# *Effect of a Constant Electric Field on Multiphoton Ionization*

A. I. Nikishov

P. N. Lebedev Physics Institute, USSR Academy of Sciences

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Differential probabilities for detachment of an electron bound by short-range forces (e.g. in $H^-$) by an electric field with an alternating dc component are obtained. In the most interesting case of a weak wave field, the presence of the constant field considerably increases the detachment probability, providing the field strength is sufficient for efficient opening of the nearest subthreshold (with respect to number of quanta absorbed from the wave) channels.

## 1. INTRODUCTION AND GENERAL FORMULA FOR PROBABILITY

In a recent note, Askar'yan and Arutyunyan[1] considered in a semiquantitative manner the interesting question of the influence of a constant electric field on multiphoton ionization of atoms by the field of the wave. A quantitative study of multiphoton ionization of atoms even by the simplest field of a monochromatic wave entails considerable difficulties, since it is necessary to take into account the joint action exerted on the electron in the final state by the field of the wave and by the long-range Coulomb forces[2-4].

On the other hand, if multiquantum detachment of electrons bound by short-range forces is considered, then one can justify the use of an approximation in which only the action of the field on the electron in the final state is taken into account[5-6]. Indeed, in this case, distances that are large in comparison with the dimensions of the bound system become effective[5-6], so one can disregard both the finite radius of the binding forces and the presence of the scattered wave in the final[1)]. The matrix element of the transition can then be written in the form[2]

$$M = -\frac{4\pi N}{(2mV)^{1/2}}\int_{-\infty}^{\infty} dt\, e^{iIt}\psi_{\mathbf{p}}^*(\mathbf{x}=0,t), \qquad N^2=\frac{c'\eta}{4\pi m}, \qquad \eta^2=2mI, \quad c'\sim 1. \tag{1}$$

Here I is the binding energy of the electron, m is its mass, $\psi_p(x)$ is the wave function of the final state, and $c'$ is a dimensionless constant of the order of unity. Formula (1) can be obtained by another method if the process is regarded as transformation of elementary particles[5,7]. The interaction that leads to the transformation is chosen in the form of a product of scalar wave functions of the particles, and the wave functions of the charge particles take into account the action of the external field. Further, recognizing that the electromagnetic field acts effectively only on a light particle, and changing over to the nonrelativistic case, we obtain (1).

We assume also that the wavelength $\lambda$ of the electromagnetic field is much larger than the effective distances of the process. Then the electromagnetic field can be described by a vector potential $\mathbf{A}(t)$ that depends only on the time. The corresponding solution of the Schrödinger equation is

$$\psi_{\mathbf{p}}(x) = \exp\left\{i\mathbf{p}\mathbf{x} - \frac{i}{2m}\int^{t}[\mathbf{p}-e\mathbf{A}(t)]^2\,dt\right\}, \tag{2}$$

$p_1$, $p_2$, and $p_3$ are the quantum numbers of the state. Substitution of (2) in (1) yields, in principle, the solution of the problem.

The sought probability turns out to be proportional to the observation time T, as it should. The multiplier of T, however, cannot result now from the $\delta$-function describing the energy conservation law; the integral with respect to t in (1) converges in the case of a field with a dc component. Accordingly, if we represent $\mathbf{A}(t)$ in the form

$$\mathbf{A}(t) = \mathcal{A}(\varphi) - \mathbf{E}t, \quad \mathcal{A}(\varphi+2\pi) = \mathcal{A}(\varphi), \quad \varphi = -\omega t + \varphi_0 \tag{3}$$

and put $\mathbf{E} = (0, 0, E)$, then $|M|^2$ should in fact be independent of $p_3$, and the integration with respect to $p_3$ should separate the multiplier of T. In classical language, $p_3$ is the integration of motion connected with the kinetic momentum $\pi_3(t)$ and with the potential (3) by the relation

$$p_3 = \pi_3(t) + eA_3(t). \tag{4}$$

Replacement of $\mathbf{E}t$ by $\mathbf{E}(t + t_0)$ in (3) does not change the field, but changes $p_3$ in accordance with (4). The dynamics of the process should therefore be independent of $p_3$.

