Quantum Effects in Electromagnetic Excitation of Sound in Semimetals

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The effect of the Landau quantization on the sound intensity excited by electromagnetic fields in semimetals is studied. The quantum effects are related to variation of the electron and hole concentration under the action of the field of the wave. The character of the oscillations and their magnitude with respect to the sound intensity, which depends monotonically on the magnetic field, are determined. A comparison is made with experiment.

INTRODUCTION

THE disruption of equilibrium in a system of conduction electrons leads in particular to the excitation of sound waves in the specimen, as a consequence of electron-phonon interaction. The excitation may turn out to be more effective in this case if the perturbation of the carriers is accompanied by a variation in their concentration n; here, according to thermodynamic considerations, a volume force, proportional to ∇n, acts on the medium. In systems with metallic conductivity, where the Debye radius is the smallest of the characteristic lengths, one deals with variation of the concentration of the individual electron groups in the presence of invariance of the total concentration (by virtue of the condition of electrical neutrality). These variations, as is shown in [1–3], play an important role in the various problems of the electrodynamics of metals; they are due to the multi-connectedness of the Fermi surfaces and to the large separation between the electron and hole "valleys" (in momentum space), thanks to which the equilibrium between the valleys is established within a time T that is much greater than the time of intravalley relaxation τ.

In the classical case, the appearance of nonequilibrium concentrations is brought about by the inhomogeneity of the variable electromagnetic field in the sample and by the conditions of flux conservation on the surfaces. Under conditions of magnetic quantization, the inhomogeneity of the magnetic induction $\mathscr{B}$ is an additional reason for the variation of the carrier concentrations: in semimetals, the equilibrium concentrations of electrons depend on $\mathscr{B}$; therefore a time-varying magnetic induction leads to their variation.[3]

It has been shown in [4–6] that the experimental data [6,7] on the excitation of sound by electromagnetic waves in semimetals located in a comparatively weak magnetic field ($B_0 \lesssim 100$ G for bismuth, $200 \lesssim B_0 \lesssim 2000$ G for antimony) at liquid helium temperatures can be explained only effects connected with variations in the electron and hole concentrations. In more intense fields, an oscillatory character has been observed experimentally (see [7]) for the amplitude of the acoustic resonances as B_0 increases.

The present work is devoted to the investigation of the electromagnetic excitation of sound in semimetals under the conditions of Landau quantization. The losses of electromagnetic energy in the creation in the sample of standing sound waves are computed; for this purpose, we solve the equation of the dynamics of an elastic medium with an exciting electromagnetic layer. As in [5], we take the damping into account by the introduction of the parameter Q—the acoustical quality factor (in accord with experimental data,[7] Q is not connected with the electron mechanism). In the research of the author and Rashba,[3] a quantum theory of the normal skin effect in semimetals was constructed, in which, along with the quantum oscillations of the magnetic susceptibility and the kinetic coefficients, the effects mentioned above, which are connected with the dependence of n on $\mathscr{B}$, are taken into account. The results of this theory are used below to find the exciting force in the equation of motion of the medium; only the force connected with the concentration gradients is taken into account (in the quasiclassical approximation, to which we limit ourselves, the role of other forces is insignificant, as it is in the classical case[5]). The simplest model of a semimetal is considered—one electron valley and one hole valley with isotropic dispersion laws.

