# Electron Scattering by Protons at Small Angles 

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Differential cross sections have been measured for elastic electron-proton scattering in the region of small momentum transfers $\left(0.195 \leq \mathrm{q}^{2} \leq 506 \mathrm{~F}^{-2}\right)$ in the internal beam of the $4.4-\mathrm{BeV}$ electron accelerator of the Erevan Physics Institute. The experiment utilized semiconductor particle detectors and a polyethylene film target. The results of the measurement permit information to be obtained on the electric form factor of the proton and its mean-square radius.

## INTRODUCTION

Experimental study of the elastic scattering of high-energy electrons by protons in the region where the one-photon approximation is valid permits information to be obtained on the electric form factor $G_{E}$ and the magnetic form factor $G_{M}$ of the proton.

A substantial part of the experiments carried out up to this time have been performed in the region of relatively large values of squared momentum transfer $q^{2}$. Data existing in the literature on proton radii are obtained in this case by extrapolation of data on form factors for $q^{2} \geq 0.5 \mathrm{~F}^{-2}$ to the region $q^{2} \rightarrow 0$, since it is precisely the derivative of the form factor for $q^{2}=0$ which determines the square of the proton radius:

$$
R_{E}^{2}=-6 d G_{E} / d q^{2} .
$$

Recently Barrett, Brodsky, Ericson, and Goldhaber ${ }^{[1]}$ have turned their attention to the desirability of increasing the accuracy of the experimental data for comparatively small $q^{2}$ values and of further advancement of experiments to the region of still smaller $q^{2}$ values than previously achieved. ${ }^{[2]}$ The results of a statistical analysis of all data on elastic ep scattering published up to May, 1970, ${ }^{[3]}$ point to the need of obtaining experimental data on the relative behavior of the differential cross section for scattering in the region of small $q^{2}$ with an accuracy better than $10 \%$.

Detection of recoil protons by the method used in the present work permits conclusions to be drawn regarding the systematic errors in various methods of measuring ep-scattering differential cross sections, which are difficult to take into account. The purpose of the present work is to report the results of the first stage of the experimental study of elastic scattering of electrons by protons in the region of small momentum transfer with detection by semiconductor detectors of low-energy recoil protons emitted at recoil angles close to $\pi / 2$. Here it is possible to advance the study to the region of previously uninvestigated small $q^{2}$ values and to increase the accuracy of the data in the region of $q^{2}$ previously studied by other methods. In a large part of the numerous experiments carried out up to this time, the scattered electrons were detected.

[^0]Only in a few studies have the authors found it possible to detect the recoil protons. ${ }^{[4-6]}$

## 1. EXPERIMENTAL ARRANGEMENT

The elastic scattering of high-energy electrons by protons with small momentum transfers has been studied in the internal electron beam of the ARUS accelerator at Erevan. Low-energy recoil protons were recorded by semiconductor (silicon) detectors. The semiconductor detectors work extremely efficiently just in the region of small momentum transfers and, consequently, low recoil-proton energies.

In the one-photon approximation the cross section for elastic ep scattering in the laboratory system can be represented as a function of the emission angle $\gamma$ in the form ${ }^{[8]}$

$$
\begin{gather*}
\left(\frac{d \sigma}{d \Omega}\right)_{p}=\left(\frac{e^{2}}{2 M}\right)^{2} \frac{1}{\cos \gamma}\left(1+\frac{T_{p}}{2 M}\right)\left\{\frac{2 M}{T_{p}} G_{x}^{2} \sin ^{2} \gamma\right. \\
\left.+\left[\left(1+\cos ^{2} \gamma\right)\left(1+\frac{T_{p}}{M}\right)-\frac{2 p}{M} \cos \gamma\right] G_{M}^{2}\right\},  \tag{1}\\
\cos \gamma \equiv \cos \theta_{e p}=\left(1+\frac{M}{E}\right)\left(\frac{T_{p}}{2 M+T_{p}}\right)^{1 / 2}, \tag{2}
\end{gather*}
$$

where $T_{p}$ is the kinetic energy and $p$ the momentum of the recoil protons. It is evident from Eq. (1) that detection of recoil protons for $\gamma \approx \pi / 2$ permits the electric form factor $G_{E}$ to be determined experimentally.

One of the advantages of recoil-proton detection instead of scattered electrons is the decrease in the magnitude of radiation corrections. Several theoretical studies ${ }^{[7-11]}$ have been made of radiation corrections

FIG. 1. Experimental apparatus: PA-preamplifier, LA-linear amplifier, LD-linear discriminator, S-shaper, PHA-pulse height analyzer.


FIG. 2. Recoil-pronton spectrum from $\mathrm{CH}_{2}$ target for two detectors: $\mathrm{a}-40 \times 5 \times 0.7 \mathrm{~mm}, \mathrm{~b}-15 \times 15 \times 1 \mathrm{~mm}$.
for the ep-scattering cross section for detection of recoil protons. We have taken radiation corrections into account according to the formulas of the most detailed studies of Bartl and Urban. ${ }^{[9-11]}$ A typical value of the radiation correction was $\sim 0.7 \%$. In the experiments we measured the recoil-proton yield from elastic ep collisions at angles close to $\pi / 2$ relative to the primary electron beam direction. The experimental arrangement is shown in Fig. 1.

