PLASMA EFFECTS IN STIMULATED COMPTON INTERACTION BETWEEN MATTER AND RADIATION

A. A. GALEEV and R. A. SYUNYAEV

Institute of Applied Mathematics, USSR Academy of Sciences

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Effects connected with stimulated Compton interaction between matter and radiation (plasma heating, distortion of the radiation spectrum, and induced light pressure) may occur at high plasma densities when the spectral radiation line width is smaller than the plasma frequency. The efficiencies of the processes are calculated as functions of plasma density and temperature and of the angular aperture of the radiation beam. It is shown that the Kompaneets differential equation describing Compton interaction between matter and spectrally broad isotropic radiation is valid in the low frequency region down to the plasma frequency. The upper limit of the brightness temperature of the radiation from cosmic masers, which is due to the line deviating from resonance via induced Compton interaction with the plasma, is found (a similar analysis is applicable to gasdynamic lasers in principle).

ALTHOUGH the probabilities of induced Compton scattering, both by free thermal electrons[1] and with allowance for screening of the charges in a plasma[2,3], were calculated relatively long ago, a detailed investigation of the physical effects described by the obtained equations has begun only most recently. Important among these effects are the heating of the electrons[4-8] and ions[9,10], the appearance of an induced force due to the pressure of light[11], distortion of the emission spectrum[12], the appearance of narrow spectral lines (solitons) in the initially broad emission spectrum[13], and evolution of spectrally narrow lines (narrowing, broadening, or drift of the line along the frequency axis, depending on its spectral profile)[7]. The qualitative picture of the evolution of the emission lines is common to the processes of induced scattering of different oscillation modes, and therefore some of the described effects were obtained earlier for the case of induced scattering of Langmuir oscillations[14,15].

The indicated effects have been widely discussed both in connection with problems of interaction of laser radiation with matter[4,6-10] and in the analysis of processes occurring in astrophysical objects, primarily in pulsars and quasars[16-18]. It should be noted that most papers (with the exception of[9,10]) deal with low-density plasmas, when the screening of the charges can be neglected. The latter takes place if the spectral width δ of the emission line (or the Doppler width $\nu_D = \nu\sqrt{2T_e/m_ec^2}$, since $\delta \gg \nu_D$ for a "broad" spectrum) exceeds the Langmuir frequency $\nu_{pe} = \sqrt{e^2N_e/\pi m_e}$, i.e., at $N_e \ll \pi m_e\delta^2/e^2$. In such a rarefied plasma, the effects of spectral distortion become manifest only at sufficiently large lengths of the effective interaction of the radiation with the electrons (astrophysical conditions). In the laboratory, on the other hand, the increase of the radiation intensity even before the appearance of spectral effects should heat the electrons to relativistic temperatures, in view of their low bulk specific heat[7].

In the present paper we generalize previously obtained results to include the case of a denser plasma, when the screening of the charges must be taken into account. What is qualitatively new in this case is the heating of the ions[9]. In addition, it becomes possible to observe the discussed spectral effects in the laboratory, for when the plasma density is increased the bulk specific heat grows rapidly and the energy released following a noticeable evolution of the spectrum can lead to a negligible heating of the plasma. For a "broad" spectrum $\delta > \nu_D\sqrt{1-\cos\theta_0}$ (where θ_0 is the angular aperture of the radiation beam), the formulas obtained earlier for a rarefied plasma remain in force so long as $\delta \gg \nu_{pe}$. When $\nu_{pe} > \delta > \nu_D\sqrt{1-\cos\theta_0}$ and $(m_e/m_i)^{1/3} < \delta/\nu_D\sqrt{1-\cos\theta_0} < 1$ and $\delta < \nu_{pe}$, screening of the charges leads to a weakening of the discussed effects in comparison with the case of a rarefied plasma, while at $\delta < \nu_{pe}$ and $\delta/\nu_D\sqrt{1-\cos\theta_0} < (m_e/m_i)^{1/3}$ it leads to their intensification (see Fig. 1).

We subsequently estimate the situation in cosmic sources of maser radiation in the lines of the molecules OH and H_2O (in principle, a similar analysis is valid also for gasdynamic lasers). We obtain the upper limit of the brightness temperature of cosmic-maser emission, imposed by the deviation of the line from resonance in the presence of induced Compton interaction with the plasma, is obtained (see Fig. 2).

1. FUNDAMENTAL EQUATIONS

We start with the equations for the occupation numbers in the phase space of the photons and with the quasilinear equation for the plasma particle velocity distribution function. The occupation numbers are defined as follows:

$$n(\nu, \mathbf{l}) = c^2 I_\nu(\nu, \mathbf{l}) / 8\pi h\nu^3, \tag{1}$$

where $\mathbf{l}$ is a unit vector in the direction of photon propagation, and the quantity I_ν is connected with the spectral density of the emission energy

$$\mathscr{E}_\nu = c^{-1}\int I_\nu(\nu, \mathbf{l})\, d^2\mathbf{l}.$$

The equation for the so-defined occupation numbers

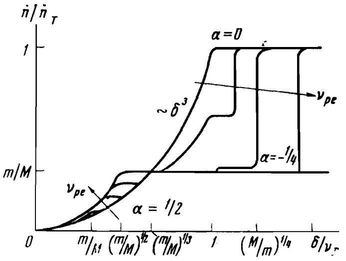

FIG. 1. Effectiveness of induced Compton processes as a function of the plasma density and of the ratio of the spectral width of the isotropic radiation δ to the Doppler width of the scattering ν_{Dj} (4). The curve is normalized to the effectiveness at a low electron density $\delta \gg \nu_{pl}$ and for spectrally broad radiation $\delta \gg \nu_{Dj}$.

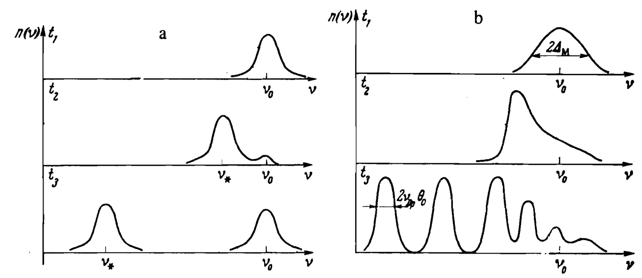

FIG. 2. Possible variants of the evolution of the spectrum of maser radiation as a result of induced Compton scattering: a—isotropic saturated maser, b—directional maser with $\Delta_M \gg \nu_{Dp}\theta_0$.

