Resonance Between Spin and Magnetohydrodynamic Waves in Antiferromagnetic Semiconductors and Metals

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Coupled oscillations of charge carriers and magnetic moments of atoms in antiferromagnetic semiconductors and metals are investigated in the case of an isotropic carrier dispersion law and for an arbitrary Fermi surface. It is shown that in such bodies resonance between the carrier and spin wave oscillations (MHD-spin resonance) occurs for a certain value of the external magnetic field strength. This resonance coupling of modes occurs in a broad frequency range (and not for a single resonance frequency). Fluctuation correlators of quantities characterizing the crystal are determined and also the cross sections for scattering of slow neutrons, electrons and electromagnetic waves with excitation (or absorption) of MHD-spin waves. It is shown that splitting of the cross section maxima occurs in the MHD-spin resonance point.

INTRODUCTION

As is well known, the oscillations of the carriers and the oscillations of the magnetic moments in antiferromagnetic metals and semiconductors are not independent; owing to the interaction between these oscillations, several branches of coupled electromagnetic-spin waves are produced in such antiferromagnets[1)]. So far, only two types of such waves were investigated: plasma-spin waves and helicon-spin waves[5]. Yet the coupling between the spin waves and the hydrodynamic oscillations of the carriers (MHD-spin waves) should also be of great interest[2)]. The reason is that both the MHD waves and one of the branches of the spin waves (the low-activation wave) are characterized by a linear dispersion law, and their phase velocities depend on the external magnetic field. At a definite value of the latter, the phase velocities of the MHd wave and of the spin wave coincide; then a resonant coupling of the waves arises in an entire range of frequencies (and not only at one resonant frequency as in the case of ordinary electromagnetic-spin resonance).

The phenomenon of MHD-spin resonance should become manifest primarily in a considerable increase of the surface impedance of the antiferromagnet. In the case of ordinary electromagnetic-spin resonance, a monochromatic wave is necessary in order to observe an increase of the impedance (with a very small frequency spread). In our case, when the resonance at a definite value of the magnetic field sets in in a wide frequency range (superposition of the dispersion curves), an increase of the impedance should take place for broad wave packets. Furthermore, this phenomenon should appear when slow neutrons, electrons, and x-rays are scattered in the antiferromagnet. Namely, when the magnetic field approaches its resonant value, a splitting of the sharp maxima of the differential scattering cross section should take place[3)].

[1)]Coupled electromagnetic-spin waves in ferromagnetic semiconductors and metals were investigated in[1-3] (see also the review[4]).

[2)]The possible existence of a unique MHD-spin resonance in the antiferromagnetic phase of metallic chromium was noted in[6,7], but coupled MHD-spin waves were not investigated thoroughly.

[3)]Raman scattering of light in antiferromagnets, with allowance for the coupling between the spin waves and ordinary sound was considered by Bolotin and one of the authors[8].

To observe MHD-spin resonance in antiferromagnetic metals, it is necessary to use temperatures lower than or of the order of 10°K, for at higher temperatures the MHD waves begin to attenuate strongly (mainly because of electron collisions). The resonant value of the magnetic field depends on the parameters of the antiferromagnet (the Neel temperature, the carrier density, etc.) and lies in the interval 10^3–10^5 Oe. The resonant coupling of the MHD and spin waves should take place in this case in the entire frequency range from 10^9 to 10^{12} sec^{-1}.