A thin (3-5 $\mu$ ) polyethylene film was used as a target in the internal beam of the Erevan ARUS accelerator. In order to increase the target life, a film in the form of a circle 140 mm in diameter was fastened by its central portion to a metallic disk of smaller diameter, rotating at 3000 revolutions per minute. The plane of the film made an angle with the electron beam direction of $45^{\circ}$. The target was located in a vacuum pipe outside the region of the circulating electron beam. At the proper moment of the cycle, the beam was diverted to the target by a special system of local perturbation of the equilibrium orbit. The duration of the burst was $\sim 1 \mathrm{msec}$.

Recoil protons with energies from 3.9 to 10.8 MeV were detected by semiconductor silicon detectors located 1.5 m from the target. Eight silicon detectors with various working dimensions and thicknesses were used (the detector lengths were $10-40 \mathrm{~mm}$, the widths $5-10 \mathrm{~mm}$, and the thicknesses $0.5-1 \mathrm{~mm}$ ). The detector pulses were amplified by a preamplifier PA, transmitted by cable to the input of a linear amplifier LA, and fed through a linear discriminator LD to a pulse-height analyzer PHA. By means of a commutating device (mixer) the channels of each of two analyzers were broken down into four groups of 100 channels each. In each group of
channels, in a certain level of background from carbon, the recoil-proton spectrum from elastic ep scattering in the hydrogen of the target was observed (see Fig. 2). The extent of the spectrum depended on the detector length (subtended angle).

In order to determine the background from carbon, measurements were made with thin carbon films. The detectors were periodically calibrated by means of an $\alpha$ source and a standard pulse generator. The energy resolution of the system, determined as the half-width of the $\alpha$ line, was 170 keV for operation in the accelerator.

## 2. RESULTS

Use of 'long"' detectors ( 4 cm ) permitted detection of recoil protons over a relatively wide energy range, inside which the ep-scattering cross section varies by a factor of two or three. After removal of the background from carbon and introduction of small corrections for the detection efficiency ( $1-2 \%$ ), the measured recoil-proton spectra gave the dependence of the differential cross sections on momentum transfer in the range $0.195 \leq q^{2} \leq 0.506 \mathrm{~F}^{-2}$.

The background from carbon was taken into account by two independent methods:

1) From the results of special measurements of the background from a carbon target of equivalent thickness. The spectra obtained in the background experiment were in this case normalized to the extreme portions of the spectra from $\mathrm{CH}_{2}$, where the contribution of recoil protons from elastic ep collisions could be neglected.
2) By approximation of the extreme portions mentioned above, of the spectra of particles recorded from $\mathrm{CH}_{2}$, by the function

$$
\begin{equation*}
\varphi(T)=A_{1} e^{-B_{1} T}+A_{2} e^{-B_{2} T}, \tag{3}
\end{equation*}
$$

where $A_{1}, B_{1}, A_{2}$, and $B_{2}$ are free parameters, and analytic continuation of this function to the region of the spectrum which receives contributions from protons from ep scattering.

The two methods of background subtraction gave identical results within experimental error.

The agreement of the relative behavior of the differential cross sections in the range $0.195 \leq q^{2} \leq 0.506 \mathrm{~F}^{-2}$ with the results of extrapolation based on analysis of the entire set of data in the literature on ep scattering argues against the hypothesis of a "proton halo."

To determine the differential cross sections in absolute units, we determined the normalization factors $\mathrm{N}_{\mathrm{k}}$ for the results of the k -th experiment (the k -th detector) by minimization of the functional

$$
\begin{equation*}
\chi^{2}=\sum_{i, k}\left[\frac{\left(d \sigma^{k} / d \Omega_{i}\right)_{\exp }-N_{k}\left(d \sigma / d \Omega_{i}\right)_{\text {theor }}}{\Lambda_{i}^{k}}\right]^{2}, \tag{4}
\end{equation*}
$$

where $\left(\mathrm{d} \sigma^{\mathrm{k}} / \mathrm{d} \Omega_{\mathrm{i}}\right) \exp$ and $\Delta_{\mathrm{i}}^{\mathrm{k}}$ are the values of the relative differential cross sections and their errors for the i-th point of the k -th detector, $\left(\mathrm{d} \sigma / \mathrm{d} \Omega_{\mathrm{i}}\right)_{\text {theor }}$ is the differential cross section for elastic ep scattering calculated at the i-th point from Rosenbluth's formula (1), in which the dependence of the form factors on $q^{2}$ is represented by the dipole formula

$$
\begin{equation*}
G_{E}\left(q^{2}\right)=\left(1+A q^{2}\right)^{-2} \tag{5}
\end{equation*}
$$