ENERGY LOSSES IN SOUND EXCITATION

We shall consider a slab of semimetal $0 \le z \le d$, irradiated on both sides by electromagnetic waves of frequency ω. For definiteness, let the conditions of radiation be such that the magnetic field of the wave $H(0) = H(d)$, and the electric field $E(0) = -E(d)$; $H \parallel \mathbf{y}$, $\mathbf{E} \parallel \mathbf{x}$. A constant external field is applied parallel to the field of the wave, the total magnetic induction inside the sample is $\mathscr{B} = B_0 + B(z, t)$. The equation for longitudinal sound oscillations $u(z)e^{-i\omega t}$ is written in the form

$$u'' + k_s^2 u = \Phi'(z); \quad k_s = \frac{\omega}{s}\left(1 + \frac{i}{Q}\right), \tag{1}$$

s is the sound velocity; the damping is introduced phenomenologically through the quality factor $Q \gg 1$; the prime denotes differentiation with respect to z. As was pointed out in the Introduction, we shall use in the expression for the force Φ' only the "deformation" part, which is connected with the concentration gradients, i.e.,

$$\Phi' = -\Lambda n' / \rho s^2, \tag{2}$$

where Λ is the difference of the deformation potentials of the electrons and holes, ρ the density of the crystal, $n(z, t)$ the total concentration of electrons (which is identical with the concentration of holes p).

We separate in n the instantaneously equilibrium part which depends on the instantaneous value of the induction $\mathscr{B}(z, t)$, i.e., we represent n in the form

$$n = n_0(\mathscr{B}) + n_1 \approx n_0(B_0) + \frac{dn}{dB}B(z,t) + n_1(z,t). \tag{3}$$

Here n_1 is the nonequilibrium concentration, dn/dB the total derivative, calculated under conditions of thermodynamic equilibrium.[3] Solving Eq. (1) with account of (2), (3) and the boundary condition

$$u'(0, d) = \Phi(0, d),$$

we find that the change in the elastic energy of the slab per unit time is

$$\dot W \equiv \frac{d}{dt}\int_0^d dz \frac{\rho}{2}[\dot u^2 + s^2(u')^2] = \frac{\rho s^2}{2}\operatorname{Re}\int_0^d dz\, \dot u'\Phi^*$$

for resonance corresponding to the establishment of an odd number of acoustic half wavelengths over the thickness of the slab d in the case considered, equal to

$$\dot W = \frac{\rho\omega^2 Q s}{\pi(N+{}^1/_2)}\left|\int_0^d dz \sin k_s z \Phi(z)\right|^2, \quad \frac{\omega d}{2s} = \left(N + \frac{1}{2}\right)\pi. \tag{4}$$

In the intermediate calculations, use was made of the property $\Phi(z) = \Phi(d - z)$ which follows from the symmetry of the problem assumed above, and only the resonance terms $\sim[\cos(k_s d/2)]^{-1}$ have been kept.

To determine $\Phi(z)$ and the energy loss (4), it is necessary to solve the set of Maxwell equations

$$\operatorname{rot}\mathbf{E} = -\frac{1}{c}\dot{\mathbf{B}}, \quad \operatorname{rot}\mathbf{H} = \frac{4\pi}{c}\mathbf{j}, \quad j_z = 0, \quad \mathbf{j} = \mathbf{j}_n + \mathbf{j}_p \tag{5}$$

and the continuity equation for electrons with account of recombination;

$$\frac{\partial n}{\partial t} - \frac{1}{e}\operatorname{div}\mathbf{j}_n + \frac{n_1}{T} = 0. \tag{6}$$

The currents j in the normal skin-effect approximation are given by the relations[3]

$$\mathbf{j}_n = \hat\sigma_n\left(\mathbf{E} + \frac{1}{e}\nabla\eta_n(\mathscr{B}, n)\right), \quad \mathbf{j}_p = \hat\sigma_p\left(\mathbf{E} - \frac{1}{e}\nabla\eta_p(\mathscr{B}, p)\right), \tag{7}$$

where η_n and η_p are the Fermi quasilevels of electrons and holes. The material equation, which connects H in semimetals not only with B but also with n_1, has the form[3]

$$H = \frac{B}{\mu} + \left(\frac{\partial H}{\partial n}\right)_B n_1, \quad \mu^{-1} = \frac{dH}{\partial B},$$
$$\left(\frac{\partial H}{\partial n}\right)_B = -4\pi\frac{dn}{\partial B}b, \quad b = \left(\frac{\partial\eta_n}{\partial n}\right)_B + \left(\frac{\partial\eta_p}{\partial p}\right)_B. \tag{8}$$

