Helicon Turbulence Spectra in Collisionless Plasma

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The effects of whistler (helicon) interaction in a collisionless plasma are calculated. It is shown that due to induced scattering by ions, evolution of the whistler vibration spectra may lead to self-trapping of the whistlers or, more precisely, to their concentration along the external magnetic field where Landau absorption vanishes. In this case a stationary turbulence spectrum expressed by a power law $W_\omega \sim 1/\sqrt{\omega}$ is formed, the exponent for ω being equal to 1/2. Conditions for stability of the spectrum with respect to decay processes are found; they show that the turbulence spectrum is stable if the total turbulence energy does not exceed the critical value.

1. GENERAL RELATIONS

In a collisionless plasma, various instabilities can excite turbulent pulsations. The stationary turbulent spectra of such oscillations are regulated by excitation of the oscillations, their nonlinear transformation, and their absorption (including collective absorption). At the present time, methods of calculating nonlinear processes, as well as excitation and damping of oscillations in a weakly-turbulent regime have been developed in detail (see [1,2]). Therefore one of the important problems is to investigate the plasma turbulence spectra. The spectra of Langmuir oscillations have been studied in [3,4]. The task of the present paper is to investigate nonlinear interactions and spectra of stationary turbulence of helicons (whistlers). It is known [5] that this term is customarily used to denote waves whose frequencies lie in the interval

$$\omega_{Hi} < \omega < \omega_{He}|x|, \quad \omega_{H\alpha} = |e_\alpha| H_0 / m_\alpha c \tag{1}$$

(ω_{He} and ω_{Hi} are respectively the gyrofrequencies of the electrons and ions), with

$$\omega = k^2c^2\omega_{He}|x|(\omega_{pe}^2 + k^2c^2)^{-1}, \quad x = \cos\theta \tag{2}$$

θ is the angle between the wave vector **k** of the wave and the external constant homogeneous magnetic field H_0, and $\omega_{p\alpha} = \sqrt{4\pi e_\alpha^2 n/m_\alpha}$ is the Langmuir frequency of the particles of species α.

We start from the following assumptions:

1) The helicon generation source is outside the investigated wave-number region;

2) $\omega_{pe} \gg \omega_{He}$;

3) $\omega_{pe} \gg kc$ and the helicon frequencies are approximated by the expression

$$\omega = k^2c^2\omega_{He}|x|\omega_{pe}^{-2}; \tag{3}$$

4) $T_e = T_i$;

5) $v_A \gg v_s$, $v_A = c\omega_{Hi}/\omega_{pi}$, $v_{T\alpha}^2 = T_\alpha/m_\alpha$, $v_s^2 = T_e/m_i$.

The first assumption makes it possible to disregard the helicon instability mechanisms, which can be either linear (excitation by anisotropic distributions of fast particles etc.), or nonlinear (conversion from Langmuir oscillations or other oscillations). The third assumption makes it possible to regard the oscillations to be mainly electromagnetically transverse. We shall show, however, that the corrections connected with the nontransversality of the oscillations turn out to be important in a number of cases when the nonlinear interactions are calculated. Finally, the fourth assumption makes it possible to exclude from consideration effects of nonlinear conversion of helicons into ion-acoustic oscillations, which attenuate very strongly in a plasma when $T_e = T_i$.

To find the helicon turbulence spectra it is necessary to set up balance equations which take into account the Landau absorption processes and the processes of nonlinear energy transformation over the spectrum. The Landau absorption, which in the case of helicons is possible only by the plasma electrons, is written symbolically in the form

$$w + e \to e'. \tag{4}$$

Among the nonlinear processes we can separate the decay interaction

$$w \rightleftarrows w' + w'' \tag{5}$$

which is allowed for helicons, and induced scattering by the electrons and ions of the plasma

$$w + e \rightleftarrows w' + e', \tag{6}$$

$$w + i \rightleftarrows w' + i'. \tag{7}$$

It should be noted that the Landau damping will be nonexponentially small only if

$$\omega < kv_{Te}|x| \approx \omega_{He}\left(\frac{v_{Te}}{c}\frac{\omega_{pe}}{\omega_{He}}\right)^2|x|. \tag{8}$$

The process (6) of scattering by electrons must be taken into account only when there is no Landau damping, i.e., when an inequality opposite to (8) holds. This follows from the general analysis contained in [6].

