}-\varepsilon_{Z,Z+1};$

c) $(A,Z)\to e^-+\tilde{\nu}+(A,Z+1),\ w_{\text{в}}^{ZA}=G_{Z,Z+1}\Omega_e(1-N_e)\Omega_{\tilde{\nu}}(1-N_{\tilde{\nu}}),$
$\varepsilon_e=-\varepsilon_{\tilde{\nu}}+\varepsilon_{Z,Z+1};$

d) $(A,Z)\to e^++\nu+(A,Z-1),\ w_{\text{г}}^{ZA}=G_{Z,Z-1}\Omega_{\tilde{e}}(1-N_{\tilde{e}})\Omega_\nu(1-N_\nu),$
$\varepsilon_{\tilde{e}}=-\varepsilon_\nu+\varepsilon_{Z,Z-1};$

a′) $\nu+(A,Z)\to(A,Z+1)+e^-,\ w_{\text{а}'}^{ZA}=G_{Z,Z+1}\Omega_e(1-N_e)\Omega_\nu N_\nu,$
$\varepsilon_e=\varepsilon_\nu+\varepsilon_{Z,Z+1};$

b′) $\tilde{\nu}+(A,Z)\to(A,Z-1)+e^+,\ w_{\text{б}'}^{ZA}=G_{Z,Z-1}\Omega_{\tilde{e}}(1-N_{\tilde{e}})\Omega_{\tilde{\nu}}N_{\tilde{\nu}},$
$\varepsilon_{\tilde{e}}=\varepsilon_{\tilde{\nu}}+\varepsilon_{Z,Z-1};$

c′) $\tilde{\nu}+e^-+(A,Z)\to(A,Z-1),\ w_{\text{в}'}^{ZA}=G_{Z,Z-1}\Omega_eN_e\Omega_{\tilde{\nu}}N_{\tilde{\nu}},$
$\varepsilon_e=|\varepsilon_{Z,Z-1}|-\varepsilon_{\tilde{\nu}};$

d′) $\nu+e^++(A,Z)\to(A,Z+1),\ w_{\text{г}'}^{ZA}=G_{Z,Z+1}\Omega_{\tilde{e}}N_{\tilde{e}}\Omega_\nu N_\nu,$
$\varepsilon_{\tilde{e}}=|\varepsilon_{Z,Z+1}|-\varepsilon_\nu.$ (3.5)

Equation (3.5) contains matrix elements, since $G_{Z,Z\pm1}=4\pi^2h^{-1}|\{H'\}|^2_{Z,Z\pm1}$, where

$$|\{H'\}|_i^2=\frac{m_ec^2}{128\pi^4c}\left(\frac{h}{m_ec}\right)^7\frac{\ln 2}{(ft)_i}F_i(\varepsilon_{e,\tilde{e}})$$

is expressed in terms of the comparative half-life $(ft)_i$. (Obviously, $G_{Z,Z\pm1}=G_{Z\pm1,Z}$). The lepton phase factors and the equilibrium distribution functions of the electrons (e) and of the positrons ($\tilde{e}$) are given by

$$\Omega_{e,\tilde{e}}=\frac{8\pi}{(hc)^3}\varepsilon_{e,\tilde{e}}\sqrt{\varepsilon^2_{e,\tilde{e}}-m_e^2c^4},\ \Omega_{\nu,\tilde{\nu}}=\frac{\varepsilon^2_{\nu,\tilde{\nu}}}{(hc)^3}, \quad (3.6)$$
$$N_e=[1+\mathscr{E}(\varepsilon_e-\psi_e)]^{-1},\ N^{\sim}=[1+\mathscr{E}(\varepsilon_{\tilde{\nu}}+\psi_e)]^{-1}, \quad (3.7)$$

where the function $\mathscr{E}(x)$ is defined in (1.8′). Formulas (3.5) include also the connection between the energy of the lepton e or $\tilde{e}$ with the energy of the lepton ν or $\tilde{\nu}$ with

$$\varepsilon_{Z,Z\pm1}=(m_{A,Z}-m_{A,Z\pm1})c^2=Q_{A,Z\pm1}-Q_{A,Z}\pm\varepsilon_{np}. \quad (3.8)$$

The energy conservation law allows at most six β processes with a given nucleus (A, Z) from (3.5),

5)At temperatures $T > 3\times10^9$ K, the role of the Coulomb barrier in nuclear reactions diminishes and the difference between the intensities of the strong and weak interactions comes fully into play [1]. This leads to an equilibrium between the direct and inverse nuclear reactions.

