# *TURBULENT FLOW OF LIQUID CRYSTALS IN AN ELECTRIC FIELD*

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A qualitative theory is presented of turbulent pulsations in a liquid crystal layer in a stationary electric field. The attenuation time of the pulsations after switching off of the electric field is estimated

As is well known, stationary flow is produced in a layer of a liquid crystal (LC) following application of an electric field E[1] if the dimensionless parameter $\lambda \sim V^2/K$ exceeds a certain threshold value $\lambda_c \sim 10^3$. Here $V = El$ ($l$ is the thickness of the LC layer) and K is the average elastic modulus of the LC. When $\lambda$ reaches a second threshold value $l_t > l_c$ the stationary regime becomes unstable and the flow of the LC becomes turbulent with further increasing of the field. The turbulent pulsations in LC at $\lambda > \lambda_t$ have a number of unique features, since the pulsational velocities and the orientations of the director are interrelated. The purpose of the present paper is to describe qualitatively the turbulence developed in the LC (at $\lambda > \lambda_t$) and to estimate the damping time of the pulsations after the electric field is turned off.

1. In the turbulized LC, the pulsational change of the angles determine the orientation n of the director are of the order of unity. Therefore the equations describing the electrohydrodynamic effect do not admit of linearization with respect to the components n, and take the following form[1]

$$\operatorname{div} \mathbf{v} = 0, \tag{1}$$

$$\rho\left(\frac{\partial v_i}{\partial t} + v_k\frac{\partial v_i}{\partial x_k}\right) = -\rho\frac{\partial\mu}{\partial x_i} - \frac{1}{8\pi}E_mE_k\frac{\partial\varepsilon_{mk}}{\partial x_i} + \rho_eE_i + \frac{\partial\sigma_{ki}'}{\partial x_k}, \tag{2}$$

$$\frac{\partial}{\partial x_k}(\varepsilon_{ki}E_i) = 4\pi\rho_e, \quad \operatorname{rot}\mathbf{E} = 0, \tag{3}$$

$$\frac{\partial}{\partial x_k}\left[-\frac{\partial}{\partial t}(\varepsilon_{ki}E_i) + 4\pi\sigma_{ki}E_i\right] = 0, \tag{4}$$

$$[\mathbf{nh}] = \Gamma. \tag{5*}$$

Here v is the velocity of the liquid, $\rho$ is the density of the LC, $\mu$ is the chemical potential of the substance in the electric field, and $\epsilon_{ki}$ and $\sigma_{ki}$ are respectively the dielectric and conductivity tensors of the LC:

$$\varepsilon_{ki} = \varepsilon_\perp\delta_{ki} + (\varepsilon_\parallel - \varepsilon_\perp)n_kn_i,$$
$$\sigma_{ki} = \sigma_\perp\delta_{ki} + (\sigma_\parallel - \sigma_\perp)n_kn_i.$$

Summation is carried out over the repeated indices in (1)–(5). Equations (1) and (2) describe the flow of an incompressible liquid dielectric containing extraneous charges with volume density $\rho_e$ in an inhomogeneous electric field **E**. The stress tensor $\sigma'_{ki}$ due to the viscosity of the LC contains the velocity gradients of the anisotropic liquid and the velocity components of the vector motion (see, e.g.,[1]). Maxwell's equations (3) and (4) are applicable to a poor conductor such as a LC, for which it is meaningful to introduce simultaneously conductivity and a dielectric constant[2]. The director motion equation (5) contains the molecular field h and the moment of the friction force **Γ**, which depends on the same quantities as $\sigma'_{kl}$[1]. The field h contains the contribution of the elastic forces (proportional to K) existing in an LC with an inhomogeneous distribution of the director, and also a contribution (proportional to $\epsilon_aE^2$, $\epsilon_a = \epsilon_\parallel - \epsilon_\perp$) of the electric field acting in an anisotropic dielectric medium.

Since the turbulized LC is characterized by large values of the parameter $\lambda$, it is possible to neglect the elastic modulus K when the pulsational motion is considered. Physically this is due to the fact that when $\lambda > \lambda_t$ the characteristic frequencies $\omega$ of such a motion are much larger than the reciprocal $\tau_s^{-1}$ of the structure relaxation time, which is proportional to the modulus K, For the same reason, we can neglect the influence of the solid walls on the pulsational motion, since at large values of $\omega$ the boundary conditions are significant only in a thin LC layer near the solid surface[3]. The thickness h of such a layer is of the order of $h \sim (K/\alpha\omega)^{1/2} \ll l$, where $\alpha$ is the average dynamic viscosity of the LC.