We introduce the turbulence energy W per cm^3, and define the spectral turbulence function $W_{\omega\Omega}$ (where ω is the frequency of the investigated oscillations and Ω is the solid angle), by normalizing $W_{\omega\Omega}$ in accordance with the relation

$$W = \int d\omega \int d\Omega\, W_{\omega\Omega} = \int W_{\omega x}\, d\omega\, dx\, d\varphi. \tag{9}$$

The quantity $W_{\omega\Omega}$ is connected with the number of quanta $N_{\mathbf{k}}$ by the relation

$$W_{\omega\Omega} = \frac{\omega k^2 N_{\mathbf{k}}}{(2\pi)^3}\frac{dk}{d\omega} = \frac{k^3 N_{\mathbf{k}}}{16\pi^3}. \tag{10}$$

(Here and throughout $\hbar = 1$.)

The general expression describing the absorption (4) and the nonlinear transformation processes (5), (6), and (7) then takes the form

$$\frac{\partial W_{\omega\Omega}}{\partial t}=\gamma_{\omega\Omega}W_{\omega\Omega} \tag{11}$$
$$+\sum_{\alpha}\int w^{(\alpha)}(\mathbf{p}_\alpha,\mathbf{k},\mathbf{k}_1)W_{\omega\Omega}W_{\omega_1\Omega_1}(k_z-k_{1z})\frac{\partial}{\partial p_z}f^{(\alpha)}(\mathbf{v})\,d\mathbf{v}\frac{d\omega_1\,d\Omega_1}{\omega_1}$$
$$+\int\frac{d\omega_1\,d\omega_2\,d\Omega_1\,d\Omega_2}{2(2\pi)^3\omega_1\omega_2}[u(\mathbf{k},\mathbf{k}_1,\mathbf{k}_2)(k^3W_{\omega_1\Omega_1}W_{\omega_2\Omega_2}$$
$$-k_2^3W_{\omega\Omega}W_{\omega_1\Omega_1}-k_1^3W_{\omega\Omega}W_{\omega_2\Omega_2})+2u(\mathbf{k}_2,\mathbf{k},\mathbf{k}_1)$$
$$\times(k^3W_{\omega_1\Omega_1}W_{\omega_2\Omega_2}+k_2^3W_{\omega\Omega}W_{\omega_1\Omega_1}-k_1^3W_{\omega\Omega}W_{\omega_2\Omega_2})].$$

Here $w^{\alpha}(\mathbf{p}_\alpha,\mathbf{k},\mathbf{k}_1)$ and $u(\mathbf{k},\mathbf{k}_1,\mathbf{k}_2)$ are the probabilities of the scattering and decay of helicons, respectively; these probabilities are calculated by standard methods described in [2,7]. The index α runs through the values of e and i corresponding to the electrons and ions of the plasma. Since the electrons, by virtue of the assumed conditions, are magnetized, the Landau-damping coefficient can be expressed in the form

$$\gamma_{\omega\Omega}=\int w^{e}(\mathbf{p}_e,\mathbf{k})k_z\frac{\partial}{\partial p_z}f^{(e)}(\mathbf{v})\,d\mathbf{v}. \tag{12}$$

The expression for the probability of helicon emission by a magnetized electron can easily be obtained by the standard method, using the expressions for the unit vectors of the oscillations in the region (3)[1)]

$$\mathbf{a}=\frac{1}{\sqrt{1+x^2}}\left\{\cos\varphi-i|x|\sin\varphi;\ \sin\varphi+i|x|\cos\varphi;\ \frac{\omega x\sqrt{1-x^2}}{\omega_{He}|x|}\right\}. \tag{13}$$

