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COLLISION BETWEEN AN ELECTROMAGNETIC WAVE AND A GRAVITATIONAL WAVE PACKET

Yu. G. SBYTOV

All-union Scientific Research Institute of Optico-physical Measurements

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The passage of a test electromagnetic wave through a plane gravitational wave packet is considered. The electromagnetic and gravitational waves propagate in opposite directions. The behavior of light rays (isotropic geodesics) in the field of the gravitational wave is investigated. It is shown that the light rays are focused by the gravitational wave; the nature of this focusing is studied as a function of the form of the functions describing the gravitational field. Analysis of the solution of the Maxwell equations, carried out under the assumption that the electromagnetic wave has a leading edge and is plane prior to entry into the gravitational field, shows that the electromagnetic wave penetrates the gravitational wave packet or reaches its back edge at a definite depth corresponding to the point where the light rays are focused. The electromagnetic wave is then reflected and subsequently propagates behind the gravitational wave.

THE investigation of the physical properties of gravitational fields is usually based on the study of their action on test bodies whose world lines are timelike geodesics. Certain elements of the action of a gravitational field on an electromagnetic wave can also be studied by studying the behavior of isotropic geodesics—the world lines of photons. The solution of the Maxwell equations, however, allows us to obtain a great deal more information about the effect of gravitational fields on an electromagnetic wave. Unfortunately, owing to the complexity of the Maxwell equations in a curved space-time, we are able to solve them in only a few high-symmetry cases. One such case, which is especially interesting since it concerns gravitational waves, is considered in the present work. We investigate the behavior of an electromagnetic wave as it passes through a plane gravitational wave packet. It is assumed that prior to its entry into the gravitational field, the electromagnetic wave is also a plane wave and propagates in the direction opposite to that of the gravitational wave.

It is convenient for the solution of the problem to use the optical tetrad-frame formalism[1,2]. We introduce at each point of 4-space a reference frame consisting of isotropic real vectors k^μ and m^μ ($k_\mu k^\mu = m_\mu m^\mu = 0$, $k_\mu m^\mu = 1$) and a complex vector t^μ ($t_\mu t^\mu = t_\mu k^\mu = t_\mu m^\mu = 0$, $t_\mu \bar{t}^\mu = -1$), such that the metric tensor $g_{\mu\nu}$ is expressed in terms of them in the following fashion:

$$g_{\mu\nu} = 2k_{(\mu}m_{\nu)} - 2t_{(\mu}\bar{t}_{\nu)}.$$

From the optical reference frame we construct the bivectors: $V_{\mu\nu} = k_\mu t_\nu - k_\nu t_\mu$, $U_{\mu\nu} = m_\mu \bar{t}_\nu - m_\nu \bar{t}_\mu$, $M_{\mu\nu} = k_\mu m_\nu - k_\nu m_\mu - t_\mu \bar{t}_\nu + t_\nu \bar{t}_\mu$ satisfying the condition $V_{\mu\nu} = -iV^*_{\mu\nu}$, etc., where the asterisk designates the dual quantity.

We shall assume that the quantities describing the gravitational wave are characterized by a dependence on the phase function $u(x^\alpha)$, while those describing the electromagnetic wave are characterized by a dependence on the phase function $v(x^\alpha)$. The phases u and v are analogs of the retarded and advanced times. Since the hypersurfaces u = const and v = const are isotropic hypersurfaces, we choose the vectors k_μ and m_μ to be tangent to them, i.e.,

$$k_\mu = u_{,\mu}, \quad m_\mu = v_{,\mu}, \tag{1}$$

in which case $k_{\mu;\nu}k^\nu = m_{\mu;\nu}m^\nu = 0$.