The mechanism of the electrohydrodynamic effect is different in the characteristic frequency regions $\omega < \tau_e^{-1} \sim (4\pi\sigma/\epsilon)$ and $\omega > \tau_e^{-1}$, where $\tau_e$ is the relaxation time of the volume electric charge, and $\sigma$ and $\epsilon$ are respectively the average conductivity and the dielectric constant of the LC. In the former case the mechanism is mainly connected with the anisotropy of the conductivity $\sigma_a = \sigma_\parallel - \sigma_\perp \sim \sigma/2$, and in the latter case it is connected with the anisotropy of the dielectric constant $\epsilon_a$ ($|\epsilon_a| \ll \epsilon$). Depending on the field E, the turbulent flow of the LC can be due to either the first or the second mechanism.

2. We now consider the region of values of E corresponding to pulsation frequencies $\omega$ satisfying the inequalities $\tau_s^{-1} < \omega < \tau_e^{-1}$. Since $|\epsilon_a| \ll a$, it follows that the anisotropy $\epsilon_a$ can be neglected if the field is not too strong. Turbulent flow of the LC corresponds to a potential difference V satisfying the condition $\tilde{V} = (\epsilon\rho/4\pi\alpha^2)^{1/2}V > 1$. We can therefore neglect the viscosity in (2), and when K = 0 and $\epsilon_a = 0$ Eq. (5) contains only ratios of the viscosity coefficients. Thus, we have at our disposal the parameters $\rho$, $l$, $(\epsilon/4\pi)^{1/2}E$, with the aid of which we obtain, from dimensionality considerations, the main parameters of the large-scale motion:

*[nh] ≡ n × h.

$$Z \sim l, \quad v_Z \sim \frac{l}{\tau_0}\tilde{V}, \quad \omega_Z \sim \frac{\tilde{V}}{\tau_0}, \quad \tau_0 = \frac{\rho l^2}{\alpha}. \tag{6}$$

By $v_Z$ we must take here to mean the change of the velocity over the extent of the main Z scale, whereas the average velocity is equal to zero.

The density $\rho_e$ of the volume electric charge, which gives rise to the large-scale turbulence in the electric field, is of the order of $\rho_e \sim \epsilon E/4\pi l$. The volume charge connected with the electric current does not play a noticeable role at scales $\zeta \ll Z$, and the pulsational motion at these scales receives energy mainly from the large-scale motion. The turbulence properties at scales $\zeta \ll Z$ are described in the spirit of ordinary turbulence theory[4], and the condition $\tilde{V} \gg 1$ actually corresponds to large Reynolds numbers $R_Z \sim \rho v_Z Z/\alpha \sim \tilde{V}$ characterizing the "developed turbulence." In the considered region of fields E and thicknesses $l$, the average dissipation q, the scale $\zeta_0$ over which the viscosity begins to play any role, and the corresponding frequency $\omega_{\zeta_0}$, as well as the total number N of degrees of freedom, are of the order of

$$q \sim \left(\frac{\alpha\tilde{V}}{\rho}\right)^3\frac{1}{l^4}, \quad \zeta_0 \sim \frac{l}{\tilde{V}^{3/4}}, \quad \omega_{\zeta_0} \sim \frac{\tilde{V}^{3/2}}{\tau_0}, \quad N \sim \tilde{V}^{9/4}. \tag{7}$$

It is seen from (2) that the term of order $\rho_e E$ is small over scales $\zeta \ll Z$ in comparison with terms that are quadratic in $v_\zeta \sim (q\zeta)^{1/3}$, so that the turbulence in question is isotropic.

The unique feature of the turbulence in the LC consists, however, in the fact that in sufficiently strong fields the viscosity begins to influence the main scale Z, which becomes a function of E, $\alpha$, and $\rho$. The reason for this is that the contribution of the dielectric mechanism, in spite of the smallness of $|\epsilon_a|$, is comparable in strong fields with the contribution of the anisotropic conductivity. Namely, in Eqs. (5) the term of order $|\epsilon_a/4\pi|E^2$ becomes comparable with terms of order $\alpha v_Z/Z \sim \alpha^2\tilde{V}/\rho l^2$ at values $\tilde{V} \sim \delta^{-1} \sim |\epsilon/\epsilon_a|$. At $\tilde{V} > \delta^{-1}$ we find from (2) that $v_Z \sim \tilde{V}l/\tau_0$, and the main scale, according to (5), is of the order of $Z \sim l/\delta\tilde{V} < l$. Accordingly, $\omega_Z \sim \delta\tilde{V}^2/\tau_0$. In this case there is also a flow of energy from the large-scale pulsations to the small-scale ones, and here $q \sim \delta\tilde{V}^4\alpha^3/\rho^3 l^4$. The smallest scale over which the viscosity in the Navier-Stokes equations becomes significant is now equal to $\zeta_0 \sim l/\delta^{1/4}\tilde{V}$.