We obtain

$$w(\mathbf{p}_e,\mathbf{k})=\frac{2\pi^2e^2\omega_{He}}{\omega_{pe}^2|x|}\left[\frac{\omega_{pe}v_\perp^2\sqrt{\omega|x|}}{2c\omega_{He}^{3/2}}-v_z\frac{\omega}{\omega_{He}}\frac{x}{|x|}\right]^2(1-x^2)\,\delta(\omega-k_zv_z^e). \tag{14}$$

Substituting (14) in (12), we obtain for $\omega\ll kv_{Te}|x|$ the result[5]

$$\gamma_{\omega\Omega}=-\frac{\sqrt{2\pi}}{2}\left(\frac{\omega}{\omega_{He}}\right)^{3/2}\frac{v_{Te}}{c}\omega_{pe}\frac{1-x^2}{\sqrt{|x|}}. \tag{15}$$

Standard methods also yield the probability of scattering by ions:

$$w(\mathbf{p}_i,\mathbf{k},\mathbf{k}_1)=\frac{\pi}{4}F(x,x_1)A(\omega_1)\frac{\delta(\omega-\omega_1-(k_z-k_{1z})v_z^i)}{n_0^2(1+T_e/T_i)^2}, \tag{16}$$

where

$$F(x,x_1)=\frac{(1+|xx_1|)^2+(|x|+|x_1|)^2}{|xx_1|},\qquad A(\omega_1)=\omega_1^2-2\omega_1k_{1z}v_z.$$

Allowance for the small terms $\sim kv_{Ti}/\omega$ is necessary, since the nonlinear increment vanishes in the zeroth approximation in this parameter. This probability holds if the condition $k_\perp v_{Ti}<\omega_{Hi}$ is satisfied, corresponding to

$$\tag{17}$$

If $\omega\gg\omega_{Hi}(v_A/v_{Ti})^2|x|$, then the unperturbed trajectories of the ions can be assumed to be straight lines. In this case the term in (11) corresponding to scattering by ions should be written in the form

$$\int w^{i}(\mathbf{p}_i,\mathbf{k},\mathbf{k}_1)W_{\omega\Omega}W_{\omega_1\Omega_1}(\mathbf{k}-\mathbf{k}_1)\frac{\partial}{\partial\mathbf{p}}f^{i}(\mathbf{v})\,d\mathbf{v}\frac{d\omega_1\,d\Omega_1}{\omega_1}, \tag{18}$$

where w^i is given by formula (16), in which $A(\omega_1)$ should be replaced by $\tilde{A}(\omega_1)=\omega_1^2-2\omega_1k_{1z}(\mathbf{k}-\mathbf{k}_1)\mathbf{v}/(k_z-k_{1z})$, and the last term under the δ-function sign by the term $-(\mathbf{k}-\mathbf{k}_1)\mathbf{v}^i$.

We transform the nonlinear equations, using a number of simplifying assumptions. One such assumption is that there are no abrupt changes in the turbulence spectrum within a frequency interval on the order of kv_{Ti}, i.e., in the frequency interval $\Delta\omega_p$, where

$$\left(\frac{\Delta\omega_p}{\omega}\right)^2\sim\frac{\omega_{Hi}}{\omega}\frac{v_{Ti}^2}{v_A^2}. \tag{19}$$

By virtue of the assumed conditions ($\omega\gg\omega_{Hi}$ and $v_A\gg v_{Ti}$) we have $\Delta\omega_p/\omega\ll1$. From the general expression for the nonlinear scattering processes and from (16) we have

$$\frac{\partial W_{\omega x}}{\partial t}=-W_{\omega x}\frac{(2\pi)^{3/2}}{32n_0m_iv_{Ti}^3}\int W_{\omega_1x_1}\,d\omega_1\,dx_1\,F(x,x_1)$$
$$\times(\omega-\omega_1)\left(\omega_1-2\omega_1^{1/2}\frac{\omega_{pe}}{c}\frac{x_1v_z}{\sqrt{\omega_{He}|x_1|}}\right) \tag{20}$$
$$\times\delta\left(\omega-\omega_1-\frac{\omega_{pe}v_z}{c\sqrt{\omega_{He}}}\left(\frac{x\sqrt{\omega}}{\sqrt{|x|}}-\frac{x_1\sqrt{\omega_1}}{\sqrt{|x_1|}}\right)\right)\exp\left\{-\frac{v_z^2}{2v_{Ti}^2}\right\}dv_z.$$