The set of equations (5) and (6) was solved in [3] for the case of a halfspace. In the problem considered here, the finiteness of the dimension of the slab is important; as applied to the symmetry assumed above, the expression for the magnetic field will be the following:

$$H(z) = H_1\frac{\cos k_1(z-d/2)}{\cos k_1 d/2} + H_2\frac{\cos k_2(z-d/2)}{\cos k_2 d/2}. \tag{9}$$

Here k_1 and k_2 are the roots of the dispersion equation

$$k^4 + k^2\left(\frac{1}{L^2} - \frac{i}{\delta_0^2}\right) - \frac{i}{L^2\delta^2} = 0, \tag{10}$$

in which

$$\delta^2 = \frac{c^2}{4\pi\omega\sigma\mu}, \quad \delta_0^{-2} = \delta^{-2}\left(1 + \frac{a^2\sigma^*}{\sigma}(1-gG)^2\right),$$
$$L^2 = \frac{\sigma^* b}{e^2}T', \quad \sigma = \sigma_{xx} + \frac{\sigma_{xz}^2}{\sigma_{zz}}, \quad \hat\sigma = \hat\sigma_n + \hat\sigma_p,$$
$$\sigma^* = \frac{\sigma_{zz}^n\sigma_{zz}^p}{\sigma_{zz}}, \quad a = \frac{\sigma_{xz}^n}{\sigma_{zz}^n} - \frac{\sigma_{xz}^p}{\sigma_{zz}^p}, \quad T' = \frac{T}{1-i\omega T},$$
$$G = \frac{B_0}{n}\frac{dn}{dB}, \quad g = \frac{enc}{aB_0\sigma^*}. \tag{11}$$

Expressing the induction B and the concentration n_1 in terms of the amplitude $H_{1,2}$ from (9), with the help of Eqs. (5)–(8), and carrying out the integration in (4), we get, after a series of transformations:

$$\dot W = \frac{QW_0\alpha^2 s}{(N+{}^1/_2)\pi}\left|\sum_{i=1,2}\frac{k_s^2}{k_s^2-k_i^2}F_i\right|^2\left(\frac{\Lambda g}{bn}\right)^2,$$
$$F_i = \frac{H_i/H(0)}{1-igG(\delta k_i)^{-2}}\left[1 - i(\delta k_i)^{-2} + \frac{L^2 gG}{\delta^2\omega T'}\left(1 - \frac{\delta^2}{\delta_0^2}\right)\right],$$
$$W_0 = |H(0)|^2/4\pi, \qquad \alpha^2 = B_0^2/4\pi\rho s^2, \tag{12}$$

H(0) is the given value of the magnetic field on the surfaces.

For the final solution of the problem, we must find the amplitudes $H_{1,2}$ from the boundary conditions. One of them is the ordinary electrodynamic condition of continuity of H on the boundary:

$$H_1 + H_2 = H(0). \tag{13}$$

The second is the condition of surface recombination, which we write phenomenologically:

$$j_{nz}(0, d) = \mp 2eSn_1(0, d), \tag{14}$$

S is the rate of surface recombination (according to [1,2] $S \sim v_F d_{np}$, d_{np} is the probability of intervalley scattering in the collision of an electron with the surface). Expressing the field E and the concentration n_1 in j_{nz} in terms of H, we obtain (14) in the form

$$\sum_{i=1,2}\frac{H_i}{1-igG(\delta k_i)^{-2}}\left[1 - i(\delta k_i)^{-2} - \frac{L^2}{\delta^2 P_i}(1 + i(\delta k_i)^2\right.$$
$$\left.-(1-gG)\left(1 - \frac{\delta^2}{\delta_0^2}\right)\right)\Bigg] = 0, \tag{15}$$
$$P_i = 2ST'k_i\operatorname{ctg}(k_i d/2).$$

after a series of transformations. The quantum effects in (12) are connected with terms containing G, (11) and the oscillating additions to μ and $\hat\sigma$. If we do not take them into account, then the results of classical theory follow from (12) and (15).[5]

