# *Analog of the Josephson Effect in a System with Electron-Hole Pairing*

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Phenomena are considered which are related to phase coherence of the electron-hole pair wave function in non-equilibrium semiconductors with electron-hole pairing. It is shown that a varying tunnel current with an oscillation frequency $\omega = 2V$ arises when voltage is applied to a semiconductor-insulator-semiconductor structure which is optically pumped by an external source. This occurs in the quasistationary state and when the conditions required for electron-hole pairing are fulfilled. In the formula presented above $V = V_r - V_l$, $V_r = (1/2)\,D_r + \mu_r$, $V_l = (1/2)\,D_l + \mu_l$, $D_l$ and $D_r$ are the forbidden band widths for the left- and right-hand side semiconductors respectively and $\mu_r$ and $\mu_l$ are the Fermi degeneracy energies of the respective semiconductors.

**1.** A few years ago, Keldysh and one of the authors[1] considered one phenomenon connected with the Coulomb interaction of the electrons from different energy bands. The gist of the phenomenon consists in the possibility of pairing the particles with the holes, as a result of which the spectrum becomes redistributed in the bands. The situation here is similar to superconductivity, particularly, the production of a pair of single-particle excitations should be regarded as a braking of a bound electron-hole pair. With changing temperature, the system undergoes a second-order phase transition, and the transition temperature $T_c$ is the condensation point of the pairs in the ground state. From the macroscopic point of view, the new coherent state into which the electron system of the crystal goes over is characterized by the existence of a certain, generally speaking complex, function $\psi = \psi(\mathbf{r}t)$, which has the meaning of the wave function of the electron-hole pair and which differs from zero below the transition point $T_c$. The presence of a single function $\psi = fe^{i\varphi}$ for the entire sample, characterizing not only one particle but the entire ensemble of electrons, leads to the occurrence of phase differences between the $\psi$-functions at any two points of the crystal that are separated by large distances; these phase differences are fixed at a given instant of time. Such a phase coherence leads to a number of specific quantum effects, the analysis of which is the subject of the present communication.

We analyze phenomena occurring in a semiconductor-insulator-semiconductor tunnel structure, and show that when a voltage is applied to the structure, which is also optically pumped from an external source, then, in the quasistationary state, when the conditions necessary to realize electron-hole pairing are produced, an alternating tunnel current begins to flow with a frequency

$$\omega = 2V, \tag{1}$$

where

$$V = V_r - V_l, \quad V_r = {}^1\!/_2 D_r + \mu_r,\; V_l = {}^1\!/_2 D_l + \mu_l,$$

$D_r$ and $D_l$ are the widths of the forbidden bands of the right and left semiconductors, respectively, and $\mu_r$ and $\mu_l$ are the Fermi-degeneracy energies of the respective semiconductors.

**2.** Thus, we consider a semiconductor-insulator-semiconductor tunnel structure, in which a stationary carrier density is produced with the aid of an external field. We confine ourselves for the time being to the case of high carrier density in the bands, when a significant redistribution of the spectrum occurs in a narrow energy layer at the Fermi surface. This case is the simplest from the mathematical point of view, being completely analogous formally to the case of superconductivity. To investigate the phenomena occurring in the tunnel structure, we use an approach based on the tunnel-Hamiltonian method[2]. Such approach was developed for the theory of the Josephson effect by Anderson[3] and by Ambeguokar and Baratoff[4].

In the tunnel-Hamiltonian method, the tunnel junction of two semiconductors is regarded as a weakly-coupled system described by a Hamiltonian that consists in the zeroth approximately of two parts corresponding to the isolated right-hand and left-hand semiconductors

$$H_0 = H_{r0} + H_{l0}, \tag{2}$$

and containing in the next higher approximation the term

$$H_W = \iint W_{lr}(\mathbf{rr}')\,(\psi_{1r}^+(\mathbf{r})\psi_{1l}(\mathbf{r}') + \psi_{2r}^+(\mathbf{r})\psi_{2l}(\mathbf{r}'))\,d\mathbf{r}\,d\mathbf{r}' + \text{h.c.}, \tag{3}$$

describing the tunnel transitions of the electrons from one semiconductor to the other. Here $\psi_{1r}^+$ and $\psi_{2r}$ are the electron creation and annihilation operators in the electron and hole bands of the right-hand semiconductor, while $\psi_{1l}^+$ and $\psi_{2l}$ are the corresponding operators of the left-hand semiconductor.