6)The problem of the kinetic analysis of β processes was posed in [18]. The most important question here is a direct comparison of the characteristic times of the strong and weak interactions. Although it is impossible to solve these problems exhaustively for all types of nuclear reactions and β transformations, a comparison of the time in principle is given in [19].

since we must have $\epsilon_{e,\tilde{e}} \geq m_e c^2$. For nucleons, three of these processes are actually realized for each. If the only difference between processes is due to a prime in the subscript, then these processes are mutually reversible. For nucleons this is seen directly, and for complex nuclei it is possible to make in (3.5) the substitutions $Z \to Z-1$ (a′, d′) and $Z \to Z+1$(b′, c′).

We introduce the mean free path of ν and $\tilde{\nu}$ with respect to absorption in the β process

$$l_i^{ZA} = [n_{A,Z}\sigma_i^{ZA}(\varepsilon_{\nu,\tilde{\nu}})]^{-1} = (n_{A,Z}w_i^{ZA})^{-1}c\Omega_{\nu,\tilde{\nu}}N_{\nu,\tilde{\nu}}$$

Taking (3.5) into account, we can group l_i^{ZA} for the different β processes into two classes

$$l_{a',b'}^{ZA} = \frac{c}{n_{A,Z}G_{Z,Z\pm1}\Omega_{e,\tilde{e}}(1-N_{\tilde{e},e})}, \quad l_{c',d'}^{ZA} = \frac{c}{n_{A,Z}G_{Z,Z\mp1}\Omega_{e,\tilde{e}}N_{e,\tilde{e}}}. \tag{3.9}$$

With the aid of (3.9) we can group pairwise into a single type the direct and inverse β processes in (3.4). Using the shift operation in (3.5), the equilibrium distributions of e and $\tilde{e}$ from (3.7), the expressions for $\epsilon_{e,\tilde{e}}$ in terms of $\epsilon_{\nu,\tilde{\nu}}$ in (3.5), as well as (3.1) and (3.8), we obtain

$$n_{A,Z}w_{a',b'}^{ZA} - n_{A,Z+1}w_{a,d}^{Z+1,A} = -\frac{c\Omega_\nu}{l_{a',b'}^{ZA}}\{\mathscr{E}(\psi_\nu-\varepsilon_\nu) - N_\nu[(1+\mathscr{E}(\psi_\nu-\varepsilon_\nu)]\},$$

$$-n_{A,Z}w_{b',c'}^{ZA} + n_{A,Z-1}w_{b',c'}^{Z-1,A} = \frac{c\Omega_{\tilde{\nu}}}{l_{b',c'}^{ZA}}\{\mathscr{E}(-\psi_\nu-\varepsilon_{\tilde{\nu}}) - N_{\tilde{\nu}}[1+\mathscr{E}(-\psi_\nu-\varepsilon_{\tilde{\nu}})]\}. \tag{3.10}$$

The two possible processes of ν absorption are combined in the first line of (3.10), and those for $\tilde{\nu}$ in the second. We have introduced in (3.10) the symbol

$$\psi_\nu = \psi_e - \varepsilon_{np} - kT\ln(n_n/n_p). \tag{3.11}$$

If the energy conservation law forbids any β process, then we must assume that the corresponding $l_i \to \infty$. By substituting (3.10) in the right-hand side of (3.4) and integrating with respect to $\epsilon_{\nu,\tilde{\nu}}$ and ω, we obtain in the proper reference frame (the limit of integration with respect to energy are established by the energy conservation law)