ANALYSIS OF THE RESULTS

In connection with existing experimental data,[7] we limit ourselves further to the case of the quasiclassical approximation and a strong magnetic field:

$$\frac{\eta_{n,p}}{\hbar\omega_{n,p}^c} \gg 1, \quad \omega_{n,p}^c = \frac{eB_0}{cm_{n,p}} \gg \frac{1}{\tau_{n,p}}$$

As was shown in [3], G here contains only terms that oscillate with the de Haas–van Alphen period. Their amplitude is

$$|G| \sim (\hbar\omega_c/\Delta)^{1/2}K \ll 1 \tag{16}$$

($\Delta = \eta_n + \eta_p$ is the magnitude of the overlap of the bands, and temperature and Dingle factors are included in $K \leq 1$). The magnetic susceptibility is determined by the expression (see [3])

$$\mu^{-1} = 1 + qG, \qquad q = 4\pi n\Delta / B_0^2. \tag{17}$$

In a strong field, we have in (11) $g \approx 1$, $a \gg 1$, so that $\delta^2 \gg \delta_0^2$ and the roots of (10) can be written in the form

$$k_1^2 = -\frac{1 - iL^2/\delta_0^2}{L^2}, \quad k_2^2 = \frac{i/\delta^2}{1 - iL^2/\delta_0^2},$$

$$L^2 = \frac{R^2 T'}{3\tau_n(1 + l_p/l_n)}, \quad \frac{L^2}{\delta_0^2} = \frac{2}{3}\omega T' q\mu, \; b = \frac{2\Delta}{3n}, \tag{18}$$

R is the Larmor radius and l the free path length.

We first consider the very realistic case for the appearance of quantum oscillations of such values of B_0 in which the parameter q, determined from the expression (17), is <1 (for bismuth, this is satisfied for $B_0 \gtrsim 300$ G). Then the oscillations of μ are unimportant and $L^2/\delta_0^2 \ll 1$. Different variants are possible, depending on the relation of the sound wavelength k_s^{-1} and the penetration depth $|k_{1,2}|^{-1}$:

1) $k_i \gg k_s$, i.e.,

$$\delta, L \ll \frac{s}{\omega} = \frac{d}{2\pi(N + 1/2)}, \tag{19}$$

the field B_0 is not strong enough for the skin layer to fill the slab for the first numbers N (we recall that $\delta \sim B_0$). Then, for different conditions of surface reflection, characterized by the parameters P_i in (15), we get for the sums entering into (12)

$$\sum_{i=1,2} \frac{k_s^2}{k_s^2 - k_i^2} F_i \approx \frac{k_s^2 \delta^2 L^2}{\delta_0^2}\left(1 - i\frac{G}{\omega T'}\right) A_1, \tag{20}$$

$$A_1 = \begin{cases} (1 + L\delta/\delta_0\sqrt{i})^{-1}, & S \to 0, \quad (20a)\\ 1, & 2T'S \gg L, \delta, \quad (20b)\\ (1 + L^2\delta/2\sqrt{i}\,T'S\delta_0^2)^{-1}, & L \ll 2T'S \ll \delta. \quad (20c)\end{cases}$$