can be obtained directly from the results of[2,3,19]

$$\frac{\partial n(\nu,\mathbf{l},t)}{\partial t}=\frac{3hN_e\sigma_T}{16\pi m_e\nu_{pe}^2c}n(\nu,\mathbf{l},t)\int \nu'^2 n(\nu',\mathbf{l}',t)\,d\nu' \tag{2}$$

$$\times\int d^2\mathbf{l}'[1+(\mathbf{l}\mathbf{l}')^2](1-\mathbf{l}\mathbf{l}')\operatorname{Im}\{|1+\varepsilon_i(\Delta\nu,\Delta\mathbf{k})|^2\varepsilon_e(\Delta\nu,\Delta\mathbf{k})$$
$$+|\varepsilon_e(\Delta\nu\ \Delta\mathbf{k})|^2\varepsilon_i(\Delta\nu,\Delta\mathbf{k})\}\pi^{-1}|\varepsilon(\Delta\nu,\Delta\mathbf{k})|^{-2},$$

where

$$\varepsilon(\Delta\nu,\Delta\mathbf{k})=1+\sum_j\varepsilon_j(\Delta\nu,\Delta\mathbf{k}),\quad \varepsilon_j(\Delta\nu,\Delta\mathbf{k})=\frac{e_j^2}{\pi m_j\Delta k^2}\int\frac{\Delta\mathbf{k}\,\partial f_j/\partial\mathbf{v}}{\Delta\nu-\Delta\mathbf{k}\mathbf{v}+i0}d^3v$$

is the dielectric constant of the plasma

$$\sigma_T=\frac{8\pi}{3}\left(\frac{e^2}{m_ec^2}\right)^2,\quad \Delta\nu=\nu-\nu'\quad \Delta\mathbf{k}=(\nu\mathbf{l}-\nu'\mathbf{l}')/c.$$

It must be borne in mind that the theory of weakly-turbulent plasma, used in the derivation of Eq. (2) and (3), is no longer valid if the vibrational energy of the particles in the high-frequency field[19] becomes of the order of their thermal energy, i.e., when

$$\int d^3\mathbf{k}\,d^3\mathbf{k}'\frac{e^2E_\mathbf{k}E_{\mathbf{k}'}}{2m_e\nu\nu'}\frac{1+\varepsilon_i(\Delta\nu,\Delta\mathbf{k})}{\varepsilon(\Delta\nu,\Delta\mathbf{k})}\approx T_e\quad\left(\frac{|\mathbf{E}_\mathbf{k}|^2}{4\pi h\nu}=n(\nu,\mathbf{l})\right).$$

This condition is important if the emission line width is less than the Doppler broadening of the beat frequency.

The quasilinear equation for the particle distribution functions takes the form of the diffusion equation in velocity space[2]:

$$\frac{\partial f_j}{\partial t}=\frac{\partial}{\partial v_\alpha}D_{\alpha\beta}{}^j\frac{\partial}{\partial v_\beta}f_j, \tag{3}$$

where

$$D_{\alpha\beta}{}^j=\frac{3h\sigma_Tc}{32\pi m_j^2}\int d^2\mathbf{l}\,d^2\mathbf{l}'\int\nu\,d\nu\,\nu'\,d\nu'[1+(\mathbf{l}\mathbf{l}')^2](1-\mathbf{l}\mathbf{l}')n(\nu,\mathbf{l},t)$$
$$\times n(\nu',\mathbf{l}',t)\Delta k_\alpha\Delta k_\beta\delta(\nu-\nu'-\Delta\mathbf{k}\mathbf{v})\{|1+\varepsilon_i(\Delta\nu,\Delta\mathbf{k})|^2\delta_{je}$$
$$+|\varepsilon_e(\Delta\nu,\Delta\mathbf{k})|^2\delta_{ji}\}|\varepsilon(\Delta\nu,\Delta\mathbf{k})|^{-2}.$$

The presented equations become much simpler in two limiting cases, when the emission line width is larger than or smaller than the Doppler broadening of the beat frequency

$$\nu_{Dj}=\frac{\nu\nu'}{c^2}v_{Tj}^2,\quad v_{Tj}^2=\frac{2T_j}{m_j}. \tag{4}$$

Actually, in the case of anisotropy of the radiation, the spectral width of the beam should be compared with the effective Doppler change of the quantum frequency by scattering, $\Delta\nu_j = \nu_{Dj}\sqrt{1-\cos\theta_0}$.

1. Broad Emission Spectrum ($\delta > \nu_D\sqrt{1-\cos\theta_0}$)

a) The case $\delta > \nu_{pe} > \nu_D\sqrt{1-\cos\theta_0}$. The relaxation of the radiation is described by Eq. (2). Since only adjacent emission components interact intensively in the emission spectrum, the integrand in this equation can be expanded in powers of the frequency difference of the interacting components. As a result we get

$$\frac{\partial}{\partial t}n(\nu\ \mathbf{l},t)=\frac{3hN_e\sigma_T}{16\pi m_ec}n(\nu,\mathbf{l},t)\sum_j\int d^2\mathbf{l}'[1+(\mathbf{l}\mathbf{l}')^2](1-\mathbf{l}\mathbf{l}') \tag{5}$$
$$\times\left[\frac{\partial}{\partial\nu'}\nu'^2n(\nu',\mathbf{l}',t)W_j\right]_{\nu'=\nu},$$

where

$$W_e=\frac{1}{\pi\nu_{pe}^2}\int x\,dx\operatorname{Im}\frac{1}{1+\varepsilon_e(x,\Delta\mathbf{k})},$$
$$W_i=\frac{1}{\pi\nu_{pe}^2}\int x\,dx\operatorname{Im}\frac{|\varepsilon_e(0,\Delta\mathbf{k})|^2}{1+\varepsilon_e(0,\Delta\mathbf{k})+\varepsilon_i(x,\Delta\mathbf{k})}.$$

It is legitimate to extend the integration over the entire frequency interval only if the width of the emission spectrum exceeds both the Doppler width and the frequency of the natural oscillations ($\delta > \nu_{Dj}, \nu_{pj}$). The integrals W_j can be easily evaluated with the aid of the theory of dispersion relations[20]. As a result we get for the effective induced-scattering cross section

$$W_e^{tot}=1,$$
$$W_i^{tot}=(m_e/m_i)[1+\nu_{De}^2(1-\cos\theta)/\nu_{pe}^2]^{-2},\quad \cos\theta=\mathbf{l}\mathbf{l}'. \tag{6}$$

The quantities W_j characterize not the scattering itself, but the effectiveness of energy transfer by scattering. This is precisely why the effective cross sections for scattering by electrons and ions are not equal but differ by a factor m_e/m_i. It follows from (5) that in the case of a "broad" spectrum $\delta \gg \nu_{pe}$ the screening effect can be neglected even when $\nu_{De}\sqrt{1-\cos\theta} < \nu_{pe}$, and the equations for a rarefied plasma can be used. In particular, the Kompaneets differential equation[1], which describes the interaction of free electrons with a spectrally broad isotropic radiation field, is valid so long as $\delta \gg \nu_{pe}$.