The turbulent LC flow considered above, is connected with the volume electric charge existing in the mesophase. It is necessary here to satisfy in the large-scale pulsations the condition $\omega_Z < \tau_e^{-1}$, which takes the forms $\tilde{V} < \tau_0/\tau_e$ and $\tilde{V} < (\tau_0/\delta\tau_e)^{1/2}$ in the regions $\tilde{V} < \delta^{-1}$ and $\tilde{V} > \delta^{-1}$, respectively. Thus, the anisotropy conductivity mechanism plays a decisive role in the phenomenon under consideration, if the electric fields are not too strong (at $E < (4\pi\alpha/\delta\epsilon\tau_e)^{1/2}$). If the characteristic frequency of the pulsational motion is much larger than $\tau_e^{-1}$, and only the dielectric mechanism operates, then the terms that depend on the electric field in the Navier-Stokes equations (2) and in the director equations (5) are proportional to the quantity $|\epsilon_a/4\pi|E^2$. In this case (at $\tilde{V} > (\tau_0/\delta\tau_e)^{1/2} \gtrsim (1/10^2\delta)^{1/2}$) analogous estimates of the parameters Z, $v_Z$, and $\omega_Z$ yield the following orders of magnitude:

$$Z \sim \frac{l}{\delta^{1/2}\tilde{V}}, \quad v_Z \sim \frac{l}{\tau_0}\delta^{1/2}\tilde{V}, \quad \omega_Z \sim \frac{\delta\tilde{V}^2}{\tau_0}. \tag{8}$$

We note that here the number $R_Z$ is of the order of unity, and consequently $\zeta_0 \sim Z$ and $\omega_{\zeta_0} \sim \omega_Z$, i.e., such a motion is characterized by a single scale. The average energy dissipation q is of the order of $\epsilon_\alpha^2 E^4/16\pi^2\rho\alpha$. The condition $\omega_Z \gg \tau_S^{-1}$, at which the foregoing estimates are valid, are satisfied here, for in this case

$$\tau_s^{-1} \sim K/\alpha Z^2 \sim \omega_Z\rho K/\alpha^2 \ll \omega_Z$$

($\rho K \ll \alpha^2$ in nematic LC). The angles $\theta$ through which the director is deflected from the initial direction are of the order of unity in the described pulsational motion. It is seen from (2) and (5) that pulsations with $\theta \ll 1$ cannot occur, for then the terms that depend on the electric field are small in the Navier-Stokes equations.

3. Let us examine the damping of the turbulent motion after the electric field is turned off. The damping of the pulsations in the LC includes two physically different processes. The first process is analogous to damping of ordinary turbulence. To determine the laws governing such damping, we can use the condition that the angular momentum contained in a region having a dimension on the order of Z be constant[4]. We note that the motion of the director makes a small contribution to the angular momentum (owing to the smallness of the moment of inertia per unit volume of the LC[5]), which is determined mainly by the velocity and scale of the liquid flow. The frequency $\omega_Z(t)$ of the damped pulsation decreases with time t, and at a certain instant $t^*$ becomes a quantity of the order of the reciprocal structural relaxation time $\tau_S^{-1}$. At $\omega_Z \lesssim \tau_S^{-1}$, the energy of the large-scale flow turns into heat, since dissipation sets in, and during the time of the dissipation the inhomogeneous distribution of the director over large scales relaxes to the unperturbed state. This interrupts the energy flow from the large-scale to the small-scale pulsations. An important role in this process is played by the elastic properties of the LC. The relaxation of the spatial distribution of the director at $t > t^*$ takes place within a time $\tau_S$, which is of the order of $\alpha Z^2(t^*)/K$ if $Z(t^*) \ll l$ or $\alpha l^2/K$ if $Z(t^*) \gtrsim l$. The last case corresponds to a wall-layer thickness of the order of $h \sim (K/\alpha\omega_Z)^{1/2} \sim l$.

