Depolarization in Elastic pn Scattering at 612 MeV

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The depolarization was measured by scattering the polarized proton beam of the Dubna synchrocyclotron $(E=612 \text{ MeV}, P_1=0.332\pm0.011)$ by deuteron neutrons. Quasielastic pn scattering events were separated out by fast electronics. The third (analyzing) target was carbon and the events were recorded with optical spark chambers. The depolarization D_{pn} at center-of-mass angles of 52, 94, and 125° was found to be 0.96 ± 0.14 , 0.67 ± 0.11 , and 0.49 ± 0.24 , respectively. Possible systematic errors have been analyzed and it was found that the experimental results were in good agreement with the predictions of the phase shift analysis of nucleon-nucleon scattering.

 ${f M}$ EASUREMENTS of depolarization in elastic pn scattering at 612 MeV (see also^[1]) were undertaken with a view to obtaining additional information on the interaction of nucleons in the isotopic spin singlet, which is necessary for the unambiguous determination of the elastic nucleon-nucleon scattering amplitude at 600-650 MeV. Figure 1 shows two curves giving the depolarization D_{pn} as a function of the CM scattering angle calculated from the two sets of phase shifts which were available at 635 MeV when this work was begun.^[2] It is clear from the figure that curves 2 and 3 are quite different and that by measuring the depolarization at 52° and 125° one should be able to reject one of the two possibilities. Measurements of the depolarization at 94°, where the two sets of phase shifts lead to essentially the same predictions, can be used as a check.

1. FORMULATION OF EXPERIMENT

Depolarization in proton-neutron scattering is usually determined by carrying out triple-scattering experiments. The first scatter is used to produce a polarized proton beam which is then scattered by a neutron target (second scatter). The depolarization Dpn is determined from the asymmetry in the angular distribution resulting from the third (analyzing) scatter.

The Wolfenstein formula^[3] relates the differential cross section I₂ for the scattering of polarized nucleons by nucleons and the polarization $\langle \sigma_2 \rangle$, on the one hand, and the triple-scattering parameters, on the other. When the planes of the first and second scatters are parallel, i.e., the normals n₁ and n₂ to these planes are parallel, the Wolfenstein formulas assumes the form

$$l_2\langle \sigma_2 \rangle = \{ I_{02}(P_2 + D(\mathbf{P}_1\mathbf{n}_2))\mathbf{n}_2 \}, \qquad (1$$

where \mathbf{P}_1 is the initial polarization of the incident nucleons, I_{02} and \mathbf{P}_2 are the differential scattering cross section and the polarization produced during the scattering of unpolarized nucleons by an unpolarized target, and D is the depolarization which is a measure of the change in the initial polarization. On the other hand, it is well known that

$$I_2 = I_{02}[1 + P_1P_2]$$
 (2)

and, consequently, we have from Eqs. (1) and (2)

$$\langle \sigma_2 \rangle = \frac{[P_2 + D(\mathbf{P}_1 \mathbf{n}_2)]\mathbf{n}_2}{1 + \mathbf{P}_1 \mathbf{P}_2}.$$
 (3)

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FIG. 1. Depolarization D_{pn} as a function of scattering angle for two sets of phase shifts taken from $[^2]$.



For the third scatter we have, by analogy with Eq. (2),

$$I_3 = I_{03}[1 + \mathbf{P}_3 \langle \sigma_2 \rangle]. \tag{4}$$

Taking into account the directions of P_1 , P_2 , P_3 , and n_2 , we have from Eqs. (3) and (4)

$$I_{\mathfrak{s}}(\theta_{\mathfrak{s}},\varphi_{\mathfrak{s}}) = I_{\mathfrak{s}\mathfrak{s}}(\theta_{\mathfrak{s}}) \left[1 + \frac{(P_{\mathfrak{s}} \pm DP_{\mathfrak{s}})P_{\mathfrak{s}}(\theta_{\mathfrak{s}})\cos\varphi_{\mathfrak{s}}}{1 \pm P_{\mathfrak{s}}P_{\mathfrak{s}}} \right],$$
(5)

where θ_3 and φ_3 are the polar and azimuthal angles in the third scatter, and $P_3(\theta_3)$ is the analyzing power of the third target. The upper and lower signs represent the "up" and "down" polarizations.²⁾

By measuring the angular distribution of the particles after the third scatter, $I_3(\theta_3, \varphi_3)$, we can determine the parameter D either from the left-right asymmetry of the distribution, or by the maximum probability method, using Eq. (5). The latter enables us to determine the triple-scattering parameters at low particle flux and low detection efficiency. It was, therefore, chosen for our own measurements of depolarization in pn collisions. Optical spark chambers were used to determine the angular distribution of protons after the third scatter.