To verify this, we transform (2), by making the substitution $t = u - p_3/eE$, into

$$\psi_{\mathbf{p}}^*(x) = \exp\left\{-i\mathbf{p}\mathbf{x} + \frac{i}{2m}(p_\perp^2 + e^2\overline{\mathcal{A}^2})u + \frac{ie^2E^2u^3}{6m}\right\}\sum_s A(s)e^{is\varphi},$$

$$e^{f(\varphi)} = \sum_{s=-\infty}^{\infty} A(s)e^{is\varphi}, \qquad A(s) = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\varphi' e^{f(\varphi')-is\varphi'}, \tag{5}$$

$$f(\varphi) = \frac{i}{2m}\int_0^u du[e^2\mathcal{A}^2(\varphi) - e^2\overline{\mathcal{A}^2} - 2ep_\perp\cdot\mathcal{A}_\perp(\varphi) - 2e^2Eu\mathcal{A}_3(\varphi)], \tag{6}$$

$$p_\perp\cdot\mathcal{A}_\perp = p_1\mathcal{A}_1 + p_2\mathcal{A}_2, \qquad \overline{\mathcal{A}^2} = \frac{1}{2\pi}\int_{-\pi}^{\pi}\mathcal{A}^2(\varphi)\,d\varphi.$$

[1)]Even when the radius of the binding forces tends to zero, the potential that leads to the bound state $Ne^{-\eta r}/r$ produces states $\psi_p = e^{i\mathbf{p}\cdot\mathbf{x}} - \exp(ipr)/(\eta + ip)r$ with scattered S-waves in the continuous spectrum. In the approximation under consideration, the scattered wave is not taken into account. Therefore, for very strong fields (and large probabilities), when $r_{\text{eff}} \sim 1/\eta$, such an approximation describes the process only qualitatively.

In the integral with respect to u in (6), $\varphi = -\omega u + \omega p_3/eE + \varphi_0$. Since $f(\varphi)$ depends on u not only via the periodic varaible $\varphi$, it follows that A(s) depends on u, i.e., $\varphi'$ in the integral with respect to $\varphi'$ in (5) should be regarded as independent of u. The lower limit of integration with respect to u in (6) corresponds to the choice of an arbitrary lower limit of integration in (2) in the form $-p_3/eE$. Now substitution of (5) in (1) yields

$$M = -\frac{4\pi N}{(2mV)^{1/2}} e^{-iIp_3/eE} \sum_s \exp\left\{is\left(\frac{\omega p_3}{eE} + \varphi_0\right)\right\} \tilde{B}(s), \tag{7}$$

$$\tilde{B}(s) = \int_{-\infty}^{\infty} du\, A(s) \exp\left\{i\left[I - s\omega + \frac{p_\perp^2 + e^2\overline{\mathcal{A}^2}}{2m}\right]u + \frac{ie^2E^2u^3}{6m}\right\} \tag{8}$$

and $\tilde{B}$ does not depend on $p_3$. From this follows the probability of detachment of the electron

$$\int dW = \int |M|^2 \frac{Vd^3p}{(2\pi)^3},$$

$$|M|^2 = \frac{8\pi^2N^2}{mV} \sum_{s,s'} \exp\left\{i(s-s')\left(\frac{\omega p_3}{eE} + \varphi_0\right)\right\} |\tilde{B}(s)|^2. \tag{9}$$

The terms of the sum (9) with $s \neq s'$ do not make any contribution in the integral of $|M|^2$ with respect to $p_3$. (These terms vanish in general after averaging over the initial phase $\varphi_0$.) Since the integral with respect to u in (8) is evaluated on finite values of u independent of $p_3$, the effective values for a given $p_3$ are $t_{eff} = u_{eff} - p_3/eE$. It follows therefore that $\int dp_3 = eET$, and we finally obtain

$$\frac{1}{T}\int dW = \frac{8N^2}{\eta}\left(\frac{B_0}{2E}\right)^{1/3} \sum_{s=-\infty}^{\infty} \int |B(s)|^2 dp_1 dp_2,$$
$$B(s) = \frac{1}{2\sqrt{\pi}}\left(\frac{e^2E^2}{2m}\right)^{1/3} \tilde{B}(s), \quad B_0 = \frac{\eta^3}{em}. \tag{10}$$

The s-th term of the sum yields here the probability of detachment of s photons with absorption from an alternating field (or emission in the field at $s < 0$), and the integral expression gives the differential distribution with respect to $p_1$ and $p_2$.