2) There are very strong fields where $k_1 \gg k_s \gg k_2$ i.e.,

$$L \ll d/2\pi(N + 1/2) \ll \delta. \tag{21}$$

Then

$$\sum_i \frac{k_s^2}{k_s^2 - k_i^2} F_i \approx -i\frac{L^2}{\delta_0^2}\left(1 - i\frac{G}{\omega T'}\right) A_2, \tag{22}$$

$$A_2 = \begin{cases} \left(1 + \frac{Ld}{2\delta_0^2}\right)^{-1}, & S \to 0, \quad (22a)\\ 1, & 2T'S \gg L, d/2, \quad (22b)\\ \left(1 + \frac{L^2 d}{4\delta_0^2 T'S}\right)^{-1}, & L \ll 2T'S \ll d. \quad (22c)\end{cases}$$

The quantum corrections are determined by the quantity $G/\omega T'$ in (20) and (22). For small values $\omega T \ll 1$, the quantum additions to the results of the classical consideration can be significant; with increase in B_0, their role increases. The quantities A_1 (20) and A_2 (22) are sensitive to the properties of the scattering surfaces. This is connected with the presence of two waves (9), thanks to which even in the considered case of the normal skin effect the amplitudes of the waves H_1 and H_2 depend on the scattering of electrons from the surface. The case (22) corresponds to complete penetration of the second wave (9) into the sample; however, the effect of the first wave, as can be shown, becomes insignificant for such values B_0 for which $Ld/2\delta_0^2 \ll 1$ in (22a) and $L^2 d/4\delta_0^2 T'S \ll 1$ in (22c) (the case (22b) is evidently unreal because of the condition $4T'S \gg d$). When these conditions are satisfied, the quantum corrections in (22) actually describe homogeneous magnetostriction, which is discussed in [7]. The corresponding critical values of B_0^*, the surpassing of which gives the transition to homogeneous magnetostriction, are the following:

for (22a):

$$B_0^* \sim \frac{d}{\delta_0}(\omega T n\Delta)^{1/2}, \tag{23a}$$

for (22c):

$$B_0^* \sim \left(\frac{\omega d}{S} n\Delta\right)^{1/2} \text{ for } TS > \frac{\delta_0^2}{d}. \tag{23c}$$

We now consider the case of so-called "strong" diamagnetism, when μ (17) oscillates in order of magnitude. This is possible for $q \gg 1$, when $qG \sim 1$, i.e., in rather weak fields (for which, however, $\omega_c\tau \gg 1$ and $\hbar\omega_c \gg 2\pi^2 T_0$, T_0 the temperature). Since this case in semimetals can exist in principle only for very low temperatures (of the order of fractions of a degree), then we shall touch on it only briefly, limiting ourselves to analysis for $k_1 \gg k_s$ and $S \to 0$. We shall also assume that $\omega Tq < 1$. Calculations with the use of (18) lead to the following result:

$$\sum_i \frac{k_s^2}{k_s^2 - k_i^2} F_i = \frac{\delta^2 L^2 k_s^2}{\delta_0^2}\left[\left(1 - i\frac{G}{\omega T} - \frac{2}{3}\mu q G\right) \times \left(1 + \frac{iGL^2}{\delta_0^2}\right) + \left(1 - \frac{2}{3}\mu q G\right)\frac{L}{\delta\sqrt{i}}\right]\left[1 + \frac{L\delta}{\delta_0^2\sqrt{i}}\left(1 - \frac{iL^2}{\delta_0^2}\right)\right]^{-1} \tag{24}$$

On this part of the period of oscillation, where $qG > 0$ in (17), the differences from (20a) are unimportant, because, in accord with the condition of low frequencies assumed above and (18), $L^2/\delta_0^2 < 1$. But on the other part of the period, where G changes sign and μ increases, L^2/δ_0^2 and μqG increase and (24) takes the form

$$-i\frac{L^2 G k_s^2}{\omega T}\left[1 + \frac{\delta\sqrt{i}}{L}\left(1 + \frac{iGL^2}{\delta_0^2}\right)\right]. \tag{25}$$