From this we can obtain an approximate interaction, differential in the frequencies, by assuming $\Delta\omega_p$ to be physically infinitesimally small:

$$\frac{\partial W_{\omega x}}{\partial t}=\frac{\pi^2}{8}\frac{\omega_{pe}^2}{n_0m_ic^2\omega_{He}}\int F(x,x_1)$$
$$\times\left[\omega^2\frac{\partial W_{\omega x_1}}{\partial\omega}\left(\frac{x}{\sqrt{|x|}}-\frac{x_1}{\sqrt{|x_1|}}\right)^2+\omega W_{\omega x_1}\left(\frac{x}{\sqrt{|x|}}-\frac{x_1}{\sqrt{|x_1|}}\right)\frac{x}{\sqrt{|x|}}\right]dx_1. \tag{21}$$

It should be borne in mind that the interaction (21) is approximate, for actually the frequency interval in which the transformation of the turbulent energy $\Delta\omega_p$ is most probable is finite and not infinitesimally small. Consequently, the use of Eq. (21) is justified only for spectra sufficiently smooth on $\Delta\omega_p$. At the same time, the smallness of $\Delta\omega_p$, especially in strong magnetic fields, indicates that (21) is valid in a wide class of turbulent spectra. We shall show that the solutions of (21) are frequently functions in powers of ω. In this case the necessary conditions for the utilization of (21) are clearly satisfied, since the actual change of the spectrum occurs over a $\Delta\omega_p$ interval on the order of ω. We note that in accordance with (21) the interaction vanishes at $x=x_1$, i.e., for pulsations that are parallel to one another.

The interaction of the pulsations with the electrons is described by a probability estimated at

$$w(\mathbf{p}_e,\mathbf{k},\mathbf{k}_1)\approx\frac{\pi}{2}\frac{\omega\omega_1}{n_0^2|xx_1|}\left(\frac{v_{Te}}{c}\frac{\omega_{pe}}{\omega_{He}}\right)^4\delta(\omega-\omega_1-(k_z-k_{1z})v_z^e). \tag{22}$$

At $\omega\gg kv_{Te}|x|$, the only case when such an interaction should be considered, the nonlinear and Compton scatterings compensate each other. What remains uncompensated are the terms $\sim k^2v_{Te}^2/\omega\omega_{He}$ and this results in a factor $(k^2v_{Te}^2/\omega\omega_{He})^2$ in the expression for the probability. The corrections connected with the screening of the nonlinear scattering by the ions are of the order of $(m_e/m_i)^{3/2}$ (see [2] concerning analogous effects for Langmuir waves). Thus, the probability (22), in which the last corrections are neglected, can be used when

1)We present here expressions that take into account the small (of the order of ω/ω_{He}) longitudinal components that are needed in the calculation of the nonlinear interactions.

$$v_A < v_{Te}(m_e/m_i)^{1/8}. \tag{23}$$

The physically infinitesimally small quantity is $\Delta\omega \sim kv_{Te}$, i.e., $(\Delta\omega/\omega)^2 \sim (v_{Te}/v_A)^2 \times \omega_{Hi}/\omega$. This quantity is small if the following condition is satisfied:

$$\omega \gg \omega_{Hi} v_{Te}^2 / v_A^2, \tag{24}$$

i.e., precisely in the region where the linear damping is small and it is meaningful to consider scattering by electrons. The condition (24) is the condition that the nonlinear interaction be differential in the frequencies in the case of scattering by electrons. When (23) and (24) are satisfied, we obtain from (22)