1. ISOTROPIC CARRIER DISPERSION. SPECTRA

We are interested in coupled MHD-spin waves in the region

$$\nu \ll \omega \ll \omega_H, \qquad kv \ll \omega_H \cos^{-1}\theta,$$
$$\omega_H = \frac{eH}{(m_1+m_2)c}, \quad \nu = \frac{m_1\nu_1+m_2\nu_2}{m_1+m_2}, \quad v = \frac{m_1v_1+m_2v_2}{m_1+m_2}, \tag{1}$$

where m_1, m_2 and ν_1^{-1}, ν_2^{-1} are the effective masses and free-path times of the conduction electrons and of the holes; v_1 and v_2 are the limiting Fermi velocities in the metal or the thermal velocities in the semiconductor; **k** and ω are the wave vector and frequency of the wave, and θ is the angle between **k** and **H** (the constant magnetic field **H** is directed along the anisotropy axis z). If[4)] $n_1 = n_2 \equiv n$ (n_1 and n_2 are the electron and hole densities) then the conductivity tensor σ is diagonal in the chosen coordinate system, and the longitudinal conductivity (along the field) is large compared with the transverse one. The transverse-conductivity components are given by[9]

$$\sigma_{ij} = \sigma_\perp \left(\delta_{ij} + \frac{3\pi}{8} e_i e_j \frac{kv\cos\theta\,\mathrm{tg}^2\theta}{\nu - i\omega}\right), \tag{2}^*$$
$$\sigma_\perp = n(m_1+m_2)c^2H^{-2}(\nu - i\omega), \quad \mathbf{e} = (kH)^{-1}[\mathbf{kH}].$$

The nonzero components of the high-frequency magnetic susceptibility tensor of an antiferromagnet with easy-plane anisotropy are[10]

$$\chi_{xx} = \frac{\chi_0\Omega_H^2}{\Omega_1^2-\omega^2}, \quad \chi_{yy} = \frac{\chi_0\Omega_1^2}{\Omega_1^2-\omega^2},$$
$$\chi_{xy} = -\chi_{yx} = \frac{i\chi_0\omega\Omega_H}{\Omega_1^2-\omega^2}, \quad \chi_{zz} = \frac{\chi_0\Omega_2^2}{\Omega_2^2-\omega^2};$$

[4)]We note that this condition is satisfied at low temperatures by many antiferromagnetic metals (for example, Cr and Mn).

*[kH] ≡ k × H.

$$\Omega_1^2 = \Omega_H^2 + (gH_1)^2 + \Omega_2^2 \sin^{-2}\Theta, \tag{3}$$

$$\Omega_2 = kv_m, \quad v_m = gH_1(\alpha - \alpha')^{1/2}(\beta - \beta')^{-1/2}\sin\Theta;$$
$$\Omega_H = gH, \quad H_1 = [2\delta(\beta - \beta')]^{1/2}M_0, \quad \delta = \chi_0^{-1}, \tag{4}$$

where χ_0 is the static magnetic susceptibility ($\chi_0 \ll 1$), M_0 is the magnetic moment of the sublattice, g is the gyromagnetic ratio, α and α' are the exchange constants, and β and β' are the magnetic-anisotropy constants. If $H < H_2$, where $H_2 = 2\delta M_0$, then in the ground state the magnetic moments of the sublattices are oriented at an angle Θ to the anisotropy axis ($\cos\Theta = H/H_2$, the y axis is chosen to be perpendicular to the equilibrium directions of the magnetic moments of the sublattices). We present also expressions for the nonzero components of the tensor $\hat{\chi}$ in the case when $H_1 < H < H_2$:

$$\chi_{xx} = \chi_{yy} = \frac{\chi_0\Omega_H^2}{\Omega_H^2 - \omega^2},$$
$$\chi_{xy} = -\chi_{yx} = \frac{i\chi_0\omega\Omega_H}{\Omega_H^2 - \omega^2}, \quad \chi_{zz} = \frac{\chi_0\Omega_2^2}{\Omega_2^2 - \omega^2} \tag{5}$$

(the formulas in (5) are valid if $\alpha k^2 \ll 1$, which is always the case by virtue of the inequalities (1)).