Table I. Differential cross section for elastic ep scattering as a function

| $\mathrm{q}^{2}, \mathrm{~F}^{-2}$ | $\begin{gathered} (\mathrm{do} / \mathrm{d} / \mathrm{S})_{p} . \end{gathered} .$ | $\mathrm{q}^{2}, \mathrm{~F}^{-2}$ | $\underset{\mathrm{bd} / \mathrm{br} \mathrm{sr}}{(\mathrm{~d} / \mathrm{p}},$ |
| :---: | :---: | :---: | :---: |
| 0.195 | $171.8 \pm 5.0$ | 0.300 | $93.0 \pm 1.2$ |
| 0.200 | $174.2 \pm 5.1$ | ${ }^{0.325}$ | $88,7 \pm 1.2$ |
| 0,210 0.225 | $160.9 \pm 5.1$ $158.5 \pm 5.0$ | 0.350 0.375 | $75.0 \pm 1.4$ $69.6 \pm \pm .4$ |
| 0.230 | 139.6 ${ }^{138.6}$ | 0.400 | $64.0 \pm 1.3$ |
| 0.250 | $126.8 \pm 4.0$ | ${ }_{0}^{0.450}$ | $52.5 \pm 1.2$ |
| 0.275 | $114.9 \pm$ 7.5 | 0.506 | $43.4 \pm 1.1$ |

Table II. Electric form factors of the proton

| $\mathrm{q}^{2}, \mathrm{~F}^{-2}$ | $G_{E}\left(q^{2}\right)$ |  | $\mathrm{q}^{2}, \mathrm{~F}^{-2}$ | $G_{E}\left(q^{2}\right)$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Present work | According to ref. 2 |  | Present work | According to ref. 2 |
| 0.200 | $0.9783 \pm 0.0010$ | - | 0.400 | $0.9575 \pm 0.0017$ | - |
| 0.250 | $0.9733 \pm 0.0010$ | - | 0.450 | $0.9526 \pm 0.0019$ | - |
| 0.300 | $0.9680 \pm 0.0013$ | $0.9731 \pm 0.0054$ | 0.500 | $0.9470 \pm 0.0021$ | - |
| 0.350 | $0.9623 \pm 0.0015$ | - | 0.600 |  | $0.9399 \pm 0.0061$ |

and the scaling law

$$
\begin{equation*}
G_{M}\left(q^{2}\right)=\mu_{p} G_{E}\left(q^{2}\right) \tag{6}
\end{equation*}
$$

For the parameter A in Eq. (5) we took the value

$$
A=1.382 \pm 0.016 \mathrm{BeV}^{-2}
$$

which was obtained by Bilen'kaya et al. ${ }^{[3]}$ on the basis of analysis of the experimental data existing in the literature on ep scattering.

Minimization of the functional (4) was carried out by the method of linearization according to the program FUMILI. ${ }^{[12]}$ Individual points which appeared at random outside of the general set of experimental data and gave a contribution $\Delta \chi^{2} \geq 9$ to the minimized functional were discarded from further consideration.

From the differential cross-section values found we determined the electric radius of the proton. For this we again minimized the functional (4), but this time in addition to the norm N , which now was the same for the entire set of points, we varied also the parameter A in Eq. (5), which is simply related to the proton radius: $\mathrm{R}_{\mathrm{E}}^{2}=12 \mathrm{~A}$. In Table I we have given some values of the differential cross sections obtained by this procedure for elastic ep scattering, as a function of $q^{2}$.

The total number of $q^{2}$ values for which differential cross-section values were obtained in 46 (see Fig. 3). In this case we have for the electric radius of the proton

$$
R_{E}=0.76 \pm 0.09 \mathrm{~F}
$$

If in determination of the proton radius in the functional (4) we fix the norm $N=1$ and vary only the parameter $A$, then we obtain for the proton radius

$$
R_{E}{ }^{\prime}=0.81 \pm 0.02 \mathrm{~F} .
$$

In essence, the latter means that the proton radius is extracted from the differential cross sections measured by us, normalized to the absolute values with inclusion of the world data (the parameter A in Eq. (4) was fixed on the basis of the data of Bilen'kaya et al. ${ }^{[3]}$ ).


FIG. 3. Cross section for elastic electron-proton scattering as a function of four-momentum transfer. The errors shown are statistical. The solid curve was calculated from Rosenbluth's formula for a dipole parametrization of the form factors according to the data of ref. 3.

The values of $\mathrm{R}_{\mathrm{E}}$ and $\mathrm{R}_{\mathrm{E}}^{\prime}$ obtained are in good agreement with each other and with the generally accepted value of the proton electric radius 0.8 F .

In Table II we have presented the results of the present work for the electric form factor of the proton in the region $0.195 \leq \mathrm{q}^{2} \leq 0.506 \mathrm{~F}^{-2}$; the dipole approximation for the form factor can be considered valid. ${ }^{[3]} \mathrm{We}$ have also shown in Table II for comparison the value of $R_{E}$ for $q^{2}=0.300 F^{-2}$ obtained by Drickey and Hand. ${ }^{[2]}$ It can be seen that within the experimental error there is good agreement between the results of the two experiments, but the accuracy achieved in the present work is higher. Values of the form factors $G_{E}$ were obtained for a number of values of $q^{2}$.

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