$$\begin{aligned}\frac{\partial W_{\omega x}}{\partial t} = &-\beta W_{\omega x}\frac{\omega}{|x|}\int\frac{dx_1}{|x_1|}\Big\{-\omega\frac{\partial W_{\omega x_1}}{\partial\omega}\Big(\frac{x}{\sqrt{|x|}}-\frac{x_1}{\sqrt{|x_1|}}\Big)^2\\ &\times\Big[\frac{4xx_1}{\sqrt{|xx_1|}}(|x|+|x_1|)(x^2+x_1^2-2x^2x_1^2)\\ &+|xx_1|(16-14x^2-14x_1^2+16x^2x_1^2)+(1-x^2)(2+x_1^2-2x^2x_1^2-x_1^4)\\ &+(1-x_1^2)(2+x^2-2x^2x_1^2-x^4)+4x^2x_1^2\Big]+W_{\omega x_1}\Big(\frac{x}{\sqrt{|x|}}-\frac{x_1}{\sqrt{|x_1|}}\Big)\\ &\times\Big[(-x^4+3x^4x_1^2-x^2+6x^2x_1^2-11x^2x_1^4+x_1^2+3x_1^4)\frac{x}{\sqrt{|x|}}\\ &+(8-2x^2-6x_1^2)\frac{x}{\sqrt{|x|}}|xx_1|+(4-2x^4+6x^2x_1^4-2x_1^2-2x_1^4)\frac{x_1}{\sqrt{|x_1|}}\\ &+(24-24x^2+24x^2x_1^2-20x_1^2)\frac{x_1}{\sqrt{|x_1|}}|xx_1|\Big]\Big\},\end{aligned} \tag{25}$$

$$\beta = \pi^2\frac{\omega_{Hi}}{n_0 m_e v_A^2}\Big(\frac{v_s}{v_A}\Big)^4. \tag{26}$$

Finally, we present here the probability of the process (5)

$$\begin{aligned}u(\mathbf{k},\mathbf{k}_1,\mathbf{k}_2) = &\frac{(2\pi)^4}{64}\frac{\delta(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\,\delta(k^2|x|-k_1^2|x_1|-k_2^2|x_2|)}{n_0 m_i \omega_{pi}^2}\\ &\times k_2^{-2}c^2\left|\frac{xx_1}{x_2}\right|\Big(k\frac{x}{|x|}-k_1\frac{x_1}{|x_1|}\Big)^2\Big(k\frac{x}{|x|}+k_1\frac{x_1}{|x_1|}+k_2\frac{x_2}{|x_2|}\Big)^2\\ &\times(2k^2k_1^2+2k^2k_2^2+2k_1^2k_2^2-k^4-k_1^4-k_2^4).\end{aligned} \tag{27}$$

It is obvious that when helicons propagate strictly along the field, the decays do not work, and they are therefore also ineffective at sufficiently small scatter relative to the magnetic field. Therefore rough comparisons of the efficiencies of the decays and the scatterings are impossible.

2. HELICON TURBULENCE SPECTRA

Let us see what types of helicon turbulence spectra can be established in a plasma if the main mode of nonlinear interaction is scattering, and let us then estimate the influence of the decays.

We compare first scattering by ions and by electrons. The characteristic time of helicon-energy frequency transformation by $\Delta\omega \sim \omega$ in nonlinear scattering by ions is

$$\gamma_\omega^i \approx \omega_{Hi} W / n_0 m_i v_A^2, \tag{28}$$

and in scattering by electrons

$$\gamma_\omega^e \approx \omega_{He}\Big(\frac{v_s}{v_A}\Big)^4\frac{W}{n_0 m_i v_A^2}. \tag{29}$$

If (23) is not satisfied, the factor $(v_S/v_A)^2$ is replaced by $(m_e/m_i)^{3/2}$.