b) The case $\nu_{pe} > \delta > \nu_{De}\sqrt{1-\cos\theta}$. If we are interested in scattering by particles, then it is necessary to separate from W_j^{tot} the fraction corresponding to virtual oscillations of the medium. Allowance for the latter process in (2) corresponds to a contribution to the integral from the zeroes of the dielectric constant[1)]

[1)]The way the effective cross sections are expressed in (5), the scattering by ion sound is due entirely to ions, whereas part of this scattering is in fact from electrons.

$$W^{coll} = \frac{1}{\nu_{pe}^2}\int_{-\infty}^{+\infty} x\,dx\,\delta[\varepsilon(x, \Delta k)]. \qquad (7)$$

Weakly damped electron Langmuir oscillations appear when the wavelength of the beats becomes much larger than the Debye radius. An emission line that is not broad enough ($\nu_{pe} > \delta > \nu_{De}$) relaxes only via scattering by electrons, and the induced Compton interaction becomes weaker. In this case W_e is determined by the difference between (17) and (18):

$$W_e^{part} = W_e^{tot} - W^{coll} = 6(\nu_{De}/\nu_{pe})^4(1-\cos\theta)^2. \qquad (8)$$

In addition, in a non-isothermal plasma with hot electrons, an appreciable contribution is made to the scattering by ion-sound oscillations. Subtracting the corresponding contribution from (6), we obtain the effective cross section for scattering by ions

$$W_i^{part} = 6T_e^2/T_i^2[1 + \nu_{De}^2(1-\cos\theta)/\nu_{pe}^2]^2. \qquad (9)$$

2. Induced Pressure of Light in a Plasma

The relaxation of the particle distribution under the influence of radiation is described by the quasilinear equation (3). The first moment of this equation yields the force of the induced pressure of light. The expression for the latter can be simplified by representing the momentum transfer in one scattering act in the form

$$(h/2c)[(\nu'-\nu)(\mathbf{l}'+\mathbf{l}) + (\nu'+\nu)(\mathbf{l}'-\mathbf{l})].$$

The contribution from the first term turns out to be even in the frequency difference and can be integrated directly, while in the second term the integrand is expanded in the small frequency difference; as a result we get

$$F_{ind}^j = \frac{3h^2\sigma_T N_e}{32\pi m_e c^2}\int d^2\mathbf{l}\,d^2\mathbf{l}'\int \nu^4\,d\nu\Big\{n(\nu,\mathbf{l},t)n(\nu,\mathbf{l}',t)W_j(\nu,\nu;\theta)\mathbf{l} + n(\nu,\mathbf{l},t)\left[\frac{\partial}{\partial\nu'}W_j(\nu,\nu';\theta)n(\nu',\mathbf{l}',t)\right]_{\nu'=\nu}\nu(\mathbf{l}'-\mathbf{l})\Big\}. \qquad (10)$$

In the limit of a rarefied plasma ($\nu_{pe} \ll \nu_{De}$), this equation yields the result of[11].

3. Plasma Heating

The particle-velocity diffusion causes plasma heating. The rate of change of the particle energy can be easily obtained already from the decrease in the radiation-energy density

$$\frac{d\mathscr{E}_j}{dt} = \frac{3h^2\sigma_T N_e}{32\pi m_e c}\int d^2\mathbf{l}\,d^2\mathbf{l}'\int \nu^4\,d\nu[1+(\mathbf{l}\mathbf{l}')^2](1-\mathbf{l}\mathbf{l}')W_j(\nu,\nu;\theta) \times n(\nu,\mathbf{l},t)n(\nu,\mathbf{l}',t). \qquad (11)$$

It must be borne in mind here that in a spectrally narrow radiation beam the particle energies increase more effectively in a plane perpendicular to the radiation beam. The resultant anisotropy of the plasma velocity distribution can be automatically lifted as a result of the development of various types of plasma instabilities (see, e.g., Mikhaĭlovskiĭ's book[21]), so that ultimately the distribution relaxes to a more or less isotropic distribution with effective temperature of the order of the energy input per particle. An example of instability of a plasma without a magnetic field is the instability observed by Fried[22], with a characteristic growth time $\tau^{-1} \approx \nu_{pe}v_{Te}/c$, which is always small in the applications considered. The foregoing gives grounds for hoping that the plasma can be effectively heated even by a narrowly-directed radiation beam.

4. Spectrally Narrow Line ($\delta \ll \nu_D\sqrt{1-\cos\theta_0}$)

It is most convenient to begin the analysis of the interaction of a narrow radiation line with matter by discussing the asymptotic form of the particle distribution function following heating by radiation for a sufficiently long time. The evolution of the particle distribution is described by a quasilinear equation that becomes much simpler in the limiting cases when the radiation beam has a small angular aperture and when the radiation is isotropic. We assume that the narrow radiation beam has axial symmetry with respect to the beam direction. Then the action of the radiation reduces to heating of the particles in a plane transverse to the radiation beam and to their acceleration by the induced pressure force along the beam. Then the distribution function that is formed during the course of the heating also has axial symmetry. The latter makes it possible to reduce the quasilinear equation (neglecting the slower acceleration of the particles along the beam) to the diffusion equation

$$\frac{\partial f_j}{\partial t} = \frac{4}{25}\frac{\partial}{v_\perp\partial v_\perp}D_j\frac{\partial}{v_\perp\partial v_\perp}f_j, \qquad (12)$$

where

$$v_\perp^{-1}D_j = \frac{75h\sigma_T c}{128\pi m_j m_e}\iint d^2\mathbf{l}\,d^2\mathbf{l}'\int \nu\,d\nu\int \nu'\,d\nu'[1+(\mathbf{l}\mathbf{l}')^2](1-\mathbf{l}\mathbf{l}') \times n(\nu,\mathbf{l},t)n(\nu',\mathbf{l}',t)W_j(\nu-\nu')^2/\mathscr{R}(\nu,\nu'),$$

$$\mathscr{R}(\nu,\nu') = [2\nu_{Dj}^2(1-\mathbf{l}\mathbf{l}')v_\perp^2/v_{Tj}^2 - (\nu-\nu')^2]^{1/2}.$$

We exclude from the expressions for the effective cross sections the contribution due to scattering by virtual ion sound:

$$W_e = \begin{cases} \left(1-\dfrac{\nu_{pi}^2}{\Delta\nu^2}\right)^2\left(1+\dfrac{\nu_{pe}^2}{\nu_{De}^2}(1-\cos\theta)^{-1}-\dfrac{\nu_{pi}^2}{\Delta\nu^2}\right)^{-2}, \\ \nu_{De} > \Delta\nu \equiv \nu-\nu' > \nu_{Di}, \\ \left(1-\cos\theta+\dfrac{\nu_{pi}^2}{\nu_{Di}^2}\right)^2\left(1-\cos\theta+\dfrac{\nu_{pe}^2}{\nu_{De}^2}+\dfrac{\nu_{pi}^2}{\nu_{Di}^2}\right)^{-2}, \\ \Delta\nu < \nu_{Di}, \end{cases}$$

$$W_i = \frac{m_e}{m_i}\left(\frac{\nu_{pe}}{\nu_{De}}\right)^4\left[1-\cos\theta+\frac{\nu_{pe}^2}{\nu_{De}^2}+\frac{\nu_{pi}^2}{\nu_{Di}^2}\right]^{-2}, \quad \Delta\nu < \nu_{Di}. \qquad (13)$$