$$-\frac{\rho}{m_p}\frac{1}{(1+\vartheta)^2}\frac{d\vartheta}{dt} = -\iint\frac{c\Omega_{\nu0}}{l_{\nu0}^\beta}\{\mathscr{E}(\psi_\nu-\varepsilon_{\nu0}) - N_\nu[1+\mathscr{E}(\psi_\nu-\varepsilon_{\nu0})]\}$$
$$\times d\varepsilon_{\nu0}d\omega_0 + \iint\frac{c\Omega_{\tilde{\nu}0}}{l_{\tilde{\nu}0}^\beta}\{\mathscr{E}(-\psi_\nu-\varepsilon_{\tilde{\nu}0}) - N_{\tilde{\nu}}[1+\mathscr{E}(-\psi_\nu-\varepsilon_{\tilde{\nu}0})]\}\,d\varepsilon_{\tilde{\nu}0}d\omega_0. \tag{3.12}$$

The necessary summation over all different pairs of nuclei has led to the appearance in (3.12) of the total mean free paths of ν and $\tilde{\nu}$ with respect to absorption in the β processes:

$$\frac{1}{l_{\nu0}^\beta} = \Sigma\left(\frac{1}{l_{a'}^{ZA}} + \frac{1}{l_{d'}^{ZA}}\right), \quad \frac{1}{l_{\tilde{\nu}0}^\beta} = \Sigma\left(\frac{1}{l_{b'}^{ZA}} + \frac{1}{l_{c'}^{ZA}}\right). \tag{3.13}$$

We note that, generally speaking, the following inequalities hold

$$l_{\nu0}^\beta \gg l_{\nu0}, \quad l_{\tilde{\nu}0}^\beta \gg l_{\tilde{\nu}0}, \tag{3.14}$$

because the possible mechanisms for absorption (and scattering) of ν and $\tilde{\nu}$ are not connected with the β transformations of the nucleons.

From (3.12) we determine the necessary and sufficient conditions for complete thermodynamic equilibrium. The equilibrium distribution functions of ν and $\tilde{\nu}$ from (1.8) and (1.9) cause the two curly brackets under the integral signs in (3.12) to vanish only if the chemical potential of the neutrino satisfies relation (3.11). We note that the definition of the chemical potential of the antineutrino (1.17) is by the same token kinetically justified.

4. DIFFUSION EQUATION FOR LEPTON CHARGE

To obtain the missing equation for the chemical potential of the neutrino, we use the approximation of the neutrino thermal conductivity in the kinetic equation (3.12).

With the aid of (1.2) and (1.8), (2.3), and (2.5), we represent the function N_ν, taken in the second approximation, only in terms of quantities in the proper reference frame:

$$N_\nu = N_{e0} - \frac{c^2h^3}{\varepsilon_{\nu0}^3}L^2 l_{\nu0}^*[D(I_{e0}L^{-3}) - D_{r\mu}(l_{\nu0}^* D_{r\mu}I_{e0})]. \tag{4.1}$$

Substituting N_ν from (4.1) and an analogous expression for $N_{\tilde{\nu}}$ into (3.12) we obtain, assuming for simplicity the equal sign in (3.14) and taking (1.8), (1.11), and (3.6) into account,

$$\frac{\rho}{m_p}\frac{1}{(1+\vartheta)^2}\frac{d\vartheta}{dt} = 2\pi\int_{-1}^{+1}d\mu_0\int_0^\infty d\varepsilon_{\nu0}\frac{L^2}{\varepsilon_{\nu0}}[D(I_{e0}L^{-3}) - D_{r\mu}(l_{\nu0}^* D_{r\mu}I_{e0})]$$
$$-2\pi\int_{-1}^{+1}d\mu_0\int_0^\infty d\varepsilon_{\tilde{\nu}0}\frac{L^2}{\varepsilon_{\tilde{\nu}0}}[D(I_{e0}'L^{-3}) - D_{r\mu}(l_{\tilde{\nu}0}^* D_{r\mu}I_{e0}')], \tag{4.2}$$