²⁾Here and henceforth, the "up" direction is assumed to be that of the normal to the plane of scattering in a right-handed coordinate set:

$$\mathbf{n} := [\mathbf{k}_1 \mathbf{k}_2] / |[\mathbf{k}_1 \mathbf{k}_2]|,$$

where \mathbf{k}_1 and \mathbf{k}_2 are the momenta of the particles before and after scattering.

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FIG. 2. Method of producing the polarized proton beam.

2. CHARACTERISTICS OF THE POLARIZED PROTON BEAM

Polarization.³⁾ The vertically polarized beam was produced by scattering 667 MeV protons from carbon. The experimental arrangement is illustrated schematically in Fig. 2. Protons extracted from the Dubna synchrocyclotron chamber were deflected by iron blocks placed in the fringe field of the accelerator magnet. They were then focused by the quadrupoles Q, deflected through $6.3 \pm 0.4^{\circ}$ by the auxiliary magnet M1, and finally intercepted by the carbon polarizer C (23.4 g/cm^2). The solid line in Fig. 2 shows the path of protons with upward polarization, whereas the broken line shows the path for protons with downward polarization. The scattered beam was collimated, deflected through 28.2° by the magnet M2, and focused. This removed neutral and low-energy charged particles. The beam was then passed through the steel collimator (length 4 m, diameter 5 cm) in the shielding wall, and was deflected through 1.8° by the magnet M3 onto the second target.

Proton-proton scattering was used for analyzing purposes to determine the beam polarization. The choice of hydrogen as the analyzer was motivated by the fact that the polarization produced in pp scattering at 595–635 MeV has been measured with high accuracy.^[5-7] Moreover, if the polarized-beam intensity is high enough, asymmetry measurements after second (analyzing) scatter can be carried out at relatively large angles, where the polarization due to pp scattering reaches a maximum and the product $I_2(\theta_2, \varphi_2)P(\theta_2)$ is a slow function of the scattering angle. This tends to reduce the probability of spurious asymmetry.

In addition to statistical errors, the following systematic uncertainties were taken into account in the polarization measurements:

1) error in the asymmetry measurement due to the uncertainty in the first scattering angle—not more than 0.005 per run;

2) spurious asymmetry due to the nonuniform distribution of protons in the beam along the target in the horizontal direction—not more than 0.04;

3) spurious asymmetry due to asymmetric positioning of the counters with respect to the beam axis and uncertainty in the second scattering angle—not more than 0.001;

4) error in the determination of the beam polariza-



FIG. 3. Diagram illustrating depolarization measurement.

tion due to the energy spread in the proton beam-not more than 0.004.

Measurements of the "upward" and "downward" polarizations yielded the following results:

$$P_x^{\dagger} = 0.329 \pm 0.012, \quad P_x^{\downarrow} = 0.341 \pm 0.020$$

Since these quantities were statistically consistent, their average was taken to be the beam polarization P_1 , i.e.

$$P_1 = 0.332 \pm 0.011$$
.

Energy and density. According to^[8], where the experimental conditions were similar to our own, the measured value of the polarization refers to 612 \pm 9 MeV. A direct estimate of the mean energy of the proton beam was carried out by using a current-carrying wire drawn through the deflecting magnet M2 (see Fig. 2). The energy was found to be 614.3 \pm 5 MeV, which is in good agreement with^[8]. This is also confirmed by estimates of the beam energy losses in the first target.

The proton beam intensity was determined from the counting rate in a number of independent scintillation counter systems connected in coincidence, which identified pp events when the beam was scattered by a CH_2 target. The proton detection efficiency was assumed to be 100%. The intensities obtained with the different counter systems were found to be close to one another, and the mean polarized beam intensity was estimated as

$$I_0 \approx 2.4 \cdot 10^5 \,\mathrm{protons/cm^2 \cdot sec}$$
.