From Maxwell's equations we can obtain with the aid of (2)—(5) a dispersion equation for coupled MHD-spin waves with a linear dispersion law. In the cases of greatest interest (if the vector **k** is perpendicular to the x axis or if $kv \ll \omega$, the only case when weakly-damped waves are possible) this equation takes the form

$$(\omega^2 - k^2v_a^2)(\omega^2 - k^2v_m^2) - 4\pi\chi_0 k^4 v_a^2 v_m^2 \sin^2\theta = 0, \tag{6}$$

where $v_a = c\omega_H/\Omega_L$ is the Alfven velocity, and $\Omega_L^2 = 4\pi e^2 n/(m_1 + m_2)$. (We neglect the small terms proportional to ν/ω, which describe the damping of the oscillations.) Equation (6) describes the coupling of one of the spin waves with a fast magnetosonic (FMS) wave. The other spin wave (in the first approximation in χ_0) does not interact with the MHD oscillations. The Alfven MHD wave also turns out to be uncoupled with the oscillations of the spin density (in the case when $kv \gg \omega$ this wave can propagate only at the angle $\theta = 0$). The waves described by the dispersion equation (6) are in general transverse and elliptically polarized.

At a magnetic field $H = H_r$, where

$$H_r = [2(\alpha - \alpha')/e^2\chi_0]^{1/2}(m_1 + m_2)\Omega_L g M_0, \tag{7}$$

the phase velocities of the unperturbed oscillations coincide, $v_a = v_m$ (MHD-spin resonance). For antiferromagnetic metals with $n_1 = n_2$ (for example, Cr and Mn), the value of the resonant field (depending on the Neel temperature, the effective masses, and the carrier densities) lies in the interval 10^3—10^5 Oe.

At the resonant value of the magnetic field, the frequencies of the coupled waves differ from the frequencies of the spin and the FMS waves by an amount on the order of $\chi_0^{1/2}$,

$$\omega_\pm = kv_m(1 \pm \Delta), \quad \Delta = (\pi\chi_0)^{1/2}\sin\theta. \tag{8}$$

Far from the MHD-spin resonance, the corrections to the frequencies are much smaller (proportional to χ_0).

FLUCTUATIONS AND PARTICLE SCATTERING

To calculate the correlation functions, it is convenient to use the general methods of fluctuation theory[11]. Omitting the cumbersome calculations, we present the final expressions for the Fourier components of the correlators of the fluctuations of the magnetic induction for the coupled waves far from resonance (the charge density in the MHD-spin waves does not fluctuate, because these waves are transverse). For the FMS wave we have

$$\langle b^2\rangle_{\omega k} = 8\pi^2\hbar(N_\omega + 1)\omega^2\delta(\omega^2 - k^2v_a^2), \tag{9}$$

and for the spin wave

$$\langle b^2\rangle_{\omega k} = 32\pi^3\hbar(N_\omega + 1)\chi_0\frac{v_a^4\sin^2\theta}{(v_a^2 - v_m^2)^2}\omega^2\delta(\omega^2 - k^2v_m^2), \tag{10}$$

where $N_\omega = (e^{\hbar\omega/T} - 1)^{-1}$, and T is the sample temperature.

The differential cross section for the scattering of slow neutrons by a FMS wave (far from resonance), per atom, is

$$d\sigma_n = \frac{N_\omega + 1}{\pi v_n N_0}\left(\frac{\mu_n F(\mathbf{k})}{\hbar^2}\right)^2\omega^2\delta(\omega^2 - k^2v_a^2)\,dp', \tag{11}$$

where v_n and μ_n are the velocity and magnetic moment of the neutron, $F(\mathbf{k})$ is the magnetic form factor, $\mathbf{p}'$ is the momentum of the scattered neutron, $\hbar\omega$ and $\hbar\mathbf{k}$ are the change of the energy and momentum of the neutron upon scattering, and N_0 is the density of the atoms of the antiferromagnet. For scattering by a spin wave (far from resonance) we have

$$d\sigma_n = 4\chi_0\frac{N_\omega + 1}{v_n N_0}\left[\frac{\mu_n F(\mathbf{k})v_a^2\sin\theta}{\hbar^2(v_a^2 - v_m^2)}\right]^2\omega^2\delta(\omega^2 - k^2v_m^2)\,dp'. \tag{12}$$

Comparing (11) and (12), we see that far from the MHD-spin resonance the cross section for scattering of neutrons by a spin wave is smaller, in order of magnitude, by a factor χ_0^{-1} than the cross section for scattering by a FMS wave.