If the relaxation of the emission line constitutes a drift in frequency towards lower frequencies[7], and not a cascaded transfer via satellites located at distances $\Delta\nu \sim \nu_D$ or $\Delta\nu \approx \nu_{pe}$[15], then the quasilinear equation describes plasma heating by slow electrostatic waves—beats with a phase velocity much lower than the thermal velocity of the particles. The asymptotic form of its solution, in the limit of large t, was obtained first in[23] (see also[24,8]):

$$f_j = \frac{5}{\Gamma(2/5)}\left[\int_0^t D_j(t')\,dt'\right]^{-2/5}\exp\left\{-\frac{v_\perp^5}{\int_0^t D_j(t')\,dt'}\right\}. \qquad (14)$$

The same solution remains in force also in the case of isotropic radiation, the only difference being that the energy input is now to all the degrees of freedom. The distortion of the distribution function, naturally, leads to a weakening of the heating in comparison with the case when the distribution is Maxwellian (the ratio of the average energies in these cases is $(\pi/2)^{1/5}\Gamma(4/5)/3\Gamma(2/5) \approx 0.19$).

In the next order in the ratio of the thermal velocity to the light velocity, it is necessary to take into account the particle acceleration under the influence of the induced-pressure force. The expression for the latter is (compare with[7]):

$$F^{j}_{ind} = \frac{3h\sigma_T}{32\pi m_e} \iint d^2\mathbf{l}\, d^2\mathbf{l}' \int \nu\, d\nu \int \nu'\, d\nu' [1 + (\mathbf{l}\mathbf{l}')^2](1 - \mathbf{l}\mathbf{l}') \times n(\nu, \mathbf{l}, t)\, n(\nu', \mathbf{l}', t)\, W_j \int [\nu\mathbf{l} - \nu'\mathbf{l}'] \frac{\Delta\nu\, \partial f_j / v_\perp \partial v_\perp}{\mathscr{R}(\nu, \nu')} d^3v. \tag{15}$$

Using the already noted symmetry of the distribution function, we can easily obtain a simple expression for the occupation numbers

$$\frac{\partial n(\nu, \mathbf{l}, t)}{\partial t} = -\frac{3hc\sigma_T N_e}{32\pi m_c \nu} \int d^2\mathbf{l}' \int \nu'\, d\nu' [1 + (\mathbf{l}\mathbf{l}')^2] n(\nu, \mathbf{l}, t) \times n(\nu', \mathbf{l}', t) \sum_j W_j \frac{\nu - \nu'}{\mathscr{R}(\nu, \nu')} \frac{\partial f_j}{v_\perp \partial v_\perp} d^3v. \tag{16}$$

The obtained equations can be easily analyzed for the particular case of an isotropic emission spectrum, for which the integration with respect to the scattering angles is carried out in explicit form in Appendix A. It turns out that even at a low plasma density the radiation with a narrow spectrum $\delta \ll \nu_{Dp}$ relaxes not on the electrons but on the protons of the plasma. Comparing (A.3) with (A.4), we find that the latter takes place under the condition

$$\nu_{De} > \nu_{pe} > \nu_{Dp}, \quad \delta < \nu_{pe} \left(\frac{m_e}{m_p}\right)^{1/2} \left[\sqrt{\frac{T_p}{T_e}} + \sqrt{\frac{T_e}{T_p}}\right].$$

At high densities, when $\nu_{pe} > \nu_{De}(m_p/m_e)^{1/4}$, the plasma protons make the main contribution to the relaxation of the radiation with a spectrum whose width δ is narrower than the plasma frequency at any ratio of δ to ν_{De}. In the opposite limiting case, $\delta > \nu_{pe}$, the relaxation proceeds in the same manner as in a plasma without screening of the electron charge, and is described by the well known Kompaneets equation. The dependence of the probability of induced scattering in an isothermal plasma on the emission line width is shown in Fig. 1.

In a non-isothermal plasma, the relaxation of radiation with sufficiently narrow spectra on the cold component of the plasma becomes weaker in accordance with a law that can be easily obtained from the formulas of the Appendix.

2. RELAXATION OF COSMIC MASER EMISSION LINE

The induced Compton interaction of spectrally narrow radiation with free electrons influences strongly the line spectrum[7]. If the line has a Doppler profile with $\delta \ll \nu_{De}$, then the interaction leads to its downward drift along the frequency axis at a constant rate $d\nu/dt$, while its shape remains the same. If the profile is steeper than the Doppler profile (weak line wings), the spectrum shifts towards the low-frequency boundary of the profile so that the line becomes narrower. Further transfer of quanta into the low-frequency region can occur via a system of satellites[15].

We consider below the influence of these effects on the emission spectrum produced in cosmic sources of maser radiation. The plasma density in the sources is quite large ($\nu_{pe} \gg \nu_{De}$), so that plasma effects occurring in the scattering must be taken into account. Collisions with neutral particles have time to establish a Maxwellian distribution of the plasma particles with identical temperatures, so that the question of the relaxation of the particle distribution does not arise.

For simplicity, we consider a homogeneous isotropic medium filled with OH and H_2O molecules, neutral atoms and molecules, electrons, and protons. It is assumed that there exist pumping mechanisms that invert the populations of the levels of the H_2O or OH molecules responsible for the emission in the discussed lines. Being interested in the situation at high radiation intensity, we shall not stop to discuss the initial stages of the enhancement of the radiation in the line: we assume for simplicity that the intrinsic spontaneous emission of the molecules is amplified. The width Δ_M of the maser emission line should be narrower than the Doppler scattering with ν_{Dp} for protons, since the upper limit for Δ_M is the Doppler width of scattering by molecules whose mass is 17–18 times larger than the proton mass.

