Investigation of the Heating of a Plasma lon Component by a Collisionless Shock Wave

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The energy spectra of ions in a plasma heated by a collisionless shock wave are obtained by the technique of passive corpuscular diagnostics. Under conditions in which an aperiodic shock front with resistive dissipation is formed in the plasma, ion heating is manifest in the appearance of a small group of ions ($\sim 10\%$) with mean energies of the order of the electron temperature. The remaining ions are cold. The results are in agreement with the theoretical model which explains the origin of this particle group as being due to Landau linear damping of ion acoustic oscillations excited in a shock wave by resonance ions located at the "tail" of the distribution function. The main energy content of the plasma is determined by the electron component. Under conditions in which breaking of the shock front occurs, predominant heating of the ion component of the plasma is observed, the ion distribution function being close to the isotropic Maxwellian distribution.

I HE various methods currently in use for the study of microfluctuations in the front of a collisionless shock wave in a plasma give consistent results, indicating the existence of superthermal fluctuations of the electromagnetic field^[1]</sup> and the density,^{<math>[2]} the characteristic</sup></sup> spatial dimensions of which extend into the shortwave region down to Debye lengths. The evidence obtained on the characteristic scale, level and threshold of excitation of the oscillations, together with the results of macroscopic measurements^[1,3]</sup> (the shock front thicknesses, electric conductivity, etc) furnish a basis for assuming that the anomalous dissipation in a collisionless shock wave is connected with the ion acoustic instability. However, full clarity has not been obtained in understanding the microprocesses that take place in the front. In particular, there remains the question as to the proper connection of the estimate of the equilibrium noise level in the theory: what sort of physical mechanism limits the growth of the instability? Comparison of the results of nonlinear theory of weak turbulence^{L4} with experiment has shown that one must evidently seek an additional mechanism, through which we can lower the level of saturation of the ion acoustic oscillations. Such a stabilizing action can show a transfer of the energy of the ion acoustic noises to the resonance ions found in the "tail" of the distribution function. In this model, which was proposed by Vekshtein and Sagdeev.[4] the ratio of the concentrations of cold and resonance ions will be

$$n_1/n_2 \approx (m_i/m_e)^{1/4},$$
 (1a)

the mean energy of the latter is of the order of the electron temperature

$$\langle E_2 \rangle \approx T_e,$$
 (1b)

and the shock front width $\Delta \approx (c/\omega_0)(m_i/m_e)^{1/4} (\omega_0 \text{ is the electron Langmuir frequency}).$

The present research is devoted to the investigation of the energy spectrum of plasma ions heated by a transverse shock wave. This investigation is of special interest in that it makes possible the experimental testing of the quantitative relation (1), and consequently the correctness of the proposed model of [4]. Moreover, information on the energy spectrum of the ions is of interest in understanding processes in shock flow in transition through the so-called "critical Mach number" $M_{C2} = 4.5 - 5.5$: in this case, according to various indirect experimental data of [3,6], one can expect a significant change in the effectiveness of the ion component of the plasma as a result of the breaking of the shock front.

The energy spectrum of the ions was found from the energy distribution of the flow of neutral charge-ex exchange atoms emitted from the plasma which was subjected to shock heating.^[7,8] The application of this method to the study of the collisionless shock wave is very effective because of the existence of plasma flows, whose velocity shifts the energy spectrum of the thermal motion of the ions into the energy region >100 eV, where the diagnostic apparatus used has a sufficient sensitivity level. This makes it possible to record particles with energies of thermal motion amounting to several electron volts.

EXPERIMENTAL ARRANGEMENT

The experiments were carried out on the apparatus UN-4^[9] (Fig. 1). The hydrogen plasma with a density $n = 10^{13}-10^{14}$ cm⁻³, placed in a cylindrical volume in a quasistationary magnetic field $H_0 = 10^2-10^3$ Oe, was subjected to a rapid compression by the variable magnetic field ($H_{\sim} = (2-3) \times 10^3$ Oe, T/4 $\approx 0.5 \mu$ sec). The cylindrical shock wave that develops moves to the axis of the system and accumulates. The diagnostic apparatus consists of a stripping chamber (stripping on the hydrogen, $p \approx (4-5) \times 10^{-4}$ mm Hg) and a differential electrostatic analyzer (for separating the ions according to their energy) located at the base of the cylindrical capacitor. Recording of the ions was done by means of an electron multiplier. Similar apparatus and the results of its calibration are described in^[8].