Formula (10) contains as particular cases both the probability of detachment of an electron by the alternating field of the wave[5-6] and the probability of tunneling in a constant field[8]. For example, turning off the field of the wave, we obtain

$$\frac{dW_c(E)}{T} = \frac{8N^2}{\eta}\left(\frac{B_0}{2E}\right)^{1/3} v^2(z)\, dp_1\, dp_2, \tag{11}$$
$$z = \left(\frac{B_0}{2E}\right)^{2/3}\left[1 + \frac{p_1^2 + p_2^2}{\eta^2}\right], \quad v(z) = \frac{1}{2\sqrt{\pi}}\int_{-\infty}^{\infty} dt\, e^{i(zt + t^3/3)},$$
$$B(s) = v(z)\,\delta_{0s}.$$

Using the relations obtained by Aspnes[9] for the Airy functions v(z), we can readily carry out the integration in (11):

$$\frac{W_c(E)}{T} = 8\pi N^2\eta\left(\frac{2E}{B_0}\right)^{1/3} \int_0^\infty v^2(y+t)\,dt = \frac{8\pi N^2}{\sqrt{y}}[v'^2(y) - yv^2(y)], \tag{12}$$
$$y = \left(\frac{B_0}{2E}\right)^{2/3},$$
$$t = \left(\frac{B_0}{2E}\right)^{2/3} \frac{p_\perp^2}{\eta^2}.$$

When $y \gg 1$ (and $c' = 1$) we obtain from this the result of Demkov and Drukarev[8]:

$$\frac{W_c(E)}{T} = I\frac{E}{B_0}\exp\left\{-\frac{2}{3}\frac{B_0}{E}\right\}.$$

## 2. LINEARLY POLARIZED WAVE

We now consider the case when the potential (3) is of the form

$$\mathbf{A}(t) = \mathbf{a}\cos\varphi - \mathbf{E}t, \quad \mathbf{a} = (a_1, 0, a_3),$$
$$a\omega = B, \quad a = \sqrt{a_1^2 + a_3^2}, \tag{13}$$

B is the amplitude of the wave field. From (5) and (6) we get

$$A(s) = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\varphi \exp\{i[\alpha\sin\varphi - \beta\sin 2\varphi + \gamma\cos\varphi - s\varphi]\},$$
$$\alpha = \alpha_1 + \alpha_2 u, \quad \alpha_1 = ea_1p_1/m\omega, \quad \alpha_2 = e^2Ea_3/m\omega,$$
$$\beta = e^2a^2/8m\omega, \quad \gamma = -e^2Ea_3/m\omega^2. \tag{14}$$

For the function A(s) we readily obtain the representation

$$A(s) \equiv A(s,\alpha,\beta,\gamma) = \sum_{l=-\infty}^{\infty} J_l(\beta) J_{s+2l}(\sqrt{\alpha^2+\gamma^2})\, e^{i(s+2l)\psi},$$
$$\operatorname{tg}\psi = \gamma/\alpha, \tag{15}$$

if we use the relation

$$e^{ix\sin\varphi} = \sum_{l=-\infty}^{\infty} J_l(x)\, e^{il\varphi}.$$

Using the same relation for the factor $\exp(i\alpha_2 u\sin\varphi)$ and changing over in (8) from the variable u to the variable $t = (e^2E^2/2m)^{1/3}u$, we obtain

$$B(s) = \sum_{n=-\infty}^{\infty} A(s-n, \alpha_1, \beta, \gamma)\, C(n),$$
$$C(n) \equiv C(n,z,z') = \frac{1}{2\sqrt{\pi}}\int_{-\infty}^{\infty} dt\, e^{i(zt + t^3/3)} J_n(z't),$$
$$z = \left(\frac{2m}{e^2E^2}\right)^{1/3}\left[I - s\omega + \frac{p_\perp^2}{2m} + \frac{e^2a^2}{4m}\right]$$
$$= \left(\frac{B_0}{2E}\right)^{2/3}\left[\frac{p_\perp^2}{\eta^2} - (2+\xi^2)\frac{s-s_0}{2s_0}\right], \tag{16}$$
$$z' = \left(\frac{2m}{e^2E^2}\right)^{1/3}\alpha_2 = 2\left(\frac{2Es_0^{3/2}}{B_0}\right)^{1/3} \frac{\sqrt{s_0}\,\xi_3}{1+\xi^2/2}, \quad \xi = \frac{ea}{\eta}, \quad \xi_3 = \frac{ea_3}{\eta},$$
$$s_0 = \frac{I}{\omega}\left(1 + \frac{\xi^2}{2}\right) = \frac{B_0}{2B}\xi\left(1 + \frac{\xi^2}{2}\right).$$