The plane gravitational wave is described in our problem by the Riemann tensor, which admits of the representation

$$R_{\mu\nu\lambda\sigma} - iR_{\mu\nu\lambda\sigma} = \Psi V_{\mu\nu} V_{\lambda\sigma}, \tag{2}$$

the expansion parameter of the rays of the gravitational wave[1] $\rho = -\frac{1}{2}k^\mu_{;\mu} = k_{\mu;\nu}t^\mu \bar{t}^\nu$ being equal to zero. It is precisely in this sense that the gravitational wave is called plane. In the expression (2) Ψ is some complex scalar function.

Let us choose as the coordinates x^0, x^1 the phases u and v respectively; x^A (A is equal to 2 and 3) are the two other coordinates. Then it follows from the system of the Newman-Penrose equations[3], which are equivalent to the Einstein equations, that $\Psi = \Psi(u, x^A)$. We shall consider the particular case of a homogeneous gravitational wave packet, when $\Psi = \Psi(u)$ and $\Psi(u) \neq 0$ for $u_1 < u < u_2$, and $\Psi = 0$ for $u < u_1$, $u > u_2$. This means that when $u < u_1$ (region Ω_0) and $u > u_2$ (region Ω_2) the space-time is flat, and only when $u_1 < u < u_2$ (region Ω_1) is the space-time curved. We can, for simplicity, assume that $\Psi(u)$ is a continuous function of u.

Let us concretize the coordinate system (u, v, x^A) in such a way that in the region Ω_0 the vectors k^μ, m^μ and t^μ have the form

$$k^\mu = \delta_1^\mu, \quad m^\mu = \delta_0^\mu, \quad t^\mu = 2^{-1/2}(\delta_2^\mu + i\delta_3^\mu),$$
$$k_\mu = \delta_\mu^0, \quad m_\mu = \delta_\mu^1, \quad t_\mu = -2^{-1/2}(\delta_\mu^2 + i\delta_\mu^3), \tag{3}$$

while the metric has the form

$$ds^2 = 2dudv - (dx^2)^2 - (dx^3)^2. \tag{4}$$

In the region Ω_1

$$k^\mu = \delta_1^\mu, \quad m^\mu = \delta_0^\mu + Q_2\delta_2^\mu + Q_3\delta_3^\mu, \quad t^\mu = 2^{-1/2}(\delta_2^\mu + i\delta_3^\mu), \tag{5}$$
$$k_\mu = \delta_\mu^0, \quad m_\mu = \delta_\mu^1, \quad t_\mu = 2^{-1/2}Q\delta_\mu^0 - 2^{-1/2}(\delta_\mu^2 + i\delta_\mu^3), \quad Q = Q_2 + iQ_3;$$
$$ds^2 = -Q\bar{Q}du^2 + 2dudv + 2Q_2dudx^2 + 2Q_3dudx^3 - dx^{2^2} - dx^{3^2}. \tag{6}$$

The functions Q_A satisfy a system of equations obtained from the Newman-Penrose system, in which Ψ and the Ricci rotation coefficients also figure as unknown functions

$$\mu = \bar{\mu} = -m_{\mu;\nu}t^\mu\bar{t}^\nu, \quad \lambda = -m_{\mu;\nu}\bar{t}^\mu\bar{t}^\nu:$$
$$\frac{\partial Q}{\partial \bar{z}} = \bar{\lambda}, \qquad \frac{\partial Q}{\partial z} = \mu, \tag{7}$$

$$\frac{\partial\mu}{\partial u} + Q\frac{\partial\mu}{\partial z} + \bar{Q}\frac{\partial\mu}{\partial\bar{z}} = -\mu^2 - \lambda\bar{\lambda}. \tag{8}$$
$$\frac{\partial\lambda}{\partial u} + Q\frac{\partial\lambda}{\partial z} + \bar{Q}\frac{\partial\lambda}{\partial\bar{z}} = -2\mu\lambda + \frac{1}{2}\Psi,$$

In Eqs. (7) and (8)

$$z = x^2 + ix^3, \qquad \frac{\partial}{\partial\bar{z}} = \frac{1}{2}\left(\frac{\partial}{\partial x^2} + i\frac{\partial}{\partial x^3}\right).$$