Neutral charge-exchange particles were introduced from the plasma volume through a ceramic tube with

¹⁾According to^[5], in the formulation of collisionless shock waves in a plasma, it is necessary to distinguish two critical Mach numbers: $M_{c1}=2.8-3$ and $M_{c2}=4.5-5.5$. At $M < M_{c1}$, a shock front is formed with resistive dissipation; at $M_{c1} < M < M_{c2}$, a shock front is formed with an isomagnetic discontinuity. At $M > M_{c2}$, destruction of the isomagnetic discontinuity takes place, accompanied by the formation of a multispeed plasma flow, which can be interpreted as the breaking of the shock wave.



FIG. 1. Schematic diagram: 1-probkotron magnetic trap; 2-shock loop; 3-preionization loop; 4-vacuum chamber; 5-magnetic probe; 6-ceramic tube; 7-vacuum valve; 8-deflecting capacitor; 9-stripping chamber; 10-needle-leak valve; 11-ion energy analyzer; 12-electron multiplier.

internal diameter 5 mm, set along the radius in the central cross section of the shock loop. In order to eliminate cumulative effects from consideration, the tube was extended beyond the axis of the system (the distance from the entrance port to the axis was $r_1 = 22$ mm). The determination of the basic parameters of the shock wave (velocity u, relative amplitude $h = (H_0 + H_{\sim})/H_0$, profile of the electric and magnetic fields) was made by means of probe pickups, ^[1,3] located along the radius ($r_1 = 22$ mm, $r_2 = 32$ mm). We note that the presence of the tube in the plasma volume did not distort the profiles of the electric and magnetic fields in the shock front. Figure 2 shows typical oscillations of the signals with the magnetic and electric probes and the output of the electron multiplier.

EXPERIMENTAL RESULTS

The method of determination of the energy spectrum of the ions under the conditions of a collisionless shock wave has been set forth in detail $in^{[8,10-12]}$. Analysis of the shape of the signals from the electron multiplier and the dependence of their length on the energy of the recorded particles show that the energy spectra obtained are averages over all states of the shock wave taken on by the shock in its path from the periphery of the plasma volume to the entrance into the tube.^{2) [10,12]} On this path, its velocity, and consequently the directed motion of the ions $V_I + U(1 - 1/h)^{[3]}$ increase continuously, reaching maximal values at the mouth of the tube. Thus, the recorded energy spectrum dn/dE = f(E) represents the sum of the energy spectra of the thermal motion of the ions moving in the direction of observation with various velocities. Consequently, the shape of the spectrum dn/dE = f(E) is determined both by the heating of the ions in the shock front and by the law of change of their directed velocity on the interval of wave propagation.³⁾ As was shown $in^{[13,14]}$, in the case of cylindrical geometry, there is in this section a sufficiently extended region where the wave velocity and the shock jump amplitude remain practically unchanged. The presence of a quasistationary region furnishes a basis for expecting that the energy spectrum of the directed motion, which the ions in this region have, cannot be monotonically decreasing (under the condition that the end of the tube is in this zone). The subsequent behavior of the spectrum, the characteristic mark of which is a more or less rapid decay, will be determined by the heating of the ions.