The scattering of electrons by MHD-spin waves (not accompanied, as indicated, by charge-density fluctuations) is due to the interaction of their spins with the fluctuations of the magnetic induction, and is therefore fully analogous to the scattering of slow neutrons. The expressions for the scattering cross sections are obtained from formulas (11) and (12) by making the substitutions

$$v_n \to v_e, \quad \mu_n \to \mu_e, \tag{13}$$

where v_e and μ_e are the velocity and the magnetic moment of the electron.

When the MHD-spin resonance point is approached ($v_a \to v_m$), the sharp maximum of the differential scattering cross section, corresponding to scattering by the FMS wave, splits into two sharp maxima that are close to each other. To obtain the correlator of the magnetic-induction fluctuations and the particle scattering cross section at the resonance point, it suffices to make in expressions (9)—(12) the substitution

$$\delta(\omega^2 - k^2v_a^2) \to {}^1/_2\{\delta(\omega^2 - \omega_+^2) + \delta(\omega^2 - \omega_-^2)\}, \tag{14}$$

where the frequencies $\omega_\pm$ are determined by formulas (8). The splitting of the maximum takes place, obviously, only in the case of weak damping of the coupled waves, $\Delta\omega \gg \nu$, where $\Delta\omega = |\omega_+ - \omega_-|$. If this condition is not satisfied, then both maxima become superimposed on one another. In this case the correlation functions and the scattering cross sections near the resonance are determined by the same formulas as in the nonresonant region.

Besides the splitting of the maxima of the scattering cross sections, the phenomenon of MHD-spin resonance should become manifest also by an appreciable increase of the surface impedance of the antiferromagnet. In the case of ordinary electromagnetic-spin resonance (intersection of the dispersion curves at a definite value of the frequency) a monochromatic wave (with a very small frequency spread) is necessary in order to observe the growth of the impedance. In our case, when the resonance at $H = H_r$ occurs in a wide frequency range (superposition of the dispersion curves), the increase of the impedance should take place for broad wave packets.

2. ARBITRARY CARRIER DISPERSION. SPECTRA

If the inequalities (1) are satisfied and $\omega \ll kv$ (this occurs in metals at $H \gtrsim 10^3$ Oe), then the transverse conductivity (relative to the field) may turn out to be large in comparison with the longitudinal conductivity in definite ranges of magnetic fields. In this case, resonance between electromagnetic and spin waves is possible for antiferromagnets with anisotropy of the easy-axis type (at $H < H_1$). Using Maxwell's equations and the known expressions for the longitudinal conductivity[12]

$$\sigma_{zz} = \frac{e^2}{k^2 \cos^2\theta} \sum (\nu - i\omega) \left|\frac{dn}{d\varepsilon_F}\right| \tag{15}$$

and for the magnetic susceptibility of the crystal.[10]

$$\begin{gathered}\chi_{xx} = \chi_{yy} = \frac{1}{2}\chi_0 \left[\frac{\Omega_3(\Omega_3 - \Omega_H)}{\Omega_3^2 - \omega^2} + \frac{\Omega_4(\Omega_4 + \Omega_H)}{\Omega_4^2 - \omega^2}\right],\\ \chi_{xy} = -\chi_{yx} = -i\chi_0\omega\left[\frac{\Omega_3 - \Omega_H}{\Omega_3^2 - \omega^2} - \frac{\Omega_4 + \Omega_H}{\Omega_4^2 - \omega^2}\right]\\ \chi_{xz} = \chi_{zx} = \chi_{yz} = \chi_{zy} = 0;\end{gathered} \tag{16}$$