Thus, when $v_A > v_{Te}(m_e/m_i)^{1/8}$, scattering by ions exceeds scattering by electrons. The linear damping of the helicons is then exponentially small in the entire interval (1). When $v_A < v_{Te}(m_e/m_i)^{1/8}$ the scattering by ions exceeds the scattering by electrons in the entire interval (1), if

$$v_A > v_{Te}(m_e/m_i)^{1/4}. \tag{30}$$

In the case of the opposite inequality in the interval

$$\omega_{Hi} < \omega < \omega_{Hi} v_{Te}^2 |x| / v_A^2 \tag{31}$$

it is necessary to consider scattering by ions and damping by electrons, and in the interval

$$\omega_{Hi} v_{Te}^2 |x| / v_A^2 < \omega < \omega_{He}|x| \tag{32}$$

it is necessary to consider scattering by electrons.

If the scattering by ions predominates, then the stationary solution of (21), with separation of the variables ω and x, is given by

$$\overline{W}_{\omega x} = \omega^{-\nu}\overline{W}_x, \quad \nu = 1/2, \tag{33}$$

$$\overline{W}_x = W_1\delta(x-x_0) + W_2\delta(x+x_0). \tag{34}$$

The solution (33) can be obtained by equating to zero the right-hand side of Eq. (21). The last equality takes place only if all the coefficients in the resultant linear combination of the linearly-independent functions of x vanish. This gives the unique value $\nu = 1/2$.

Waves with $x = x_0$ and a wave with $x = -x_0$ interact in each elementary scattering act. The solution (33) is of interest because self-channeling of the helicon energy towards directions parallel and antiparallel to the external magnetic field takes place in the course of time; this can be seen by considering the stability of this solution. Let us examine the time behavior of small perturbations. Substituting in (21) a solution of the form

$$W_{\omega x} = \overline{W}_{\omega x} + \tilde{W}_{\omega x}, \tag{35}$$

where $W_{\omega x}$ is given by (33), we obtain

$$\begin{aligned}\frac{\partial\tilde{W}_{\omega x}}{\partial t} = &\frac{\alpha}{2}\omega^{1/2}\tilde{W}_{\omega x}\int dx_1 F(x,x_1)\overline{W}_{x_1}(|x|-|x_1|)\\ &+\alpha\omega^{1/2}\overline{W}_x\int dx_1 F(x,x_1)\Big[\omega\frac{\partial\tilde{W}_{\omega x_1}}{\partial\omega}\Big(\frac{x}{\sqrt{|x|}}-\frac{x_1}{\sqrt{|x_1|}}\Big)^2\\ &+\tilde{W}_{\omega x_1}\Big(\frac{x}{\sqrt{|x|}}-\frac{x_1}{\sqrt{|x_1|}}\Big)\frac{x}{\sqrt{|x|}}\Big], \quad \alpha = \frac{\pi^2}{8}\frac{\omega_{Hi}}{n_0 m_i v_A^2}.\end{aligned} \tag{36}$$

An analysis of (36) shows that the solution (33) is unstable for perturbations with $|x| > x_0$ and stable when $|x| < x_0$. The spectrum becomes one-dimensional. The characteristic self-channeling time is determined by the quantity γ^{-1}, where γ corresponds to (28). The one-dimensional character of the spectrum is subsequently conserved (this is seen when $x_{10} = 1$ is substituted in (36)).

If the scattering by electrons predominates, then it is necessary to use Eq. (25). Its stationary solution also takes the form (33) and (34). This solution, however, is unstable against small perturbations in definite angle intervals, and the locations of these instability intervals depend on the value of x_0. In particular, at $x_0 = 1$, the evolution of the small perturbations is described by the equation

$$\begin{aligned}\frac{\partial\tilde{W}_{\omega x}}{\partial t} = &-\tilde{W}_{\omega x}\beta\frac{\omega^{1/2}}{|x|}\Big\{W_1\Big[4\frac{x}{\sqrt{|x|}}(2x^2-1+|x|)-|x|(3x^2+2)\\ &-6x^2+1\Big]+W_2\Big[-4\frac{x}{\sqrt{|x|}}(2x^2-1+|x|)\end{aligned}$$

$$-|x|(3x^2+2)-6x^2+1\Big]\Big\}(1-x^2), \quad |x|\neq 1. \tag{37}$$

We see therefore that a continuous redistribution of the helicon energy over the angles takes place.