The initial condition for Eqs. (7) and (8) is:

$$\text{for } \ u \leqslant u_1 \quad \mu = \lambda = Q = 0. \tag{9}$$

Since $\Psi = \Psi(u)$, then for the initial conditions (9) the functions μ and λ will depend only on u and the system (8) can be written as:

$$\frac{d\lambda}{du} = -2\mu\lambda + \frac{1}{2}\Psi, \qquad \frac{d\mu}{du} = -\mu^2 - \lambda\bar{\lambda}. \tag{10}$$

It follows from Eqs. (7) that

$$Q = \bar{\lambda}(u)\bar{z} + \mu(u)z + q(u). \tag{11}$$

By means of the coordinate transformation

$$z' = z - a(u),$$

where

$$da/du = \mu a + \bar{\lambda}\bar{a} + q,$$

we can make the function q(u) vanish. We shall assume this below. In the region Ω_2 the functions Q, λ, and μ satisfy by continuity Eqs. (7) and (10), with the exception that $\Psi = 0$.

Before solving the Maxwell equations in the gravitational field, let us investigate the behavior of the rays of the electromagnetic wave, i.e., of the isotropic geodesics with the tangent vectors m^μ. The behavior of these rays is clearly described by the expansion parameter

$$\mu = {}^1\!/_2 m_{;\mu}{}^\mu = -m_{\mu;\nu}t^\mu\bar{t}^\nu$$

and the deformation parameter λ[1]. The expansion parameter is the rate of change of the area of a small section of the wave surface as the wave propagates divided by the area itself, while the deformation parameter characterizes the deformation of a small section of the wave surface. Under wave surface, corresponding to a definite phase v = const, is to be understood in the present case a two-dimensional surface defined by the conditions u = const, v = const, the various successive positions of the same wave surface as it propagates with the velocity of light in 3-dimensional space corresponding to different increasing values of the coordinate u for v = const.

If a gravitational wave of the type (2) is incident on a set of test particles or photons, then the action of each of its phases is such that some of the particles which at a given moment are on the gravitational wave surface receive positive relative accelerations, while the others gain negative relative accelerations. In other words the gravitational wave brings some particles together and makes others move away from one another. The action of the gravitational wave over a finite interval of time may intensify this effect, but it may turn out to be more complex, depending on the form of the function $\Psi(u)$ in the interval (u_1, u_2).

Let us to begin with consider the case when the real and imaginary parts of the function Ψ have a definite sign in the interval (u_1, u_2). It can be seen from the second equation of the system (10) that $\mu < 0$ when $u > u_1$, while it follows from the first that Re λ and Im λ have the same signs as Re Ψ and Im Ψ respectively. In the region Ω_2 Eq. (10) is easy to solve:

$$\mu = [u^* - {}^1\!/_2(R_1 + R_2)][(u^* - R_1)(u^* - R_2)]^{-1}, \tag{12a}$$

$$\lambda = \lambda_2 R_1 R_2[(u^* - R_1)(u^* - R_2)]^{-1};$$
$$u^* = u - u_2, \quad R_1 = (|\lambda_2| - \mu_2)^{-1}, \quad R_2 = -(\mu_2 + |\lambda_2|)^{-1}, \tag{12b}$$
$$\mu_2 = \mu(u_2), \quad \lambda_2 = \lambda(u_2).$$

If the functions μ and λ are nonsingular in the interval (u_1, u_2), then $\mu_2 < 0$, and we can show that $|\lambda_2| > |\mu_2|$ in the case under consideration (definiteness of Re Ψ and Im Ψ). Then $R_1 > 0$, $R_2 < 0$. This means that at the point $u = \bar{u} = u_2 + R_1$ the functions μ and λ are singular. The existence of the singularity is due to the fact that the gravitational field causes some points of the electromagnetic wave surface to draw together and other points to move away from each other. The relative velocities gained by the points of the electromagnetic wave surface (the accelerations are equal to zero) result, when the surface emerges from the gravitational field, in the surface collapsing into a line at the instant $u = \bar{u}$. From the point of view of an observer in the gravitational wave or behind its back edge, the electromagnetic wave surface is a convexo-concave surface, although this effect is produced not by the linear curvature of the surface but by the nature of the variation of the distance between the points of the wave surface.