At high intensity of the isotropic radiation in the homogeneous medium, the transport equation takes the form

$$\frac{\partial n(\nu, t)}{\partial t} = \left(\frac{C}{\sqrt{2\pi}\,\Delta_M} + \frac{A}{\sqrt{2\pi}\,\Delta_M}\right) \exp\left\{-\frac{(\nu - \nu_0)^2}{2\Delta_M^2}\right\} - n(\nu, t) \int \nu'^2 n(\nu', t) K(\nu - \nu')\, d\nu', \tag{17}$$

where C = $Bn(\nu, t)$ for an unsaturated maser, and the constant C is determined only by the pumping rate in the case of a saturated maser; $K(\Delta\nu) = hN_p\sigma_T\gamma_p(\Delta\nu)/m_p\nu_{Dp}^2 c$ is a coefficient characterizing the effectiveness of the induced Compton interaction with the plasma protons (see Appendix A). When $\Delta_M \ll \nu_{Dp}$, we have $\gamma_p(\Delta\nu) \approx \gamma_p'(0)\Delta\nu$, where

$$\gamma_p'(0) = 11\sqrt{\pi}/5\nu_{Dp}(1 + T_e/T_p)^2.$$

The coefficients A and B are expressed in terms of the spontaneous transition probability A_{mn}, the level population f_m, the statistical weights g_m, and the density N_M of the active molecules[25]:

$$B = \frac{c^3}{8\pi\nu^2} A_{mn} \left(f_m - \frac{g_m}{g_n} f_n\right) N_M, \qquad A = \frac{f_m N_M c^3}{8\pi\nu^2} A_{mn}.$$

The first term on the right-hand side of (17) describes the spontaneous emission of the molecules, the second describes their induced emission, and the third (nonlinear in n) describes the induced Compton interaction with the plasma. Since the effect of induced Compton scattering is quadratic in $n(\nu, t)$, it follows that regardless of the maser parameters there are values of $n(\nu, t)$ such that the influence of the Compton processes predominates.

1. Saturated Maser

In the case of a saturated maser, the stimulated emission probability is determined only by the pumping rate. Recognizing that $\Delta_M \ll \nu_{Dp}$, we rewrite (17) in the simpler form

$$\frac{\partial n(\nu,t)}{\partial t}=\frac{Bn_{\text{sat}}}{\sqrt{2\pi}\,\Delta_M}\exp\left\{-\frac{(\nu-\nu_0)^2}{2\Delta_M^2}\right\}+K'(0)n(\nu,t)\int\nu'^2(\nu-\nu')n(\nu',t)\,d\nu', \tag{18}$$

$K'(0) = 11\sqrt{\pi}\hbar N_p\sigma_T/5m_p\nu_{Dp}^3c(1 + T_e/T_p)^2$; $n_{sat} = C/B$ is the occupation number at which the maser saturates. The qualitative picture of the emission-line relaxation, described by Eq. (18), is quite clear. The emission intensity increases linearly with time until it reaches at the line center a critical value on the order of $n_{cr}(\nu) \sim (n_{sat}B/K'(0)\nu_0^2\Delta_M^3)^{1/2}$. At this value, the induced Compton scattering by the plasma protons leads to a drift of the line with the Doppler profile towards lower frequencies. The emission is present as before at the frequency ν_0, and is connected with the continuing operation of the maser. Near ν_0, however, the intensity decreases, so that the Compton effect rapidly transfers the produced quanta to the drifting line. In principle, it becomes possible to transfer the maser to an unsaturated regime or even to stop it entirely. The emission intensity is low between the moving line and ν_0; the quanta from this region are rapidly transferred to the moving line.

When the line shifts through a distance on the order of ν_{Dp}, it becomes possible for a new line to be formed near the frequency ν_0, and the entire process is repeated. After a sufficiently large time interval, there should be observed in the medium a series of lines of nearly equal intensity, but shifted in frequency.

Although it is impossible to obtain an analytic solution of (18), it is nonetheless possible to trace the asymptotic behavior of the solutions describing the time variation on the intensities of the moving and standing lines. We obtain first an equation for the rate of motion of the line over the spectrum. To this end, following[7], we write down the solution of (18) in implicit form in terms of the energy density $\mathscr{E}$ of the radiation and the number of the quanta N_γ:

$$\mathscr{E}(t)=\frac{8\pi h}{c^3}\int\nu^3n\,d\nu,\qquad N_\gamma(t)=\frac{8\pi}{c^3}\int\nu^2n\,d\nu; \tag{19}$$

$$n(\nu,t)=\frac{Bn_{\text{sat}}}{\sqrt{2\pi}\,\Delta_M}\exp\left\{-\frac{(\nu-\nu_0)^2}{2\Delta_M^2}\right\}$$

$$\times\int_0^t dt'\exp\left[-\frac{c^3K'(0)}{8\pi h}\int_{t'}^{t}(h\nu N_\gamma(t'')-\mathscr{E}(t''))\,dt''\right].$$

The profile of the moving line is determined by recognizing that on the left of its central frequency the main contribution to the integral (19) is made by the region of small t'

$$n(\nu,t)=\frac{Bn_{\text{sat}}t}{\sqrt{2\pi}\,\Delta_M}\exp\left[-\frac{(\nu-\nu_0)^2}{2\Delta_M^2}-\frac{c^3K'(0)}{8\pi}\int_0^t\left(\nu N_\gamma(t')-\frac{\mathscr{E}(t')}{h}\right)dt'\right]; \tag{20}$$

from this we get

$$\nu_*=\nu_0-\Delta_M^2\frac{c^3K'(0)}{8\pi}\int_0^t N_\gamma(t')\,dt', \tag{21}$$

$$N_\gamma(t)\approx\frac{8\pi}{c^3}\nu_0^2Bn_{\text{sat}}t.$$

In the case of a saturated maser, the number of quanta increases linearly with time, and the emission line is accelerated to the left.

The transfer of the quanta to the moving line shuts the maser off rapidly: The number of quanta at the frequency ν_0, in accordance with (21), decreases in inverse proportion to the time cubed:

$$n(\nu,t)\approx\frac{Bn_{\text{sat}}}{\sqrt{2\pi}\,\Delta_M}\exp\left\{-\frac{(\nu-\nu_0)^2}{2\Delta_M^2}\right\}\Big/\frac{c^3}{8\pi}K'(0)\,[\nu_0-\nu_*(t)\,]N_\gamma(t). \tag{22}$$

The subsequent evolution of the spectrum depends on the relation between the maser parameters and those of the medium. It is convenient to introduce two characteristic times: 1) the growth time of the intensity in the unsaturated regime, $t_M = \sqrt{2\pi}\,\Delta_M B^{-1}$; 2) the time of distortion of the spectrum as a result of the induced Compton effect at the minimal intensity n_{sat} sufficient to saturate the maser, $t_c^{-1} = \nu_0^2 n_{sat}\Delta_M^2 K'(0)$. In addition, we introduce a special symbol I_* for the emission intensity at the frequency ν_0 at which t_M becomes comparable with the time of the stimulated Compton scattering by the plasma ions:

$$\frac{8\pi h\nu^3}{c^2}n_*\equiv I_*=\frac{kT_p}{\sigma_Tt_M}\frac{\nu_{Dp}}{\Delta_M^2}\frac{1}{N_p\lambda^3},\qquad \Delta_M\ll\nu_{Dp}. \tag{23}$$

It is assumed in this section that $n^* \gg n_{sat}$, so that the maser becomes saturated before the stimulated Compton scattering comes into play. Depending on the relation between n_* and n_{sat}, there are two possibilities.