 $In^{[12]}$, a numerical calculation was performed and expressions obtained which approximate the experimental distribution functions of the ions for two cases: when the velocity of the directed motion of the ions on a portion of the wave propagation is constant and when it increases linearly from zero to some maximum value $v_{d max}$. Here it was assumed that the distribution function of the thermal velocities of the ions is isotropic Maxwellian. The expressions obtained have the following form: for the first case

$$\frac{dn}{dE} \sim \sqrt{E} \exp\left[-\frac{(\sqrt{E} - \sqrt{E}_{\rm d})^2}{T_{\rm i}}\right], \qquad (2)$$

and for the second,

$$\frac{dn}{dE} \sim \sqrt{E} \left[\operatorname{erf} \left(-\sqrt{\frac{E}{T_i}} \right) - \operatorname{erf} \left(-\frac{\sqrt{E} - \sqrt{E_{d \max}}}{\sqrt{T_i}} \right) \right]. \quad (3)$$

Numerical calculation shows that in spite of the different character of the laws of change of the directed velocity of the ions as the shock wave moves to the axis of the system, the maximum of the distribution function in both cases will be observed for $E\approx E_{d\,max}$ (and the closer to $E_{d\,max}$ the larger $E_{d\,max}$ is than T_i) and the rate of the decrease of dn/dE for $E \gtrsim E_{d\,max}$ for the same temperature will be practically the same. Thus we reach the conclusion that the law of change of the directed velocity of the ions has a slight effect on the determination of the value of $E_{d\,max}$ from the maximum of the distribution function and of the temperature of the ions from its decay.



FIG. 2. Typical oscillograms of signals from the magnetic probe (1), from the electric probe (2), from the electronic amplifier from ions with energies of 1 kV (3) $(a-M < M_{c1}, b-M \gtrsim M_{c2})$.

²⁾At the same time, it is necessary to note that the accuracy of determination of the momentum and the location of its "start" (as a consequence of the reduction in the time of flight along the analyzer) increases with increase in the energy of the recorded particles. For ions with energies larger than 1 keV, one can say with assurance that they "start" from the plasma, arriving at the entrance to the ceramic tube (and not at the periphery of the plasma volume).

³⁾The distribution along the plasma column radius of the neutral atom density, which are ion charge-exchange centers, also affects the shape of the recorded spectrum. However, in first approximation, it can be regarded as close to the equilibrium distribution, inasmuch as the gas desorbed (under the action of the initial plasma) from the walls of the chamber and the tube, at the starting moment of the shock loop, manages to distribute itself throughout the entire volume.

Figure 3 shows the energy spectrum of the ions that is typical for a regime with small Mach numbers $(M < M_{c_1})$. It has three characteristic sections: the first with relatively weak changes of dn/dE = f(E), the portion of rapid decay and the high energy tail. If we take E = 400 - 450 eV for the maximum value of the directed energy, with which the rapid fall off of the spectrum begins, then it is excellently confirmed by the data of the probe measurements ($E_{d max} \approx 420 \text{ eV}$). The temperature of the ions, determined by means of the approximation of the rapid decay region by the expressions (2) and (3), lies in the range 4-9 eV. If we take into account the adiabatic compression of the plasma in the shock front, then the temperature of the ions T_{io} in the initial plasma corresponds to the obtained values of T_i, equal to 1-2, 2-4 or 2.5-5 eV depending on the number of degrees of freedom (three, two or one) in the thermal distribution. For such temperatures and densities of 10^{13} - 10^{14} cm⁻³, the initial plasma can be assumed to be equilibrium-as estimates show, the time between the beginning of the pre-ionization chain and the shock loop (>30 μ sec) is sufficient to establish such a distribution. (We note that the temperature of the electrons in the initial plasma, measured by means of Langmuir probes, amounts to 0.5-2 eV.)

It is seen from Fig. 3 that the approximation of the experimental distribution by functions obtained on the basis of a Maxwellian isotropic velocity distribution of the ions is possible only up to some value of the energy $E_1 \approx 650$ eV. Two facts must be noted in the subsequent behavior of the experimental distribution. First, much larger values of dn/dE are observed than follow from the equilibrium distribution. Second, as calculation shows, the ion energy $E_p = (\sqrt{E_{d \max}} + \sqrt{T_c/2})^2$ which has, in the reference frame of the plasma flow, a velocity equal to the phase velocity⁴) of the ion-acoustic oscillations, $c_s \approx \sqrt{T_e/m_i}$, excited in the shock front, is always greater than $\rm E_{i}.$ (Thus, in the case under consideration, $\rm E_{p}\approx 10^{3}~eV>E_{1}.)$ These two most reliably established experimental facts are in qualitative agreement with the Vekshtein-Sagdeev theoretical model.[4]