At $\xi \ll 1$, when it seems that not more than $s_0$ terms are significant in the sum over n in (16), it is convenient to calculate the function C(n) by representing $J_n(z't)$ in the form of the series. Then the integrals with respect to t are expressed in final analysis in terms of the Airy function v(z) and its derivative, since the n-th derivative of the Airy function, by virtue of the relation $v''(z) = zv(z)$ is again expressed in terms of v and v′. We note, finally, that at $\gamma = 0$ the function A(s) coincides with $A_0(s, \alpha, \beta)$ the asymptotic forms of which were investigated earlier[10,11,5]. At $\alpha = 0$ we obtain from (14) or (15)

$$A(s, 0, \beta, \gamma) = e^{is\pi/2} A_0(s, \gamma, -\beta). \tag{17}$$

In the absence of a dc component of the field (E = 0) by virtue of energy conservation, the summation in (10) proceeds only over $s > s_0$. At finite E, the "subthreshold'' channels with $s < s_0$ are also open[2]. To obtain a clear idea of their role, we consider the simple case when $a_3 = 0$, i.e., **a** is orthogonal to **E**. Then

$$\gamma = z' = 0, \quad C(n) = v(z)\delta_{0n},$$
$$\frac{W}{T} = \frac{8N^2}{\eta}\left(\frac{B_0}{2E}\right)^{1/3}\sum_{s=-\infty}^{\infty}\int A_0^2(s,\alpha,\beta)v^2(z)\,dp_1\,dp_2,$$
$$z = \left(\frac{B_0}{2E}\right)^{2/3}\left[\frac{p_\perp^2}{\eta^2} - (2+\xi^2)\frac{s-s_0}{2s_0}\right]. \tag{18}$$

The asymptotic form of $A_0^2(s, \alpha, \beta)$ at

$$s_0 \gg 1, \quad \frac{|s-s_0|}{2s_0} \ll 1, \quad \frac{p_1^2}{\eta^2} \lesssim \frac{1+\xi^5}{s_0}. \tag{19}$$

was obtained in[5]. If the expression obtained there is rewritten in a form that does not presuppose the presence of the energy conservation law, then we obtain

$$A_0^2(s,\alpha,\beta) = \frac{1}{\pi|f''|}e^{2\,\mathrm{Re}\,f}[1-\cos(2\,\mathrm{Im}\,f - \arg f'')],$$
$$2\,\mathrm{Re}\,f = 2s\left\{\frac{\sqrt{1+\xi^2}}{2+\xi^2} - \mathrm{Ar\,sh}\frac{1}{\xi} - \frac{s-s_0}{s_0}\frac{\sqrt{1+\xi^2}}{2+\xi^2}\right.$$
$$\left. + \frac{p_1^2}{\eta^2\sqrt{1+\xi^2}(1+\xi^2/2)} + \ldots\right\}, \tag{20}$$
$$2\,\mathrm{Im}\,f = 2s\left\{\frac{\pi}{2} - \frac{4\sqrt{1+\xi^2}}{2+\xi^2}\frac{p_1}{\eta} + \ldots\right\},$$
$$\arg f'' = \pi + \frac{p_1}{\eta(1+\xi^2)} + \ldots, \quad |f''| = \frac{4s\sqrt{1+\xi^2}}{2+\xi^2} + \ldots \tag{21}$$

The dots denote terms $\sim(s - s_0)/2s_0$ relative to the smallest terms that have been written out.

Using (20) and changing over in (18) to the variables $p_\perp = \sqrt{p_1^2 + p_2^2}$ and $\varphi$, we can readily carry out the integration with respect to

$$\int_0^{2\pi} d\varphi\, e^{2Q\cos^2\varphi} = 2\pi e^Q I_0(Q), \quad Q = \frac{sp_\perp^2}{\eta^2\sqrt{1+\xi^2}(1+\xi^2/2)}, \tag{22}$$
$$p_1 = p_\perp\cos\varphi.$$

$I_0(Q)$ is a Bessel function. The contribution of the oscillating term in (20) to the integral can be neglected. The remaining integral with respect to $p_\perp^2$ is given by

$$\int_0^\infty \frac{dp_\perp^2}{\eta^2}e^Q I_0(Q)v^2(z), \tag{23}$$

and the integrand determines the form of the distribution with respect to $p_\perp^2$ at fixed s. The obtained expressions are simple enough to be able to trace the influence of a constant field on the multiquantum ionization.