If the time t_{sw} necessary to switch off the maser and determined by Eq. (22) is longer than the line drift time t_R (see (20))

$$t_{sw}=t_c^{2/3}t_M^{1/3}\gg t_R=\left(\frac{\nu_{Dp}}{\Delta_M}t_ct_M\right)^{1/2}, \tag{24}$$

then the line deviates by more than the Doppler frequency of the plasma ions, and ceases to limit the maser. In this regime, a line splits off from the fundamental frequency every t_R seconds and subsequently grows to the value

$$n=n_{\text{sat}}\frac{t_R}{t_M}\approx\left(n_*n_{\text{sat}}\frac{\nu_{Dp}}{\Delta_M}\right)^{1/2},\qquad n_{\text{sat}}<n_*\left(\frac{\Delta_M}{\nu_{Dp}}\right)^{3/2}. \tag{25}$$

If $t_R \gg t_{sw}$, the maser goes over to an unsaturated regime after a time t_{sw}, and the rate of extinction of the maser is considerably decreased:

$$n(\nu_0,t)=\frac{n_{\text{sat}}t_{sw}}{t\ln^{2/3}(t/t_{sw})},\qquad t\gg t_{sw}. \tag{26}$$

The number of quanta in the moving line changes in this case in proportion to $\ln^{3/5}(t/t_{sw})$, i.e., the line practically ceases to become amplified. The limiting occupation number of the radiation in the line is in this case

$$n=n_{\text{sat}}\frac{t_{sw}}{t_M}\approx(n_*^2n_{\text{sat}})^{1/3},\qquad n_*\left(\frac{\Delta_M}{\nu_{Dp}}\right)^{3/2}<n_{\text{sat}}<n_*. \tag{27}$$

2. Unsaturated Maser

In the case of an unsaturated maser, the exponential amplification of the spontaneous emission leads to the formation of a narrow emission line near the central frequency

$$n(\nu,t)=\frac{A}{\sqrt{2\pi}\,\Delta_M}\exp\left\{-\frac{(\nu-\nu_0)^2}{2\Delta_M^2}\right\}\int_0^t\exp\left[\frac{(t-t')}{\tau_M(\nu)}\right]dt', \tag{28}$$

$$[\tau_M(\nu)\,]^{-1}=\frac{B}{\sqrt{2\pi}\,\Delta_M}\exp\left\{-\frac{(\nu-\nu_0)^2}{2\Delta_M^2}\right\}.$$

The exponential growth continues for a time

$$t_M' = \tau_M(0)\ln(\nu_0^2 K'(0) A\tau_M)^{-1} \approx \Lambda\tau_M(0), \tag{29}$$

until the Compton scattering comes into play and causes the emission frequency to drift. In (23) it is necessary to use for the limiting value of n_* the effective line width

$$\Delta_M' = \Delta_M \Lambda^{-1/2}. \tag{30}$$

It must be emphasized that the wings of the line (28) decrease much more steeply than in the Gaussian profile. It therefore can not drift in frequency, without change of shape, farther than the Δ_M'. It must be borne in mind, however, that the wings of the line (28) are strongly raised by the presence of spontaneous emission. All the components of the spectrum in the raised wings to the left of the central emission line increase exponentially, since the stimulated Compton scattering transfers quanta from the central line to the left wing. The latter makes it possible to transfer all the quanta rapidly and thus switch the maser off (such a process for plasma oscillations was considered in[15]). The position of the new line is determined by the competition of two factors, the increased rate of scattering (its maximum is reached at $\nu = \nu_m$) and the simultaneous exponential decrease of the spontaneous emission at lower frequencies:

$$\nu_* - \nu_0 = (\nu_m - \nu_0) \Big/ \left[1 + \left(\left[\frac{d^2 K(\nu - \nu_0)}{d\nu^2} \right]_{\nu=\nu_m} \Delta_M^2 \nu_0^2 \Delta_M' n_*' t_{*C} \right)^{-1} \right],$$

where $t_{*C}^{-1} = K(\nu_m - \nu_0)\nu_0^2 \Delta_M' n_*'$ is time of growth of the radiation at the frequency ν_m as a result of the Compton scattering.

Since the line produced in this manner has a Compton profile with a new width on the order of

$$\delta^{-2} = \Delta_M^{-2} + \left[\frac{d^2}{d\nu^2} K(\nu - \nu_0) \right]_{\nu=\nu_m} \nu_0^2 \Delta_M' n_*' t_{*C},$$

it follows that after all the quanta are transferred to it it will drift in frequency and will cease to limit the maser after a certain time. Therefore, just as in the case of the saturated maser, the maser will emit not one but a series of lines, in each of which the occupation number is of the order of

$$n_*' \approx \frac{kT_p}{8\pi h\nu^3 \Delta_M t_M \sigma_T} \frac{\nu_{Dp}}{\Delta_M} \frac{\Lambda}{N_p \lambda^3}. \tag{31}$$

3. Case of Anisotropic Emission

We assume that anisotropic emission with a quantum divergence angle $\theta_0 \ll 1$ is amplified in an infinite homogeneous medium filled with a plasma and with molecules with inverted level population. At $1 - \cos\theta_0 > \Delta_M^2/\nu_{Dp}$, all the qualitative conclusions of the preceding subsections remain in force, except for the appearance of an additional dependence on θ_0. According to the formulas of Appendix B, it is necessary to substitute $K'\theta_0$ for K' in all formulas.

On the other hand, if $\theta_0 \ll \Delta_M/\nu_{Dp} < 1$ the emission spectrum can be regarded as broad in comparison with the maximum possible change of the frequency by stimulated scattering. The spectrum relaxation satisfies the equation[7]

$$\frac{\partial n(\nu,\alpha,t)}{\partial t} = \frac{C(\alpha)}{\sqrt{2\pi}\,\Delta_M} \exp\left\{ -\frac{(\nu-\nu_0)^2}{2\Delta_M^2} \right\} + \frac{3}{8} \frac{h\sigma_T N_p \theta_0^4}{m_p c} \frac{1}{\nu^2} \frac{\partial \nu^4 n^2(\nu,\alpha,t)}{\partial \nu} \tag{32}$$

It was assumed in (32) that

$$n = \begin{cases} n_0 & \text{for} \quad \alpha < \theta_0 \\ 0 & \text{for} \quad \alpha > \theta_0 \end{cases}. \tag{33}$$

Equation (32) with large n describes the motion of the maser-produced quanta downward along the frequency axis, namely, a rapid increase of the slope of the spectrum near the low-frequency boundary of the profile[12]. The Compton processes should subsequently lead to a splitting of the line into solitons[13] with a spectral width $\delta \sim \nu_{Dp}\theta_0$, and to a drift of these solitons down the frequency axis, described by an equation of the type (17). Owing to the large line width, complete switching off of the maser is not very likely in this case. It is more likely that a stationary spectrum will become established within the limits of the line and will increase towards lower frequencies; oscillations with frequency $\nu_{Dp}\theta_0$ will become superimposed on this spectrum. Outside the limits of the line, at $\nu < \nu_0 - \Delta_M$, the emission spectrum should consist of solutions that drifted constant velocity down the frequency axis. The limiting occupation number of the line, starting with which the line spectrum is appreciably distorted by the Compton processes, can be readily obtained from (32):

$$n_{cr} \leqslant \left(\frac{n_{\text{sat}}}{\theta_0^4} \frac{m_p c^2}{h\nu_0} \frac{\Delta_M/\nu_0}{\sigma_T N_p c t_M} \right)^{1/2} \tag{34}$$

in the case of a saturated maser and

$$n_{cr} \leqslant \frac{1}{\theta_0^4} \frac{m_p c^2}{h\nu_0} \frac{1}{\sigma_T N_p c t_M} \frac{\Delta_M}{\nu_0}$$

in the case of an unsaturated one.