The test of the ensuing quantitative relations (1) unfortunately encounters difficulties. These are connected with the fact that the ion distribution function in the moving set of coordinates⁵⁾ corresponding to the Maxwellian value v_d of the ions cannot be obtained from the averaged distribution in general form. However, if we take the model proposed in^[4] as a working model in the analysis of the experimental data, then it is easy to show that the estimate of the relations (1) is possible.

First of all, we note that the heating of the resonance ions leads to changes only in the high energy "tail" of the initial ion distribution. This region, occupied in the spectrum by resonance ions, should be bounded on the low energy side by some value E'_i , which will be determined both by the spectrum and by the amplitude of the



FIG. 3. Typical energy spectrum of ions for the case $M < M_{c1}$. The parameters are: M = 1.8, h = 2, $n = 1.4 \times 10^{13}$ cm⁻³, $H_0 = 520$ Oe. 1– approximation by means of Eq. (2), $T_i = 6 \text{ eV}$, $E_{d \max} = 408 \text{ eV}$; 2approximation by means of Eq. (3), $T_i = 7.5 \text{ eV}$, $E_{d \max} = 480 \text{ eV}$.



FIG. 4. Typical energy spectrum of ions for the case $M \gtrsim M_{c2}$. The parameters are: M = 4.5, h = 5.4, $n = 7 \times 10^{13}$ cm⁻³, $H_0 = 280$ Oe. The continuous line is the computed curve. $T_i = 12 \text{ eV}, T_2 =$ 130 eV, $E_{d \max} = 380 \text{ eV}, n_2/n_1 \approx 8.$

ion-acoustic noise. If all the ions with $E' > E'_i$ are attributed to resonance, then, independent of the shape of their distribution, there is an unambiguous connection between the value of E'_1 and the value of the ratio of the densities of the "cold" and "hot" (resonance) ions n_1/n_2 :

$$\frac{n_1}{n_2} = \frac{I(0, E_1')}{I(E_1', \infty)}, \qquad I(A, B) = \int_A^B \sqrt[y]{E'} \exp\left(-\frac{E'}{T_1}\right) dE'.$$

Thus the problem of the determination of n_1/n_2 is materially simplified-it reduces to finding the temperature of the cold ions and the boundary of the resonance region E'_1 in its distribution.

If we speak of specific changes that occur in the initial (equilibrium) distribution functions of the ions, then one can undoubtedly assume the following:⁶⁾ at the point $E' = E'_1$ a more rapid decrease in the value of dn/dE should be observed; however, in what follows, this goes over into a more gently sloping section. This fact gives a basis for connecting the characteristic turning point observed at the end of the region of fall off in the experimental distribution with the boundary energy $E_1 = (\sqrt{E_d \max} + \sqrt{E'_1})^2$.^[12] Analysis of five investigated regimes with $M < M_{c_1}$

⁴⁾The value of the electron temperature necessary for its calculation was found from independent measurements of $n_i T_e^{[5]}$ (here n_i was determined from the Mach number and the velocity of the shock wave).

⁵⁾We shall call this the set of coordinates connected with the plasma flow, and all quantities in it will be denoted by primes.

⁶⁾This follows from the law of conservation of the number of particles.

according to the presented scheme showed that the value of the ratio n_1/n_2 falls within the limits 10 ± 5 (i.e., it has the order of the theoretical estimate $n_1/n_2 \approx 7$).

With the help of graphic integration, we can estimate approximately the mean energy E_2' of the resonance ions. Here, however, we are obliged formally to assume that the high energy ''tail'' of the experimental distribution is formed by ions only with maximum E_d . Such an estimate, as it is easy to see, can only be decreased (a more accurate calculation^[12] shows that the decrease does not amount to more than 15%). The values of E_2' found in this fashion have the same order of magnitude as the electron temperature of the plasma. Thus, in the case considered, $\langle E_2' \rangle \approx 120 \ \text{eV}$ and $T_e \approx 240 \ \text{eV}$.