3. DEPOLARIZATION MEASUREMENTS

<u>Apparatus</u>. The apparatus for depolarization measurements is illustrated schematically in Fig. 3. It can

³⁾Measurements of the beam polarization were described in greater detail in ^[1].

be used for measurements at two angles simultaneously, or at a single angle with two sets of statistics [scatter-ing through $\theta_2(\varphi_2 = 0)$ and $\theta_2(\varphi_2 = \pi)$ in the laboratory system].

The polarized proton beam produced as described above was passed through a collimator and was intercepted by the "neutron" target. The latter was formed by the deuteron neutrons and measurements were carried out of the CD_2 and C difference. The CD_2 target thickness was 4.65 g/cm² and that of the carbon target was 9 g/cm².

The system which detected quasielastic pn events from one part of the apparatus (the second part was completely identical with the first; in Fig. 3 the elements corresponding to it are indicated by letters with primes) consisted of four scintillation counters. The telescope $C_1 + C_2$ defined the recoil protons. The neutron detector C_3 was arranged in coincidence with this telescope, and the anticoincidence counter AC in front of the neutron detector excluded pp events.

The change in the initial proton polarization due to second scatter was analyzed by a thin-walled optical spark chamber SC1, which contained the carbon analyzer C. The analyzer was divided into two parts in order to reduce multiple-scattering effects. The thickness of the carbon target was varied, depending on the energy of the analyzed protons. At angles of 52° and 94° the analyzer target thickness was 9 g/cm^2 (the mean energy losses were 21 and 33 MeV at 52 and 94° , respectively) and at 125° the target thickness was 4.5and 2.7 g/cm^2 (mean energy losses of 34 MeV in each).

The small spark chamber SC2 was placed in front of the anticoincidence counter at the associated (neutron) angle. It was used as a check on the efficiency of the anticoincidence channel: the absence of tracks in this chamber was used as a criterion for selecting pn events from the photographs. Tracks in the large and small spark chambers were photographed on the same frame of a high-sensitivity 35-mm film.

A block diagram of the electronics is shown in Fig. 4. The pn events were defined by coincidences between C1, C2, and C3 and the absence of the gating pulse from the anticoincidence counter AC. Fast coincidence and anticoincidence circuits were employed. The time resolution of the coincidence circuits was about 4 nsec.

The fact that the direction of the primary-beam polarization could be varied, and the measurements could be performed at an angle θ_2 to the left and right of the beam, enabled us to determine the parameter D_{pn} for all four combinations of the first and second scatters, i.e., in the notation of^[3] we could examine left-left (LL), left-right (LR), right-left (RL), and right-right (RR) scatters (Fig. 5).



FIG. 4. Block diagram of the electronics.



FIG. 5. Combinations of first and second scatters which could be used in the experiment.

<u>Measurements</u>. The experimental conditions were as follows. The protons were detected with nearly 100% efficiency, and the neutron detection efficiency was ~10%. The anticoincidence channel suppressed the pp events by a factor of 1000, and the background in the absence of the target was 0.01%.

The beam polarization was not varied during a particular run, and runs with different directions of P_1 were taken in succession. The experimental data at 52° were obtained for both parts of the apparatus in equal volume. Most of the data (~80%) at 125° were accumulated on one side of the setup (LR and RR scattering), whilst the statistical material at 94° refers to LL and RL scattering alone. Altogether 45 000 m of film were exposed (about 2.25×10^{6} frames).

Analysis. As noted above, the particle tracks in the large and small spark chambers were photographed in two projections onto mutually perpendicular planes. Useful events were selected from these films on the basis of the following criteria:

a) only those frames were taken in which the large chamber contained a proton track scattered from one of the targets, while the small chamber showed no track, or the inclination of the track to the axis of the small chamber was greater than allowed by the geometry of the experiment $(\pm 5^{\circ})$;

b) frames containing "clearly inelastic" scattering events were not included (forked tracks in a large chamber):

c) frames showing scattered tracks of two or more protons and scattering from the spark chamber plates were also rejected.