4. Upper Limit of Brightness Temperature of Cosmic-Maser Emission

Compact sources of maser emission of H_2O with wavelength λ = 1.35 cm, located in the HII regions (e.g. in W49) have the following parameters[26]: $\delta/\nu \sim 10^{-6}$ (from which it follows that the molecule temperature is $T_M \sim T_p \lesssim 10^{3\circ}$ K), and the emission brightness temperature $T_b = nh\nu_0/k$ exceeds $10^{15\circ}$K. The last figure was obtained under the assumption that the dimension of the application zone is $l < 10^{13}$ cm, a value determined by radio-interference methods and also obtained from the recently observed rapid ($\Delta t \sim 5$ min) variability[27] of one of the sources in W49. These methods provide a reliable estimate of the dimensions only in the case of a saturated maser. We shall assume below that the maser is at the saturation limit, $n \sim n_{sat}$, and $t_M \sim l/c$ is the effective time of departure of the quanta from the source. In the case of strong saturation, this limit will drop in proportion to $n_s/n = A_p/A_{mn}n$, where A_p characterizes the rate of the pumping processes and $A_{mn}n$ the rate of the stimulated decay.

The maximum brightness temperature of an isotropic maser is obtained from (17):

$$\frac{kT_b}{m_p c^2} \leqslant \frac{1}{\sigma_T N_p l} \left(\frac{kT_p}{m_p c^2} \right)^{3/2} \left(\frac{\nu}{\delta} \right)^2. \tag{35}$$

If the maser is not isotropic and the angular aperture of the emission beam is equal to $\theta_0 > \sqrt{m_p/m_M} = 1/4, 5$, then the right-hand side of (35) must be divided by θ_0. When $\theta_0 < \sqrt{m_p/m_M}$, we obtain the upper limit from (32) and (34), putting $\partial n^2\nu^4/\partial\nu \approx n^2\nu_0^3\nu_0/\delta$,

$$\frac{kT_b}{m_pc^2} \leqslant \frac{1}{\sigma_T N_p l}\frac{\delta}{\nu}\theta_0^{-4}. \tag{36}$$

At $\theta_0 < 1/200$, the principal role is assumed by scattering by electrons, and m_p in (36) must be replaced by m_e. Substituting in (35) and (36) the cited cosmic-maser parameters, we find that at $N_e = N_p > 10^5\,\theta_0^{-1}$ cm^{-3} in the case of a quasi-isotropic maser, and at $N_e = N_p > 10^3\,\theta_0^{-4}$ cm^{-3} for a directional maser, the emission spectrum should become strongly distorted[2)]. This is not observed, although the density of the H_2O molecules in the sources of W49 is estimated[28] at $N_M \gtrsim 10^{11}\pi\theta_0^2$ cm^{-3}. At the normal cosmic abundance of the elements, such a molecule concentration corresponds to $N_H + N_{H_2} \gtrsim 10^{14}\pi\theta_0^2$cm^{-3}. Ultraviolet radiation, soft x-ray background radiation, and cosmic rays should maintain at $T \sim 10^{3\circ}$K a degree of ionization $N_p/(N_H + N_{H_2})$, which greatly exceeds 10^{-9} (by several orders of magnitude). Therefore the presented estimates can be reconciled with the observation data only in two cases: If $\theta_0 \ll 1$, i.e., the maser is strongly anisotropic, or else if the medium in the cosmic maser has an anomalous chemical composition—it is poor in hydrogen[28]. Unfortunately, these conclusions are not final, owing to the considerable uncertainty in the values of l and $T_b \sim 1/l^2$, if the cosmic masers are not saturated (the question of the degree of their saturation has not yet been solved[26]). In addition, if the maser radiation is coherent, then the theoretical conclusions obtained in the random-phase approximation are no longer valid.

Formulas (25), (27), and (34) impose appreciable limitations on the brightness temperature of the cosmic-maser and gas dynamic-maser emissions, namely, if $kT_b \gg m_pc^2$ and the saturation is strong, it suffices to have a negligible degree of ionization of the gas medium in which the radiation is produced in order to effect appreciable changes in the maser emission spectrum.

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APPENDIX A

RELAXATION OF ISOTROPIC RADIATION

In the case of isotropic radiation, the integration over the angles in (5) and (6) can be carried out in explicit form, and these equations can be reduced to the following standard equations:

a) for a broad radiation spectrum ($\delta > \nu_{Dj}$)

$$\left(\frac{\partial n}{\partial t}\right)_j = \frac{h\sigma_T N_j}{m_j c} n(\nu,t)\left[\frac{\partial}{\partial\nu}\alpha_j(\nu,\nu')\nu'^2 n(\nu',t)\right]_{\nu'=\nu}; \tag{A.1}$$

b) for a narrow radiation spectrum ($\delta < \nu_{Dj}$)

$$\left(\frac{\partial n}{\partial t}\right)_j = \frac{h\sigma_T N_j}{m_j c\nu_{Dj}^2} n(\nu,t)\int \beta_j(\nu,\nu')\nu'^2 n(\nu',t)\gamma_j(\nu-\nu')\,d\nu'. \tag{A.2}$$

2)The bremsstrahlung absorption becomes appreciable at $N_e > 10^7$ cm^{-3}.

The expressions obtained for α, β, and γ turn out in general to be very cumbersome, and we therefore confine ourselves to the limiting cases of high and low plasma electron densities (the plasma is assumed to be quasineutral and homogeneous).