$$\Omega_{3,4} = gH_1\left[1 + \frac{\alpha - \alpha'}{\beta - \beta'}k^2\right]^{1/2} \pm \Omega_H, \tag{17}$$

we obtain the dispersion equation of the coupled (low-activation) MHD-spin waves

$$(\omega^2 - \omega_f^2)(\omega^2 - \Omega_4^2) - 2\pi\chi_0\omega^2\Omega_4\Omega_H = 0, \tag{18}$$

$$\omega_f = k^2 c r_D |\sin\theta\cos\theta|, \qquad r_D = \left|4\pi e^2 \sum \frac{dn}{d\varepsilon_F}\right|^{-1/2}, \tag{19}$$

where r_D is the Debye radius and ϵ_F is the Fermi energy. (The summation in (15) and (19) is over all types of carriers.) We note that Eq. (18) describes weakly-damped waves only if $H_1 - H \ll H_1$. (The high-activation spin wave does not interact with oscillations of the electron fluid.)

At a specified value of the magnetic field and of the angle θ, the electromagnetic-spin resonance takes place at wave-vector values $k = k_0$, where

$$k_0^2 = \frac{g(H_1 - H)}{c r_D |\sin\theta\cos\theta|} \tag{20}$$

($k_0 \sim 10^{14}$ cm^{-1} and $\sim 10^5$ cm^{-1} for a metal and for a semiconductor, respectively). At the point of the electromagnetic-spin resonance, the frequencies of the coupled spin waves are given by

$$\omega_\pm = \Omega_4(1 \pm \Delta), \qquad \Delta = \left(\frac{\pi}{2}\frac{\chi_0 H_1}{H_1 - H}\right)^{1/2}. \tag{21}$$

We note that these formulas are valid at not too low values of the difference $H_1 - H$ ($H_1 - H \gg g^{-1}\nu$), for otherwise the inequalities (1) are violated. Recognizing that $\nu \gtrsim 10^9$ sec^{-1} in metals, we see that the frequencies ω_+ and ω_- are close to each other, and $\Delta \ll 1$.

FLUCTUATIONS AND SCATTERING PROCESSES

We present expressions for the charge-density and magnetic-induction fluctuation correlators for the electromagnetic wave far from resonance and for the cross section for the scattering of slow neutrons by this wave:

$$\langle\rho^2\rangle_{\omega\mathbf{k}} = \hbar(N_\omega + 1)\frac{\omega^4}{2c^2}\operatorname{ctg}^2\theta\,\delta(\omega^2 - \omega_f^2), \tag{22}$$

$$\langle b^2\rangle_{\omega\mathbf{k}} = 8\pi^2\hbar(N_\omega + 1)\,\omega^2\delta(\omega^2 - \omega_f^2), \tag{23}$$

$$d\sigma_n = \frac{N_\omega + 1}{\pi v_n N_0}\left(\frac{\mu_n F(\mathbf{k})}{\hbar^2}\right)^2 \omega^2\delta(\omega^2 - \omega_f^2)\,d\mathbf{p}'. \tag{24}$$

For a spin wave far from resonance we have

$$\langle\rho^2\rangle_{\omega\mathbf{k}} = \pi\hbar(N_\omega + 1)\chi_0 k^2(kr_D)^2\frac{\Omega_4^3\Omega_H\omega_f^2}{(\omega_f^2 - \Omega_4^2)^2}\cos^4\theta\,\delta(\omega^2 - \Omega_4^2), \tag{25}$$

$$\langle b^2\rangle_{\omega\mathbf{k}} = 16\pi^3\hbar(N_\omega + 1)\chi_0\frac{\Omega_H}{\Omega_4}\frac{\Omega_4^2\omega_f^4}{(\omega_f^2 - \Omega_4^2)^2}\delta(\omega^2 - \Omega_4^2), \tag{26}$$

$$d\sigma_n = 2\chi_0\frac{N_\omega + 1}{v_n N_0}\left[\frac{\mu_n F(\mathbf{k})\,\omega_f^2\Omega_4}{\hbar^2(\omega_f^2 - \Omega_4^2)}\right]^2\frac{\Omega_H}{\Omega_4}\delta(\omega^2 - \Omega_4^2)\,d\mathbf{p}'. \tag{27}$$

We note that the scattering of neutrons polarized along a constant magnetic field by waves of the types considered is always accompanied by spin flip.