The spark chamber photographs were scanned with the semiautomatic PIP-35^[9] and PIF-1^[10] devices which produced a punch-tape record of the data. The angular distribution of the protons scattered by carbon was determined from this data on the Minsk-22 and BESM-4 computers, as described in detail in^[11]. The accuracy with which θ_3 an φ_3 were determined is also discussed in.^[11] Altogether 90 000 frames were selected and examined (51 000 at 52°, 21 000 at 94°, and 18 000 at 125°). The next step was to select events according to the angle of incidence on the spark chamber and the angular resolution of the apparatus, and on the basis of the scattering angles $\theta_{3\min}$ and $\theta_{3\max}$ $(6-30^{\circ})$. The final number of useful events is about 62 000 (see table below) of which about 22% are background events due to the carbon target.

θ_2 , deg cms	D_{pn}	β	Number of events
52 ± 5	0.96 ± 0.14	0,001±0.008	33200
94±4	0.67 ± 0.11	0,033±0.011	16000
125±5	0.49 ± 0.24	0.011±0,013	12600

The parameter D_{pn} was determined from the angular distribution of protons after the third scatter, using the method of maximum probability.^[12] The probability function has the form

$$L = \prod_{i=1}^{n} \left[1 + \frac{(P_2 \pm D_{pn}P_i)P_3(\theta_{si})\cos\varphi_{si}}{1 \pm P_1P_2} + \beta \sin\varphi_{si} \right],$$

where P_1 is the initial proton beam polarization, P_2 is the polarization after elastic pn scattering at a given angle, and $P_3(\theta_{3i})$ is the analyzing power of carbon. In contrast to Eq. (5), the probability function includes the additional term $\beta \sin \varphi_{3i}$, which is used to exhibit the possible presence of spurious up-down asymmetry in the angular distribution after third scatter. To determine D_{pn} we used values of P_2 obtained from an analysis of polarization data on elastic pn scattering at 600 MeV^[7] and 635 MeV^[5] by the least-squares method, and the $P_3(\theta_3)$ curves taken from^[13-17]. The error in D_{pn} is given by the diagonal element of the error matrix.

The background due to the carbon was subtracted in accordance with the expression

$$D_{p_n} = (1 - K)^{-1} (D_{CD_2} - KD_C)$$

where D_{CD_2} and D_C are the depolarizations measured for CD_2 and carbon, respectively, and 1 - K and K are the relative probabilities of pn scattering by deuteron neutrons and carbon, respectively. They were determined from counting rates due to pn events from these targets (K = 0.26 ± 0.01).

4. RESULTS

The results obtained for D_{pn} , averaged over the two directions of P_1 , and at 52° and 125° over the values obtained in the two different parts of the apparatus as well, are shown in the table together with the values of β which characterize the up-down asymmetry of the proton distribution in the spark chamber. The errors shown in the table include errors in P_1 and P_3 as well as purely statistical errors.

In the analysis of the data we also considered the following possible sources of systematic errors.

1. The error in the measurements of the azimuthal angle, $\Delta \varphi_3 = \pm 3^{\circ}$,^[11] which was not taken into account in the analysis of the angular distributions by the maximum probability method. To estimate the effect of the error in φ_3 , the φ_3 distributions were rotated through $\pm 5^{\circ}$. The result of this was that the value of the parameter varied within the range $\pm 4\%$. The true difference is probably smaller still because the error in φ_3 produces only a slight smearing of the distribution and not a rotation of it.

2. Errors due to multiple Coulomb scattering. These were eliminated by the choice of the minimum third scatter angle $\theta_{3min} = 6^\circ$, which exceeds the root mean square multiple-scattering angle by a factor of roughly three. 3. The difference between the initial "up" and "down" polarizations. Measurements of the beam polarizations showed (see above) that in our experiments these quantities were equal to within experimental error.

4. "Instrumental" asymmetry in the distribution of protons in the spark chamber associated with the adjustment of the optical system of the photographic equipment. Measurements performed at 52° in the two different lineups of the installation for physically identical combinations of the first and second scatters (respectively, LL and RR, and LR and RL scattering; Fig. 5) gave equal values of D_{pn} to within experimental error.^[1] This indicates the absence of a systematic uncertainty due to imprecise adjustment of the optical system.