Thus, a stable stationary helicon spectrum can exist in the entire interval (1) if (30) is satisfied, and in the interval (31) if (30) is not satisfied.

Let us examine now the influence exerted on the one-dimensional spectrum (33) with $x_0 = 1$ by decay processes and by linear damping. Using the expression (27) for the decay probability at $|x| = 1$, and taking into account the plasmon decay and coalescence processes described by the last term in (11), we obtain the following expression for the evolution of small perturbations:

$$\frac{\partial W_{\omega_1 x_1}}{\partial t} = -\gamma_{\omega_1 x_1} W_{\omega_1 x_1} + \sum_{j=2}^{5} \beta^{(j)}_{\omega_1 x_1} W_{\omega_j x_j},$$

$$\gamma_{\omega_1 x_1} = \gamma^{L}_{\omega_1 x_1} - \beta^{(2)}_{\omega_1 x_1} + \sum_{i=3}^{5} \beta^{(i)}_{\omega_1 x_1}, \tag{38}$$

$$\frac{\partial W_{\omega_j x_j}}{\partial t} = -\gamma^{L}_{\omega_j x_j} W_{\omega_j x_j} + \beta^{(j)}_{\omega_1 x_1} W_{\omega_1 x}; \quad j = 2, 3, 4, 5.$$

Here the terms with $\gamma^{l}_{\omega x}$ represent the linear damping of the helicons, and the remaining terms in the right-hand sides of (38) are the result of decay processes. The terms with $\beta^{(2)}_{\omega_1 x_1}$ and $\beta^{(3)}_{\omega_1 x_1}$ correspond to decays of the type $k \rightleftarrows k_1 + k_2$, $k_1 \rightleftarrows k + k_2$, $k = \{\mathbf{k}, \omega\}$, which are allowed by the conservation laws for all values of $|x_1|$. The terms with $\beta^{(4)}_{\omega_1 x_1}$ and $\beta^{(5)}_{\omega_1 x_1}$ correspond to the process $k_2 \rightleftarrows k + k_1$, which is allowed by the conservation laws only when $|x_1| \leq 1/7$.

An analysis of (38) shows that for the solution (33) and (34) to be stable it suffices that the linear damping at $|x_1| = |x_{min}|$ exceed the buildup due to the decay instability.

Here

$$|x_{max}| \approx (v_A / v_{Te})^2 \text{ when } v_A > v_{Te}(4m_e / m_i)^{1/6}, \tag{39}$$

$$|x_{min}| \approx (4m_e / m_i)^{1/3} \text{ when } v_A < v_{Te}(4m_e / m_i)^{1/6}. \tag{40}$$

From this we obtain the following estimate for the helicon energy:

$$\text{if (39) is satisfied} \quad \frac{W}{n_0 T_e} \lesssim \sqrt{\frac{8}{\pi}}\left(\frac{v_A}{v_{Te}}\right)^7 \sqrt{\frac{m_i}{m_e}} \tag{41}$$

$$\text{if (40) is satisfied} \quad \frac{W}{n_0 T_e} \lesssim \sqrt{\frac{128}{\pi}}\frac{v_A}{v_{Te}} \sqrt{\frac{m_e}{m_i}}. \tag{42}$$

Thus, we can assume that under the conditions (41) and (42), $T_e = T_i$, and $\omega < \omega_{He}(v_s/v_A)^2|x|$, the helicon energy becomes concentrated in the direction of the magnetic field, where there is no damping, and a stable distribution $W_\omega \sim 1/\sqrt{\omega}$ is produced. This result can be applied to many observations of helicons in the magnetosphere, and also to stochastic acceleration of fast particles by helicons in magnetic traps, inasmuch as helicons propagating along the magnetic field precominently increase the energy of the particles perpendicular to the field, and by the same token contribute to containment of the accelerated particles in the magnetic traps.

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