where I_{e0}' is the equilibrium intensity of the antineutrino. Integrating in (4.2) with respect to μ_0 by the method described in Sec. 2, and introducing the equilibrium concentrations of ν and $\tilde{\nu}$,

$$n_{\nu e} = \frac{4\pi}{c}\int_0^\infty\frac{d\varepsilon_{\nu0}}{\varepsilon_{\nu0}}I_{e0}, \quad n_{\tilde{\nu}e} = \frac{4\pi}{c}\int_0^\infty\frac{d\varepsilon_{\tilde{\nu}0}}{\varepsilon_{\tilde{\nu}0}}I_{e0}', \tag{4.3}$$

we arrive at the equation

$$\frac{\rho}{m_p}\frac{1}{(1+\vartheta)^2}\frac{d\vartheta}{dt} = \rho\frac{d}{dt}\left(\frac{n_{\nu e}-n_{\tilde{\nu}e}}{\rho}\right) - \frac{4\pi}{3}\int_0^\infty\frac{d\varepsilon_{\nu0}}{\varepsilon_{\nu0}}\frac{1}{r^2}\frac{\partial}{\partial r}$$
$$\times\left(r^2 l_{\nu0}^*\frac{\partial I_{e0}}{\partial r}\right) + \frac{4\pi}{3}\int_0^\infty\frac{d\varepsilon_{\tilde{\nu}0}}{\varepsilon_{\tilde{\nu}0}}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 l_{\tilde{\nu}0}^*\frac{\partial I_{e0}'}{\partial r}\right). \tag{4.4}$$

Just as in (2.10), $\epsilon_{\nu0}$ and $\epsilon_{\tilde{\nu}0}$ must be regarded as constants in the derivatives $\partial I_{e0}/\partial r$ and $\partial I_{e0}'/\partial r$.

It is remarkable that the entire aggregate of the first derivatives of (4.4) can be represented in the form of a derivative of the lepton charge density. We introduce the definition of the lepton charge density

$$\mathscr{L} = \frac{1}{\rho}(n_e + n_{\nu e} - n_{\tilde{e}} - n_{\tilde{\nu}e}). \tag{4.5}$$

Taking into account (3.4) as well as the electroneutrality conditions $N_p = n_e - n_{\tilde{e}}$, which can be used to determine the chemical potential ψ_e of the electrons, we obtain from (4.4) the lepton-charge diffusion equation

$$\frac{d\mathscr{L}}{dt} = -\frac{1}{\rho r^2}\frac{\partial}{\partial r}[r^2(\Lambda_\nu - \Lambda_{\tilde{\nu}})], \tag{4.6}$$

where

$$\Lambda_\nu = -\frac{4\pi}{3}\int_0^\infty\frac{d\varepsilon_{\nu0}}{\varepsilon_{\nu0}}l_{\nu0}^*\frac{\partial I_{e0}}{\partial r}, \quad \Lambda_{\tilde{\nu}} = -\frac{4\pi}{3}\int_0^\infty\frac{d\varepsilon_{\tilde{\nu}0}}{\varepsilon_{\tilde{\nu}0}}l_{\tilde{\nu}0}^*\frac{\partial I_{e0}'}{\partial r}. \tag{4.7}$$

Changing over to the Lagrangian coordinates t and m we obtain from (4.6)

$$\frac{\partial \mathscr{L}}{\partial t}+4\pi\frac{\partial}{\partial m}(r^2\Lambda)=0. \tag{4.8}$$

The flux and density of the lepton charge are given, on the basis of (4.3), (4.5), (4.7), and (1.8), by