By determining the functions Z(t) and $\omega_Z(t)$ in accordance with[4], we obtained from the relation $\omega_Z(t^*) \sim \tau_S^{-1}$ the value of $\tilde{t}^* = \gamma t^*/\tau_0$ and different regions of values of E and $l$:

$$\tilde{t}^* \sim \gamma^{1/3}\tilde{V}^{2/3}, \qquad 1 < \tilde{V} < \min\left\{\frac{1}{\gamma^{1/2}}, \frac{\tau_0}{\tau_e}\right\}; \tag{9a}$$

$$\tilde{t}^* \sim 1, \quad \frac{1}{\gamma^{1/2}} < \tilde{V} < \min\left\{\frac{\gamma^{1/3}}{\delta^{5/3}}, \left(\frac{\tau_0}{\delta\tau_e}\right)^{1/2}\right\}; \tag{9b}$$

$$\tilde{t}^* \sim \frac{\gamma^{1/3}}{\delta^{5/3}\tilde{V}}, \quad \frac{\gamma^{1/3}}{\delta^{5/3}} < \tilde{V} < \min\left\{\frac{1}{\gamma^{-1/3}\delta^{5/3}}, \left(\frac{\tau_0}{\delta\tau_e}\right)^{1/2}\right\}; \tag{9c}$$

$$\tilde{t}^* \sim \frac{1}{\gamma^{1/3}\delta^{10/3}\tilde{V}^2}, \quad \frac{1}{\gamma^{2/3}\delta^{5/3}} < \tilde{V} < \left(\frac{\tau_0}{\delta\tau_e}\right)^{1/2}; \tag{9d}$$

$$\tilde{t}^* \sim \frac{\gamma^{1/3}}{\delta^{1/2}\tilde{V}}, \quad \frac{1}{\gamma^{2/3}\delta^{1/2}} > \tilde{V} > \left(\frac{\tau_0}{\delta\tau_e}\right)^{1/2}; \tag{10a}$$

$$\tilde{t}^* \sim \frac{1}{\gamma^{1/3}\delta\tilde{V}^2}, \quad \tilde{V} > \max\left\{\frac{1}{\gamma^{2/3}\delta^{1/2}}, \left(\frac{\tau_0}{\delta\tau_e}\right)^{1/2}\right\}. \tag{10b}$$

Here $\gamma = \rho K/\alpha^2$. Formulas (9) correspond to the onset of turbulence in an electric field because of the conductivity mechanism, while formulas (10) correspond to the dielectric mechanism. In cases (9a), (9c), (9d), and (10) we have

$$R_z(t^*) \sim \rho v_z(t^*) Z(t^*) / \alpha < 1,$$

and in the case (9b) we have $R(t^*) > 1$. In cases (9a, b, c) and (10a) we have $\tau_s^{-1} \sim K/\alpha l^2$. In cases (9d) and (10b) we have $\tau_s^{-1} \sim K/\alpha Z^2(t^*) \sim \gamma/t^*$, and the structure relaxation occurs here over dimensions $Z(t^*) \sim (\alpha t^*/\rho)^{1/2} \ll l$.

Connected with the pulsations of the director n and with the corresponding pulsational change $\Delta\epsilon_{ik} = \epsilon_a n_i n_k$ of the dielectric constant is the experimentally observed "dynamic scattering" of light. The light scattering stops when the electric field is turned off. The LC layer regains its transparency effectively within the time during which the pulsations over the scales $\xi \lesssim l$ vanish. This time is of the order of $t^* + (\tau_0/\gamma)$ at $Z(t^*) \lesssim l$, or $t^*/\gamma$ if $Z(t^*) \ll l$. Relations (9) and (10) show that the quantity $\tilde{t}^* \sim \gamma t^*/\tau_0$ depends in a complicated manner on the reduced potential difference $\tilde{V}$, and does not depend on the thickness $l$. At the present time there are experimental data[6] that reveal qualitatively such a dependence of the time of vanishing of the "dynamic scattering" on the electric field and on the thickness of the LC layer.

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[1]S. A. Pikin, Zh. Eksp. Teor. Fiz. 60, 1185 (1971) [Sov. Phys.-JETP 33, 641 (1971)].

[2]L. D. Landau and E. M. Lifshitz, Elektrodinamika sploshnykh sred (Electrodynamics of continuous media), Fizmatgiz, 1959, Chap. 7 [Addison-Wesley, 1959].

[3]S. A. Pikin, Zh. Eksp. Teor. Fiz. 61, 2133 (1971) [Sov. Phys-JETP 34, 1137 (1972)].

[4]L. D. Landau and E. M. Lifshitz, Mekhanika sploshnykh sred (Fluid mechanics), Fizmatgiz, 1953, Chap. 3 [Addison-Wesley, 1958].

[5]Orsay Liquid Crystal Group, J. Chem. Phys., 51, 816 (1969).

[6]L. K. Vistin', Dissertation, Crystallography Institute of the Academy of Sciences, 1971.

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