We note one particular case. Let $B_0/2s_0^{3/2}E \sim 1$ and $\xi^2 \ll 1$. The first of these conditions denotes that in any case the first subthreshold channel is effectively open (the quantity z in (18) is of the order of unity). In this case, just as in the absence of a constant field, $p_{\perp\,\mathrm{eff}}^2/\eta^2 \sim s_0^{-1}$, $s_{\mathrm{eff}} \sim s_0$, and the integral (23) is of the order of $s_0^{-1}$. Recognizing also that

$$\mathrm{Ar\,sh}\frac{1}{\xi} - \frac{\sqrt{1+\xi^2}}{2+\xi^2} = \ln\frac{2}{\xi} - \frac{1}{2} + \frac{\xi^2}{4} - \frac{\xi^4}{32} + \ldots,$$

we have $A_0^2(s, \alpha, \beta) \sim \xi^{2s}$, $\xi^2 \ll 1$, as expected if perturbation theory with respect to the wave field is applicable. It follows therefore that the reaction proceeds via the first subthreshold channel with a probability larger by a factor $\xi^{-2}$ than via the first channel above threshold. The probability that the reaction will proceed with absorption of some number of photons $s > s_0$ is not altered in order of magnitude by the presence of the field in this case. Thus, in accordance with[1], a moderately strong constant electric field lowers the effective number of quanta absorbed from the electromagnetic wave in the ionization process.

## 3. LOW FREQUENCY LIMIT

If the process under consideration evolves within a time such that the phase of the wave has no opportunity to change appreciably, then the probability in the periodic field is connected with the probability in the constant field by the relation

$$W = \frac{2}{\pi}\int_0^{\pi/2} d\psi\, W_c(E(\psi)), \tag{24}$$

$W_c(E)$ is the probability in the constant field with intensity E (see formula (12)).

In the case of the linearly polarized wave (13) we have

$$E(\psi) = [(a_1\omega\sin\psi)^2 + (a_3\omega\sin\psi + E)^2]^{1/2}. \tag{25}$$

If $a_1 = 0$, then relation (24) holds also for the differential probability. At $a_3 = 0$ it is easy to verify that formula (18) reduces to (24) as $\xi \to \infty$ (actually at $\xi^2 \gg B_0/B$). Indeed, using the results of[11,5], we have

$$A_0^2(s,\alpha,\beta) = \frac{2}{\pi\xi^2\sin^2\psi}\left[\frac{2B\sin\psi}{B_0}\right]^{2/3}v^2(y)\,[1+\cos 2\tilde{\xi}],$$
$$y = \left[\frac{B_0}{2B\sin\psi}\right]^{2/3}\sigma, \quad \sigma = 1 + (2+\xi^2)\frac{s-s_0}{2s_0} - \frac{p_1^2}{\eta^2}, \tag{26}$$
$$\cos\psi = \alpha/8\beta = p_1/\eta\xi,$$

where the contribution from $|p_1/\eta\xi| > 1$ vanishes as $\xi \to \infty$. The oscillating term $\cos 2\tilde{\xi}$ can be omitted, since it makes no contribution upon integration. Replacing the sum over s in (18) by an integral, we obtain[9]

$$\int_{-\infty}^{\infty} ds\, v^2(y)v^2(z) = \frac{2s_0}{\xi^2}\left(\frac{2E}{B_0}\right)^{2/3}\frac{\sqrt{\pi}}{4}\left|\frac{B\sin\psi}{E}\right|^{1/3}v_1(\tilde{z}),$$
$$\tilde{z} = \left[\frac{B}{[(B\sin\psi)^2+E^2]^{1/2}}\right]^{2/3}\left(1+\frac{p_2^2}{\eta^2}\right), \quad v_1(x) = \int_x^\infty v(t)\,dt. \tag{27}$$

The integral of $v_1(z)$ with respect to $p_2$ should be transformed with the aid of the formula

$$\int_0^\infty du\, v_1(x+u^2) = \frac{\sqrt{\pi}}{2^{1/3}}\int_0^\infty dx' v^2\left(\frac{x+x'}{2^{2/3}}\right),$$

after which we obtain (24), with the integration with respect to $p_1$ playing the role of averaging over the phase $\psi$.

[2]The gist of this effect, which consists of "smearing" of the δ-function describing the energy conservation law by the constant field, is the same as in the Franz-Keldysh effect[12,13].

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69