The above-considered behavior of the light rays can be observed when the functions μ and λ are continuous in the interval (u_1, u_2) and when Re Ψ and Im Ψ do not preserve their sign in the interval (u_1, u_2). In the latter case, however, another situation when $|\lambda_2| < |\mu_2|$ and $R_1 > 0$, $R_2 > 0$ is possible. The electromagnetic wave surface will appear to an observer in the region Ω_2 as a double concave surface of unequal radii of curvature. The surface will degenerate into a line at the points $u = u_2 + R_1$ and $u = u_2 + R_2$. Physically, such an effect comes about as a result of the fact that when the signs of Re Ψ and Im Ψ are not preserved, the directions of the positive and negative relative accelerations of the photons can change, so that the subsequent phases of the gravitational wave may cause those photons which had earlier moved away from each other to draw together. Finally, the case when μ and λ have a singularity in the interval (u_1, u_2) is possible.

Let us turn to the solution of the Maxwell equations in the gravitational field in question. We shall assume that μ and λ do not have singularities in the interval (u_1, u_2), and that $|\lambda_2| > |\mu_2|$, i.e., $R_1 > 0$, $R_2 > 0$. For an arbitrary electromagnetic field the combination $F_{\mu\nu} - iF^*_{\mu\nu}$ of the electromagnetic field tensor and its dual can be decomposed[1] in terms of the bivectors $V_{\mu\nu}$, $U_{\mu\nu}$, and $M_{\mu\nu}$ introduced above:

$$F_{\mu\nu} - iF_{\mu\nu}^* = F_1V_{\mu\nu} + F_2U_{\mu\nu} + F_3M_{\mu\nu}.$$

Writing the Maxwell equations $(F^{\mu\nu} - iF^{*\mu\nu})_{;\nu} = 0$ in the system (k^μ, m^μ, t^μ), we obtain four complex equations for the scalar functions F_k $(k = 1, 2, 3)$[1]. These equations have, with allowance for the vanishing of certain Ricci rotation coefficients, the form

$$2^{1/2}\frac{\partial F_2}{\partial z} - \frac{\partial F_3}{\partial v} = 0,$$
$$2^{1/2}\frac{\partial F_1}{\partial \bar z} + \frac{\partial F_3}{\partial u} + Q\frac{\partial F_3}{\partial z} + \bar Q\frac{\partial F_3}{\partial \bar z} + 2\mu F_3 = 0,$$
$$\frac{\partial F_2}{\partial u} + Q\frac{\partial F_2}{\partial z} + \bar Q\frac{\partial F_2}{\partial \bar z} - 2^{1/2}\frac{\partial F_3}{\partial \bar z} + \mu F_2 = 0,$$
$$\frac{\partial F_1}{\partial v} + 2^{1/2}\frac{\partial F_3}{\partial z} - \lambda F_2 = 0. \tag{13}$$

The problem of the passage of an electromagnetic wave through a gravitational wave packet reduces to the solution of Eqs. (13), with the functions μ and λ defined by Eqs. (10) when $u > u_1$, while when $u \le u_1$, $\mu = \lambda = 0$. The initial condition for the functions F_k is prescribed thus:

$$F_1 = F_3 = 0, \quad F_2 = f(v) \quad \text{for } u \le u_1. \tag{14}$$

The condition (14) implies that prior to its entry into the gravitational wave packet, the electromagnetic wave is a plane wave propagating toward the gravitational wave.