$$\Lambda=-\frac{7}{8}\frac{4acT^3}{3k}4\pi r^2\rho\left(\frac{\lambda_T}{T}\frac{\partial T}{\partial m}+\lambda_\psi\frac{\partial\psi_\nu'}{\partial m}\right),$$
$$\mathscr{L}=\frac{1}{\rho}\left\{n_e-n_{\tilde e}+\frac{4\pi}{3}\left(\frac{kT}{hc}\right)^3[(\psi_\nu')^3+\pi^2\psi_\nu']\right\},$$
$$\lambda_T=\lambda_{T\nu}-\lambda_{T\tilde\nu},\quad \lambda_\psi=\lambda_{\psi\nu}+\lambda_{\psi\tilde\nu},$$
$$\lambda_{T\nu}=\frac{15}{7\pi^4}\int_0^\infty l_{\nu0}\frac{x^3\exp\{2(x-\psi_\nu')\}}{(1+\exp\{x-\psi_\nu'\})^3}dx, \tag{4.9}$$
$$\lambda_{\psi\nu}=\frac{15}{7\pi^4}\int_0^\infty l_{\nu0}\frac{x^2\exp\{2(x-\psi_\nu')\}}{(1+\exp\{x-\psi_\nu'\})^3}dx.$$

The expression for $\lambda_{T\tilde\nu}$ and $\lambda_{\psi\tilde\nu}$ are obtained from those in (4.9) by the procedure used in the discussion of (2.15). A comparison of (2.15) with (4.9) leads to the Onsager relation[14] $l_\psi = l_T$, which can be regarded as a check on the correctness of the given kinetic analysis. Equation (4.8) completes the system of neutrino gasdynamics equations in the thermal-conductivity approximation.

It is of interest to dwell on the character of the approximation used to derive (4.8). The lepton-charge flux Λ is proportional, in analogy with the energy flux H, to a small parameter raised to the first power. Nonetheless, to derive (4.8) we need the neutrino distribution function N_ν in the second approximation. The reason is that one order of smallness in N_ν from (4.1) is compensated for by the presence of the neutrino mean free path in the denominators of the integrand of (3.12). Notice also the high accuracy of (4.8), where the discarded terms are of second order of smallness relative to those retained (see footnote 4 above).

5. BOUNDARY CONDITIONS IN THE NEUTRINO THERMAL CONDUCTIVITY APPROXIMATION

Neutrino opacity sets in only in the central core of the star, where the temperature and density are both high. The shell of the star, which has an incomparably larger radial dimension, is transparent of neutrinos and antineutrinos. It is therefore necessary to consider the neutrino core of the star and formulate external boundary conditions for Eqs. (2.14) and (4.8) in a certain intermediate value of the Lagrangian coordinate M, which certainly includes the entire opaque region, but still does not subtend over the greater part of the transparent shell. It is clear that M itself has an arbitrary meaning, but errors in its choice lead to peculiar results. An overestimate of M leads to an artificial isothermal region in the envelope, and an underestimate leads to a sharp temperature gradient near the boundary M. The external boundary conditions are the consequence of the absence of neutrino-radiation sources outside the central core of the star. The condition for the vanishing of the external ν and $\tilde\nu$ fluxes on the moving boundary m = M in the proper reference frame is

$$2\pi\int_0^\infty d\varepsilon_0\int_{-1}^0\mu_0(I_{\nu0}+I_{\tilde\nu0})\,d\mu_0=0,\quad 2\pi\int_0^\infty d\varepsilon_0\int_{-1}^0\mu_0\left(\frac{I_{\nu0}}{\varepsilon_{\nu0}}-\frac{I_{\tilde\nu0}}{\varepsilon_{\tilde\nu0}}\right)d\mu_0=0. \tag{5.1}$$

Substitution of the functions $I_{\nu0}$ from (2.8) with allowance for (1.2) and subsequent integration with respect to μ_0 by the method used in Sec. 2, leads to two conditions:

$$-\frac{c}{4}U+\frac{1}{2}H=0,\quad -\frac{c}{4}n+\frac{1}{2}\Lambda=0, \tag{5.2}$$

where

$$n=n_{\nu e}-n_{\tilde\nu e}=\frac{15}{6\pi^4}\frac{aT^3}{k}[(\psi_\nu')^3+\pi^2\psi_\nu']. \tag{5.3}$$

In the theory of photon thermal conductivity, the first of the two boundary conditions (5.2) is used on the outer surface of the star[20]. The total aggregate of the boundary conditions in the neutrino thermal conductivity approximation includes, besides (5.2), the conditions at the center of the star (the absence of pointlike sources):

$$\partial T/\partial r=0,\quad \partial\psi_\nu/\partial r=0,\quad m=0. \tag{5.4}$$

The function r and v, as well as the total pressure P + K, obey the usual gasdynamic conditions (r = 0 and v = 0 at m = 0, continuity of all the functions at m = M).