5. Error in Dpn due to possible systematic error in the analyzing power P_3 connected with the fact that some data on the analyzing power of the carbon which we used in the analysis refer to elastic pC scattering. In our experiment, in addition to elastically scattered protons, we also recorded inelastically scattered protons. The true values of P_3 may therefore differ appreciably from those employed. This difference probably has a particularly pronounced effect on the measurements of D_{pn} at angles at which the recoil proton energy is sufficiently high. For example, in the analysis reported in^[1] of the data obtained at $\theta_2 = 52^\circ$ (recoil proton energy 460 MeV) with an analyzing power measured for purely elastic pC scattering,^[13,14] the values of D_{pn} for nonidentical combinations of the first and second scatter (LL and RL or RR and LR) differed by more than three standard deviations. In view of this, we were forced to carry out measurements of the analyzing power^[15] under conditions which were close to those prevailing in the present experiment. The use of values of P_3 which were thus obtained reduced the above discrepancy down to two standard deviations

 $(\,D_{pn}^{LL\,,RR}$ = 0.75 \pm 0.20 and $D_{pn}^{RL\,,LR}$ = 1.18 \pm 0.20).4)

The effect of the above factors on the value of D_{pn} measured at 94° (proton energy 266 MeV) is small in comparison with the statistical error in the result. This is so because the fraction of inelastically scattered protons is relatively small and the mean polarization in elastic pC scattering^[16] is higher by a factor of almost two, as compared with the situation at 52°.

In the analysis of the data obtained at 125° , we used values of P_3 corrected for scattering on carbon levels.^[17] However, a possible source of systematic errors in this case may be the energy dependence of the analyzing power P_3 , since the energy difference between protons incident on the first and second analyzing targets is approximately 25%. The effect of this energy dependence on D_{pn} was estimated by separate analysis of scattering in the first and second targets, using the corresponding values of the analyzing power. The results showed that the energy dependence modified the value of D_{pn} by less than 3%.

It is also important to note that by taking the average of the data obtained for different combinations of first

⁴⁾According to the adopted criteria, these values may be considered to be consistent.



FIG. 6. Experimental results: \triangle -at 635 MeV, $[^5] \bigcirc$ -at 612 MeV (present work). The curve was calculated using the phase shifts from $[^{18}]$.

and second scatters, and using the two independent lineups of the apparatus, we achieved a substantial reduction in systematic errors.

Toward the end of the experiment described above, it was pointed out in^[18] that measurements of the Wolfenstein parameter Apn could be used to discriminate against one of the two possible sets of phase shifts available earlier at $630 \text{ MeV}^{[2]}$ with a probability of errors of the first kind of $10^{-3}\%$ (set II in the notation of^[2]). The resulting values of the depolarization D_{pn} fully confirm this result. Figure 6 shows the depolarization as a function of the scattering angle θ_2 , calculated from the remaining set of phase shifts, [18] together with the experimental data obtained in the present work. It is clear that the agreement between the experimental points and the calculated curve is satisfactory to within experimental error. For comparison Fig. 6 shows also the value of D_{pn} at $\theta_2 = 112.3^\circ$, as reported in^[5].

It is important to note that the unambiguous result obtained in^[18] at 630 MeV as a result of the phase shift analysis is valid, strictly speaking, only under certain definite assumptions about the range of angular momenta within which the single-meson approximation can be employed, and the character of meson production processes. There has been considerable increase in the volume of experimental data on elastic nucleon-nucleon scattering in recent years. It has, in fact, become possible to verify the conclusions made in^[18] by carrying out phase-shift analyses under more general assumptions than was possible earlier.

It will be very interesting to perform measurements of D_{pn} for angles $\theta_2 \ge 140^\circ$, where the depolarization is expected to change sign (see Fig. 6). In this angular range one would have to carry out double scattering experiments from a polarized proton target, since the corresponding triple-scattering experiment is exceedingly difficult owing to the low energy of the scattered protons and the low analyzing power of the analyzing targets.^[19] We would like to thank A. P. Vorob'ev for his collaboration in the polarization measurements, V. S. Kiselev, G. D. Stoletov, and Yu. A. Batusov for many useful discussions, F. Legar for placing at our disposal the program for analyzing the data by the maximum probability method, and S. F. Pushkin and V. M. Sakovskii for their help in this work. Major contributions were made to the analysis of the spark-chamber photographs by the operators V. R. Abazova, V. A. Maksimova, Ts. Markova, T. I. Smirnova, and M. I. Shelaeva to whom we are greatly indebted.

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