1. Low electron density ($\nu_{pe} \ll \nu_{De}$). In this case, the screening of the electron charge can be neglected and the stimulated scattering by the electrons is described by formulas obtained in the free-electron approximation:

$$\text{a)}\ \alpha_e = 1;$$

$$\text{b)}\ \beta_e = 1,\quad \gamma_e = \frac{1}{5}\left(\frac{\nu-\nu'}{\nu_{De}}\right)^2 \operatorname{sign}(\nu-\nu')\int_1^\infty \sqrt{x-1} \tag{A.3}$$

$$\times(16-12x+11x^2)\exp\left(-\frac{(\nu-\nu')^2x}{\nu_{De}^2}\right)\frac{dx}{x^3};$$

At $\delta \ll \nu_D$ we have $\gamma_e = 11\sqrt{\pi}(\nu-\nu')/5\,\nu_{De}$.

On the other hand, scattering by the ion jacket occurs mainly without a change in the direction of the photon propagation, for when the scattering angle increases the scattering probability decreases like $|\nu^2\ /\nu^2$ $|\nu_{pe}^2/\nu_{De}^2(1-\cos\theta)|^2$. Taking this circumstance into account, we can easily integrate with respect to the angles:

$$\text{a)}\ \alpha_p = \frac{3}{4}\left(\frac{\nu_{pe}}{\nu_{De}}\right)^4 \ln\frac{2\nu_{De}^2}{\nu_{pe}^2}, \tag{A.4}$$

$$\text{b)}\ \beta_p = \frac{\nu_{pe}^2}{\nu_{De}^2},\qquad \gamma_p(\nu-\nu')$$

$$= \frac{3}{4}\frac{(\nu-\nu')}{\Delta}\int_0^\infty \left[x + \frac{(\nu-\nu')^2}{\Delta^2}\right]^{-3/2} e^{-x}x\,dx;$$

when

$$\delta \ll \Delta = \nu_{pe}\left(\frac{m_e}{m_p}\right)^{1/2}\left[\sqrt{\frac{T_p}{T_e}} + \sqrt{\frac{T_e}{T_p}}\right]$$

we have $\gamma_p(\nu-\nu') = 3/4(\nu-\nu')/\Delta$.

We note that Eq. (A.2) retains its form in the case of a narrow directional radiation beam with aperture $\theta \gg \nu_{pe}/\nu_{De}$. Comparing Eqs. (A.3) and (A.4) we find the main contribution to the relaxation of the radiation with a spectrum narrower than the value of Δ given by the plasma protons even at a low particle density:

$$1 > \frac{\nu_{pe}}{\nu_{De}} > \left(\frac{m_e}{m_p}\right)^{1/4},\qquad \delta < \Delta = \nu_{Dp}\frac{\nu_{pe}}{\nu_{De}}.$$

The contribution of the protons to the relaxation of radiation with a broad spectrum decreases with decreasing density much more rapidly and turns out to be small already at $\nu_{pe}/\nu_{De} < (m_e/m_p)^{1/8}$, if $\delta > \nu_{Dp}$.

2. High electron density ($\nu_{pe} \gg \nu_{De}$). In a dense plasma, the screening of the electron charge leads to an attenuation of the scattering by the electrons:

$$\text{a)}\ \alpha_e = {}^{33}/_{15}\nu_{De}^4 / \nu_{pe}^4,\ \nu_{De} \ll \delta \ll \nu_{pe}; \tag{A.5}$$

$$\text{b)}\ \beta_e = \frac{\nu_{De}^4}{\nu_{pe}^4};\qquad \gamma_e = \frac{(\nu-\nu')^2}{115\nu_{pe}^2}\operatorname{sign}(\nu-\nu')\int_1^\infty \sqrt{x-1}$$

$$\times(256-160x+120x^2+56x^3+27x^4)\exp\left[-\frac{(\nu-\nu')^2}{\nu_{De}^2}x\right]\frac{dx}{x^5};$$

at $\delta \ll \nu_{De}$ we have $\gamma_e \approx 27\sqrt{\pi}(\nu-\nu')/115\nu_{De}$.

The probability of scattering by protons reaches in this case its maximum value ($\alpha_p = \beta_p = 1$). The expression for the kernel of the integral equation (A.2) in an isothermal plasma turns out to be much more complicated, and coincides at $T_p \gg T_e$ with expression (A.3), where the substitution ($\nu_{De} \to \nu_{Dp}$) should be made.

APPENDIX B

RELAXATION OF ANISOTROPIC RADIATION

We are considering an anisotropic radiation beam with angular aperture $\theta_0 \ll 1$ in a homogeneous infinite medium. The distribution n(0) is given by (33). We use the notation $\nu_{De} = \nu\sqrt{2kT_e/m_ec^2}$.

Equations (5) and (6) reduce to the following standard forms:

a) for a broad spectrum ($\delta > \nu_{Dj}\theta_0$)

$$\left(\frac{\partial n(\nu, \alpha, t)}{\partial t}\right)_j = \frac{3}{4}\frac{h\sigma_T N_j}{m_j c} n(\nu, \alpha, t) \int_0^{\theta_0} (1 - \cos\theta)(1 + \cos^2\theta)\, d\cos\alpha' \times \frac{\partial}{\partial\nu} \alpha_j(\nu, \theta)\nu'^2 n(\nu, \alpha', t) \approx \frac{3}{4}\frac{h\sigma_T N_j \theta_0^4}{m_j c}\frac{1}{\nu^2}\frac{\partial}{\partial\nu}\nu^4 n_0^2 \tag{B.1}$$

where θ is the scattering angle;

b) for a narrow spectrum ($\delta \ll \nu_{Dj}\theta_0$)

$$\left(\frac{\partial n(\nu, \alpha, t)}{\partial t}\right)_j = \frac{h\sigma_T N_j}{m_j c \nu_{Dj}^2} n(\nu, \alpha, t) \int_0^{\theta_0} \frac{1}{\sqrt{1 - \cos\theta}}(1 + \cos^2\theta) \times d\cos\alpha' \int \beta_j(\nu, \nu', \theta)\nu'^2 n(\nu', t, \alpha')\gamma_j(\nu - \nu')\, d\nu' \approx \frac{h\sigma_T N_j}{m_j c \nu_{Dj}^2} \times \theta_0 n(\nu, \alpha, t) \int \beta_j(\nu, \nu')\nu'^2 n(\nu', \alpha', t)\gamma_j(\nu - \nu')\, d\nu'. \tag{B.2}$$

In the case of a low electron density ($\nu_{pe} \ll \delta$) we have $\alpha_e = \beta_e = 1$, $\alpha_p \ll \alpha_e$, $\beta_p \ll \beta_e$, $\gamma_e \approx 11\sqrt{\pi}(\nu - \nu')/ \times 5\nu_{Dj}$.

In the case of high electron density ($\nu_{pe} \gg \delta$) we have

$$\alpha_e = \beta_e = \left(\frac{\nu_{De}}{\nu_{pe}}\theta_0\right)^4, \quad \alpha_p = \beta_p = 1,$$

$$\gamma_p \approx 11\sqrt{\pi}(\nu - \nu')/5\nu_{Dj}.$$

In all formulas of Sec. I, the dependence on θ_0 can be easily obtained in analogy with the formulas given in[7].

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