At the beginning, while $GL^2/\delta_0^2 < 1$, (25) exhibits a decay in comparison with (20a), because the factor $\delta_0^2 G/\delta^2\omega T \approx G/\omega_c^2\tau^2\omega T$, which distinguishes (25) from (20a) in a rough estimate is small for $G/\omega T \sim 1$. Upon approach to the region of absolute instability, when $\mu^{-1} \to 0$, the term GL^2/δ_0^2 increases, (25) tends toward

$$\sqrt{i}\frac{\delta^2 L^2 k_s^2}{\delta_0^2}\frac{G^2 L}{\omega T\delta}$$

and can increase for $G^2L/\omega T\delta \sim G^2\mu^{1/2} > 1$.

DISCUSSION

Here we shall discuss only the results which refer to the case $q \ll 1$, which correspond to experimental conditions for which quantum oscillations of the amplitude of the effect of sound excitation in bismuth were observed.[7] Oscillations were observed, beginning with fields ~2 kOe, in a region where the amplitude of the excitation, which depends monotonically on B_0, is already greatly diminished in comparison with its value at lower fields, penetrating to a depth $\sim\delta$ (the second term in (9)). If we assume that in the region of monotonic decay of the effect, the relation $G/\omega T' \ll 1$ because $K \ll 1$ in (16), then this decay follows from (12) upon use of (22). Upon increase in B_0, the Dingle and temperature factors cease to limit the amplitudes of oscillation for the de Haas-van Alphen fundamental, the amplitude G (16) increases and it can be shown that $G/\omega T' \ll 1$, which, in accord with (12) and (22), leads to oscillations of the sound excitation that grow with the field, and that are of half its period. The character of the dependence of the maxima on B_0 is determined both by the increase in K with growth of B_0 and also by the relation between B_0 and the critical fields (23). It is difficult to distinguish the effect of these two causes; here the analysis of the temperature dependence of the generation amplitudes would be useful. If we assume the temperature and B_0 are such that $K_1 \approx 1$ in (16), but these values of $B_0 < B_0^*$, then the oscillations of $\dot{W}$ (12) should increase as B_0 in the case (22c) and as B_0^3 in the case (22c). In the following, for $B_0 > B_0^*$, the increase changes to a decay $\propto B_0^{-1}$. For the parameters of bismuth, which correspond to the conditions of experiment,[7] $B_0^* \sim 10^5$ G (22a) and $B_0^* \sim 10\,(v_F/S)^{1/2}$ G (22c). In [7] a growth of the oscillations was found over the entire interval of fields used (≤ 7 kG), but these data are insufficient to draw any conclusion as to the smallness of the value of the rate of surface recombination S.

We also note that the increase and decrease in the maxima, which is gradual with respect to the numbers, and which was observed in [7], can be explained by the contribution of the de Haas-van Alphen second harmonic to the quantity G. If the ratio of amplitudes of the second harmonic and the fundamental $K_2/K_1 = \beta \ll 1$, because of the temperature and Dingle factors, then it is easy to establish the fact (by elementary calculations) that, with accuracy to terms $\sim B$, the amplitudes of successive values of m of the quantum oscillations in (12), differ, among other things by the factors $(1 + \beta\sqrt{2}\,(-1)^m)$.

Clearly expressed quantum oscillations for the case described by Eqs. (20) were not observed in [7]. In accord with (20), they should be more effective if lower frequencies are used (thicker samples); then the case of a thin skin layer, corresponding to (20), can be important even at high fields, which also increases the role of the quantum term $G/\omega T'$. Observation of oscillations against the background of the effect of excitation that is monotonic in the field would allow us to determine directly the value of the time T. A comparison of the amplitudes of the oscillations in the region described by (22), with the value of the monotonic absorption, corresponding to Eqs. (20) without quantum corrections, can also give information on the parameters T, S, and the rest. By virtue of everything set forth above, it is demonstrated that the investigation of quantum effects for electromagnetic excitation of sound is a convenient method of study of the electronic properties of semimetals.

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