6. CASE OF HIGH TEMPERATURES

At densities and temperatures that are typical of collapsing stars[3] it is possible to simplify considerably the picture of the β processes. In the temperature interval 60×10^9 – 200×10^9°K there are practically no complex nuclei[17,21]. In matter consisting of free nucleons, the main β processes are (see[22])

$$e^-+p\rightleftarrows n+\nu\ \text{(for a, a')},\quad e^++n\rightleftarrows p+\tilde\nu\ \text{(for b, b')}. \tag{6.1}$$

In the case $m_ec^2/kT = 5.9T_9^{-1} \ll 1$ we can neglect the finite rest energy of the electron m_ec^2 and the difference between the rest energies of the nucleons $\epsilon_{np} = 2.53m_ec^2$. Then, according to (3.9) and (3.6), we have

$$l_{a'n}=\frac{m_p}{\sigma_0}\frac{1+\vartheta}{\vartheta}\frac{1}{\rho}\left(\frac{m_ec^2}{\varepsilon_\nu}\right)^2\frac{1}{1-N_e}, \tag{6.2}$$
$$l_{b'p}=\frac{m_p}{\sigma_0}\frac{1+\vartheta}{\rho}\left(\frac{m_ec^2}{\varepsilon_{\tilde\nu}}\right)^2\frac{1}{1-N_{\tilde e}}, \tag{6.3}$$

where

$$\sigma_0=\frac{1}{4\pi c}\left(\frac{h}{m_ec}\right)^3\frac{\ln 2}{(ft)_n}\approx 2\cdot10^{-44}\,\text{cm}^2. \tag{6.4}$$

In accordance with (3.3) and (3.2), we have the relation

$$\vartheta=\frac{n_n}{n_p}=\mathscr{E}(\psi_e-\psi_\nu)=\mathscr{E}(\psi_e)\exp(-\psi_\nu'). \tag{6.5}$$

Calculation of the averaged mean free paths by means of formulas (2.15) and (4.9), with the function ψ_e eliminated with the aid of (6.5) yields

$$l_T=B(1+\vartheta)\left\{-\frac{\vartheta-1}{\vartheta}\psi_\nu'+\frac{1+\vartheta}{2\vartheta}\left[(\psi_\nu')^2+\frac{\pi^2}{3}\right]\right\},$$
$$l_\psi=\lambda_T=\frac{1}{2}B\frac{(1+\vartheta)^2}{\vartheta}\left(\psi_\nu'-\frac{\vartheta-1}{\vartheta+1}\right);\quad \lambda_\psi=\frac{1}{2}B\frac{(1+\vartheta)^2}{\vartheta},$$
$$B=\frac{15}{7\pi^4}\frac{m_p}{\sigma_0}\frac{1}{\rho}\left(\frac{m_ec^2}{kT}\right)^2. \tag{6.6}$$

An essential property of the mean free paths is their decrease with rising temperature. At temperatures $\gtrsim 200\times10^9$°K, enough μ and π mesons appear. The problem becomes complicated, since the mesons interact with the nucleons and change their concentrations. In addition, muonic neutrinos are emitted[4], to which the star likewise soon becomes opaque[23]. It is then necessary to consider one more diffusion equation for the muonic lepton decay, and to modify the previously introduced diffusion equation for the electronic lepton charge. An attempt to generalize the theory in this manner was made recently[24].

We mention also a paper[25] that considers the influence of the Pauli principle for neutrinos on the volume luminosity of the star. It can be concluded from this paper that this effect takes place when the optical neutrino thickness is $\tau_\nu \gtrsim 1$. In such a case it is necessary to take into account the neutrino absorption reactions when concrete problems are solved. A sufficiently good approximation at $\tau_\nu > 1$ is the neutrino thermal conductivity approximation considered above.

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