Since the hypersurface u = const is a characteristic of the Maxwell equation, we must introduce one more condition for the determination of the solution. We shall require that the function $f(v)$ be different from zero when $v > v_1$. This implies that the electromagnetic wave has a leading edge. Thanks to the last condition, the function F_1 can be expressed in terms of the functions F_2 and F_3 thus:

$$F_1 = \int_{v_1}^{v}\left(\lambda F_2 - 2^{1/2}\frac{\partial F_3}{\partial z}\right)d\eta. \tag{15}$$

As can be seen from the equations of the system (13) and the conditions (14) and (15), the functions F_k do not depend on x^A and are of the form

$$F_3 = 0, \quad F_2 = f(v)\exp\left(-\int_{u_1}^{u}\mu(\xi)\,d\xi\right), \tag{16a}$$

$$F_1 = \lambda(u)\exp\left(-\int_{u_1}^{u}\mu(\xi)\,d\xi\right)\int_{v_1}^{v}f(x)\,dx. \tag{16b}$$

Thus, upon entry into the gravitational field the electromagnetic wave, which is heretofore a uniform field (in the sense that the invariants $F^{\mu\nu}F_{\mu\nu}$ and $F^{\mu\nu}F^*_{\mu\nu}$ vanish), ceases to be uniform. Its algebraic structure is expressed by the relation:

$$F_{\nu\mu} - iF_{\mu\nu}^* = F_1V_{\mu\nu} + F_2U_{\mu\nu}, \tag{17}$$

where F_1 and F_2 are given by the formulas (16a) and (16b). Physically the structure (17) leads to a situation in which the electric and magnetic intensity vectors determined in the local tetrad frame are not equal in magnitude and not perpendicular to each other, and the energy flux density is not equal (up to the factor c) to the energy density.

In the region Ω_2, when $u_2 < u < \bar u$, we can, using the formulas (12a) and (12b), derive explicit expressions for F_1 and F_2,

$$F_2 = f(v)\,|R_1R_2|^{1/2}[(R_1 - u^*)(u^* + |R_2|)]^{-1/2}\exp\left(-\int_{u_1}^{u_2}\mu(\xi)\,d\xi\right), \tag{18a}$$

$$F_1 = \lambda_2|R_1R_2|^{3/2}[(R_1 - u^*)(u^* + |R_2|)]^{-3/2}\exp\left(-\int_{u_1}^{u_2}\mu(\xi)\,d\xi\right)\int_{v_1}^{v}f(x)\,dx. \tag{18b}$$

The functions F_1 and F_2 become infinite at the point $\bar u = u_2 + R_1$. In order to interpret physically the singularity of the functions F_1 and F_2, let us compute the energy and the energy flux of the electromagnetic wave relative to an observer with 4-velocity $V^\mu = h^\mu(0) = m^\mu + \frac{1}{2}k^\mu$. The world lines corresponding to the chosen vector field V^μ are timelike geodesics. Relative to these observers the electromagnetic wave propagates in the direction $h^\mu(1) = m^\mu - \frac{1}{2}k^\mu$. Let us also introduce two unit spacelike vectors $h^\mu(2)$ and $h^\mu(3)$, which together with $h^\mu(0)$ and $h^\mu(1)$ form a reference frame:

$$h^\mu(2) = 2^{-1/2}(t^\mu + \bar t^\mu), \quad h^\mu(3) = -i2^{-1/2}(t^\mu - \bar t^\mu).$$

Relative to the given system of the observers the electric and magnetic vectors determined by the reference-frame components of the tensor $F_{\mu\nu}$[4] are equal to

$$E_i = F_{\mu\nu}h^\mu(i)h^\nu(0) = \left\{0, 2^{-1/2}\left(\operatorname{Re}F_1 + \frac{1}{2}\operatorname{Re}F_2\right), 2^{-1/2}\left(\operatorname{Im}F_1 - \frac{1}{2}\operatorname{Im}F_2\right)\right\}, \tag{19}$$

$$H_i = F_{\mu\nu}^*h^\mu(0)h^\nu(i)$$
$$= \left\{0, 2^{-1/2}\left(\operatorname{Im}F_1 + \frac{1}{2}\operatorname{Im}F_2\right), 2^{-1/2}\left(-\operatorname{Re}F_1 + \frac{1}{2}\operatorname{Re}F_2\right)\right\}.$$