The cross section for the scattering of electrons by coupled waves is a sum of two terms:

$$d\sigma_e = d\sigma_\rho + d\sigma_b,$$

where $d\sigma_\rho$ is the cross section for the scattering of electrons by the charge density fluctuations (electric scattering), and $d\sigma_b$ is the cross section for the scattering due to the interaction of the electron spin with the fluctuations of the magnetic induction (magnetic scattering). The cross sections for electric scattering by the electromagnetic and spin waves are

$$d\sigma_\rho = \frac{N_\omega + 1}{\pi v_e N_0}\left(\frac{e\omega^2}{\hbar^2 k^2 c}\right)^2\operatorname{ctg}^2\theta\,\delta(\omega^2 - \omega_f^2)\,d\mathbf{p}', \tag{28}$$

$$d\sigma_\rho = \chi_0\frac{N_\omega + 1}{v_e N_0}\left[\frac{e r_D\omega_f\cos^2\theta}{\hbar^2(\omega_f^2 - \Omega_4^2)}\right]^2\Omega_4^3\Omega_H\delta(\omega^2 - \Omega_4^2)\,d\mathbf{p}'. \tag{29}$$

The cross section for magnetic scattering are obtained in both cases from (24) and (27) with the aid of the substitution (13). We note that in this case

$$d\sigma_\rho/d\sigma_b \sim (mcr_D/\hbar)^2 \gg 1. \tag{30}$$

The differential cross sections for the scattering of x-rays by the electromagnetic and spin waves far from resonance are

$$d\sigma_\gamma = \hbar\frac{N_\omega + 1}{4\pi N_0}\left(\frac{e\omega^2}{mc^3}\right)^2\operatorname{ctg}^2\theta\,\delta(\omega^2 - \omega_f^2)\,d\Omega'\,do', \tag{31}$$

$$d\sigma_\gamma = \chi_0\hbar\frac{N_\omega + 1}{2N_0}\left[\frac{ek^2 r_D\omega_f\cos^2\theta}{mc^2(\omega_f^2 - \Omega_4^2)}\right]^2\Omega_4^3\Omega_H\delta(\omega^2 - \Omega_4^2)\,d\Omega'\,do'. \tag{32}$$

(Ω' is the frequency of the scattered photon).

We note that in the nonresonant region, the cross section for the scattering of neutrons, electrons, and photons by the spin waves are smaller by a factor χ_0^{-1} than the cross sections of the corresponding processes on the electromagnetic wave. At the point of the electromagnetic-spin resonance, the maximum corresponding to the intense scattering by the electromagnetic wave becomes split. To obtain the fluctuation correlators and the scattering cross sections at the resonance point, it suffices to make in formulas (22)—(24), (28), and (31) the substitution

$$\delta(\omega^2-\omega_i^2)\to {}^1/_2\{\delta(\omega^2-\omega_+^2)+\delta(\omega^2-\omega_-^2)\}, \tag{33}$$

where the frequencies $\omega_\pm$ are defined in (21).

We note in conclusion that to take into account the damping of the coupled oscillations it is necessary to make the following substitutions in all the expressions obtained for the fluctuation correlators and the scattering cross sections

$$\delta(\omega^2-\omega_k^2)\to\frac{2}{\pi}\frac{\gamma_k\omega_k}{(\omega^2-\omega_k^2)^2+(2\gamma_k\omega_k)^2} \tag{34}$$

where γ_k is the damping decrement of the wave with frequency ω_k.

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