We can thus draw the general conclusion that the experimental results are in qualitative and satisfactory quantitative agreement with the conclusions of the theory.^[4]

We note that, in spite of the small relative concentration of resonance ions, they carry the bulk of the energy of the ion component. Thus, in the case considered, the ratio of the energy-bearing resonance and cold ions amounts to ~ 4. Here the mean energy of the ions of the plasma (per degree of freedom) is of the order of 15 eV. Comparison of the energy-bearing ion and electron components of the plasma indicates that in regimes with $M < M_{c1}$ there is a preferential heating of the electrons, which agrees with the experimental data obtained by an independent method in^[6]. Thus, in the considered case, $nT_e/2n\langle E' \rangle \approx 10$.

We proceed to consider the experimental results that apply to collisionless shock waves with $M \gtrsim M_{c_2}$. The typical distribution for this case is shown in Fig. 4. The basic difference from the graph considered above for $M < M_{c_1}$ is that the path of the experimental points is well described by the isotropic Maxwell distribution of the ions in the wave front moving with the constant velocity vd over almost the entire energy range $(T_i = 130 \text{ eV under the conditions of Fig. 4})$. Some deviation of the experimental points from this curve in the region ${\tt E} \approx {\tt E}_d$ can be explained if we add in the approximating expressions a term that corresponds to ions with $T_i = 12$ eV and concentrations one order smaller than the concentration of the "hot" ions. Possible reasons for the observed deviation can be, for example, the small adiabatic heating of the ions in the "subcritical" state of flow or their incomplete thermalization in the "transcritical" state.

The energy content of the ion component under the conditions $M\gtrsim M_{C2}$ significantly exceeds the energy content of the electron component (under the conditions of Fig. 4, $nT_i/nT_e\approx 3$), which is in agreement with indirect measurements.^[6]

In conclusion, we consider the effect of the ionic pressure behind the shock on the motion of the ions. The change of the directed energy of the ion, $\Delta E = \frac{1}{2} m_{1}(u^{2} - v_{1}^{2})$ passing through the front is given by the following expression:^[3]

$$\Delta E = e\varphi + \Delta (nT_i) / n, \qquad (4)$$

where φ is the jump in the potential at the front, v_i the

М	∆E, eV	$e\varphi$, eV	$\frac{\Delta (nT_i)/n}{eV}$
$1.5(M < M_{c1})$ $4.5(M \approx M_{c2})$	500 ± 100 300 ± 100	320 ± 80 180 ± 40	$20 \pm 5 \\ 130 \pm 20$

velocity of the ion behind the front in the set of coordinates attached to the wave, $\Delta(nT_i)/n$ the increment in the thermal energy of the ions.

Experiments carried out earlier^[3] have shown that the value of $e_{\varphi}/\Delta E$ is close to unity for $M < M_{c_1}$. For $M \sim M_{c_1}$, it falls off and becomes of the order of $\frac{1}{2}$ for large Mach numbers, which indicates the important effect of the ionic pressure behind the shock front on the vionic motion. The joint application in the present research of corpuscular and probe methods, which guarantees the determination of all necessary parameters, has made it possible to verify the relation of the components in (4). As is seen from the table, the term $\Delta(nT_i)/n$, which corresponds to the ionic pressure behind the shock front, is small in comparison with e_{φ} for $M < M_{c_1}$, but of the order of $e\varphi$ for $M \approx M_{c_2}$. Thus, the conclusion arrived at $in^{[3]}$ from indirect measurements, as to the important effect of the ionic pressure behind the front of the wave on the motion of ions across the front, is confirmed for $M \gtrsim M_{c_2}$.

We can thus make the general conclusion that the results do not contradict and, more than that, confirm the current theoretical notions as well as the experimental conclusions of $[^{3,5,6}]$ concerning the processes that take place in the front of a collisionless shock wave in plasma. Furthermore, they show the great effectiveness of the method used.

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