The energy density W and energy flux density P^i are respectively equal to

$$W = \tfrac{1}{2}(E^2 + H^2) = \tfrac{1}{2}(|F_1|^2 + \tfrac{1}{4}|F_2|^2), \quad P^i = \{P, 0, 0\}, \tag{20}$$

$$P = \frac{1}{8}|F_2|^2 - \frac{1}{2}|F_1|^2 = \frac{1}{2}\exp\left(-2\int_{u_1}^{u}\mu(\xi)\,d\xi\right) \times\left[\frac{1}{4}|f(v)|^2 - |\lambda(u)|^2\left|\int_{v_1}^{v} f(x)\,dx\right|^2\right]. \tag{21}$$

The expression in the square brackets in (21) vanishes at a certain value $\tilde{u}$ ($\tilde{u} < \bar{u}$) and then becomes negative. This shows that the electromagnetic energy corresponding to a definite phase v = const penetrates the gravitational wave packet or reaches its back edge at a certain distance. The electromagnetic energy is then reflected and subsequently propagates behind the gravitational wave. The point $\bar{u} = u_2 + R_1$ determines the maximum penetration depth of the electromagnetic energy. Notice that for $u \to \bar{u}$, $|F_1| \gg |F_2|$ and the electromagnetic field becomes a uniform field, i.e., a pure wave that propagates behind the gravitational wave.

Although the system of the observers with the 4-velocities $V^\mu = m^\mu + \tfrac{1}{2}k^\mu$ is convenient for the computation of the energy and the Poynting vector, it conceals a number of distinctive features of the behavior of the electromagnetic field, since the world lines of the observers are not parallel geodesics. Let us construct in the region Ω_2 a tetrad system consisting of the vectors:

$$\begin{aligned} h^\mu(0) &= (\tfrac{1}{2} + B\bar{B})k^\mu + m^\mu + \bar{B}t^\mu + B\bar{t}^\mu, \\ h^\mu(1) &= (-\tfrac{1}{2} + B\bar{B})k^\mu + m^\mu + \bar{B}t^\mu + B\bar{t}^\mu, \\ h^\mu(2) &= -2^{-1/2}(t^\mu + \bar{t}^\mu) - 2^{-1/2}k^\mu(B + \bar{B}), \\ h^\mu(3) &= -i2^{-1/2}(t^\mu - \bar{t}^\mu) - i2^{-1/2}k^\mu(B - \bar{B}), \end{aligned} \tag{22}$$

where $B = -2^{-1/2}(\mu z + \overline{\lambda z})$. It can be shown that the vectors (22) are unit tangent vectors to the Cartesian coordinates (t, x^1, x'^2, x'^3), the latter being related to the coordinates u, v, x^2, and x^3 by the transformation

$$\begin{aligned} t &= u + x^1 = \tfrac{1}{2}u + v + \tfrac{1}{2}[\mu z\bar{z} + \tfrac{1}{2}(\lambda z^2 + \bar{\lambda}\bar{z}^2)], \\ x^1 &= v - \tfrac{1}{2}u + \tfrac{1}{2}[\mu z\bar{z} + \tfrac{1}{2}(\lambda z^2 + \bar{\lambda}\bar{z}^2)], \\ x'^2 &= x^2, \quad x'^3 = x^3. \end{aligned} \tag{23}$$

Relative to the tetrad system (22), i.e., in terms of the Cartesian coordinates (t, x^1, x^2, x^3), the electromagnetic energy density and the components of the Poynting vector are equal to

$$\begin{aligned} W &= \tfrac{1}{2}[|F_1|^2 + (\tfrac{1}{2} + |B|^2)^2|F_2|^2 - B^2F_1\bar{F}_2 - \bar{B}^2\bar{F}_1F_2], \\ P_1 &= \tfrac{1}{2}[(\tfrac{1}{4} - |B|^4)|F_2|^2 - |F_1|^2 + B^2F_1\bar{F}_2 + \bar{B}^2\bar{F}_1F_2], \\ P^2 &= 2^{-3/2}[(B + \bar{B})(\tfrac{1}{2} + |B|^2)|F_2|^2 - BF_1\bar{F}_2 - \bar{B}\bar{F}_1F_2], \\ P^3 &= -i2^{-3/2}[(\bar{B} - B)(\tfrac{1}{2} + |B|^2)|F_2|^2 + \bar{B}\bar{F}_1F_2 - BF_1\bar{F}_2]. \end{aligned} \tag{24}$$

The equation of the family of electromagnetic wave surfaces corresponding to a definite phase v = const coincides with the second equation of the system (23):

$$v = x^1 + \tfrac{1}{2}u - \tfrac{1}{2}[\mu z\bar{z} + \tfrac{1}{2}(\lambda z^2 + \bar{\lambda}\bar{z}^2)]. \tag{25}$$

It is easy to obtain from Eq. (25) the unit vector **n** of the normal to the two-dimensional wave surface corresponding to v = const and t = const. The vector **n** indicates the direction of propagation of the electromagnetic wave in the Cartesian system of coordinates:

$$\mathbf{n} = \left\{\frac{|B|^2 - \tfrac{1}{2}}{|B|^2 + \tfrac{1}{2}}, \frac{(\mu + \operatorname{Re}\lambda)x^2 - \operatorname{Im}\lambda x^3}{|B|^2 + \tfrac{1}{2}}, \frac{(\mu - \operatorname{Re}\lambda)x^3 - \operatorname{Im}\lambda x^2}{|B|^2 + \tfrac{1}{2}}\right\}. \tag{26}$$

The expression (26) shows that the direction of propagation of the electromagnetic wave at each spatial point (with the exception of the axis $x^2 = x^3 = 0$) varies in time as u increases, until, in the limit, as $u \to \bar{u}$, it coincides with the direction of propagation of the gravitational wave. The distinguishability of the axis $x^2 = x^3 = 0$ is connected with the choice of the given Lorentz reference frame; when we go over to another Lorentz reference frame another direction turns out to be the preferred direction.

Comparison of the components of the Poynting vector (24) with the vector **n** shows that the direction of energy propagation does not coincide with the direction of propagation of the wave, but the energy flux also changes its sign when $u \to \bar{u}$. Indeed, for u close to u_1 the vector $P^i \approx P^1 \approx \tfrac{1}{8}|F_2|^2$. When $u \to \bar{u}$, μ, λ, and, consequently, B are very large, and we have

$$P^i \approx P^1 \approx -\tfrac{1}{2}|B|^4|F_2|^2.$$

Thus, a "reflection" of sorts of the electromagnetic wave takes place when it penetrates into the field of the gravitational wave. The same result is obtained in the case when $|\lambda_2|^2 < |\mu_2|^2$, i.e., when $R_1 > 0$, $R_2 > 0$. The maximum penetration depth of the electromagnetic wave is determined by the singularity $\bar{u} = u_2 + R_1$ closest to u_2. The same situation evidently obtains when μ and λ become infinite inside the gravitational wave packet.

An interesting phenomenon is observed if the electromagnetic wave is itself a wave packet, i.e., if f(v) $\neq 0$ for $v_1 < v < v_2$. Then, as can be seen from Eqs. (16a) and (16b), when $v > v_2$ the quantity $F_2 = 0$, but F_1 does not vanish:

$$F_1 = \lambda(u)\exp\left(-\int_{u_1}^{u}\mu(\xi)\,d\xi\right)\int_{v_1}^{v_2} f(x)\,dx \neq 0.$$

This means that an observer in the gravitational field or behind the back edge of the gravitational wave will, when $v > v_2$, detect the "tail" of the electromagnetic radiation in the form of an electromagnetic wave propagating behind the gravitational wave.

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77