We assume for the semiconductors in the tunnel structure a simple model with isotropic dispersion laws in the electron and hole bands. We assume for simplicity that the extrema $E_1$ and $E_2$ of the electron and hole bands are located at the point $\mathbf{p} = 0$ of momentum space, and in addition the masses of the electrons and holes are equal. Then, say for the right-hand semiconductor, we can write

$$\begin{aligned} E_{1r}(\mathbf{p}) &= \mathbf{p}^2/2m + {}^1\!/_2 D_r = \mathbf{p}^2/2m - \mu_r + ({}^1\!/_2 D_r + \mu_r) = \varepsilon(\mathbf{p}) + V_r, \\ E_{2r}(\mathbf{p}) &= -\mathbf{p}^2/2m - {}^1\!/_2 D_r = -\mathbf{p}^2/2m + \mu_r - ({}^1\!/_2 D_r + \mu_r) \\ &= -\varepsilon(\mathbf{p}) - V_r, \end{aligned} \tag{4}$$

where $D_r$ is the width of the forbidden band, $\mu_r$ is the energy of the Fermi degeneracy of the electrons and holes of the right-hand semiconductor, and $\epsilon(\mathbf{p}) = \mathbf{p}^2/2m - \mu_r$.

In the case of high carrier density, $na_0^3 \gg 1$ (n is the density and $a_0$ is the Bohr radius of the exciton), the Coulomb interaction energy of the electrons and holes is much smaller than the Fermi energy. The existence of an electron-hole bound state under these conditions was already postulated in[1], but for a semi-metal with overlapping bands. This naturally raises the question whether the conclusions of[1] are applicable to the system considered by us. This question can be answered in the affirmative. The point is that the lifetime of the non-equilibrium carriers in semiconductors is usually long enough compared with the time required to establish thermodynamic equilibrium in each band separately[5,6], and also with the time during which the electron-hole pairing, accompanied by redistribution of the spectrum, takes place. In this case our system becomes completely equivalent to that considered in[1], and the appearance of terms with $V_r$ in the Hamiltonian does not lead to any physical effects (we are disregarding tunnel transitions for the time being), and their influence reduces to the appearance of trivial phase factors in the particle creation and annihilation operators and in the Green's functions[7].

Furthermore, at $na_0^3 \gg 1$, the Coulomb interaction between the electrons and the holes is strongly screened, as a result of which only the carriers in a narrow momentum-space layer near the Fermi surface take part in the production of electron-hole pairs. We shall therefore consider a model analogous to the BCS model in superconductivity theory, in which the interaction between the electrons and the holes is described by a certain effective interaction Hamiltonian

$$H_g = g\int \psi_{1r}^+(\mathbf{r})\psi_{2r}^+(\mathbf{r})\psi_{2r}(\mathbf{r})\psi_{1r}(\mathbf{r})\,d\mathbf{r}, \tag{5}$$

where

$$g = (2\pi e^2/\varepsilon_0 p_F^2)\ln(2p_F/\varkappa_D) > 0$$

is the interaction constant, and the energy cutoff occurs at a frequency $\omega_D \ll \omega_F$, $p_F$ is the Fermi momentum of the crystal, $\kappa_D^{-1}$ is the Debye screening radius, $\epsilon_0$ is the dielectric constant of the crystal, and $\psi_{1r}$ and $\psi_{2r}$ are the electron operators in the electron and hole bands.

Just as in[1], we disregard in the Hamiltonian the interaction of the carriers within a single band. As a result, the Hamiltonian of one of the semiconductors, say the right-hand one, is

$$H_{r0} = \int (\varepsilon(\hat{\mathbf{p}}) + V_r + U)\psi_{1r}^+(\mathbf{r})\psi_{1r}(\mathbf{r})\,d\mathbf{r} - \int (\varepsilon(\hat{\mathbf{p}}) + V_r - U)\psi_{2r}^+(\mathbf{r})\psi_{2r}(\mathbf{r})\,d\mathbf{r} + g\int \psi_{1r}^+(\mathbf{r})\psi_{2r}^+(\mathbf{r})\psi_{2r}(\mathbf{r})\psi_{1r}(\mathbf{r})\,d\mathbf{r}. \tag{6}$$

The letter U denotes here the applied potential difference (the contact potential difference between the semiconductors is assumed equal to zero). The expression for $H_{l0}$ is similar.

3. In the tunnel-Hamiltonian method, the current from one semiconductor to the other is determined from the rate of change in the number of particles in one of the semiconductors:

$$J(t) = e\langle \dot{N}(t)\rangle;\qquad N_r = \int \psi_{1r}^+(\mathbf{r})\psi_{1r}(\mathbf{r})\,d\mathbf{r} + \int \psi_{2r}^+(\mathbf{r})\psi_{2r}(\mathbf{r})\,d\mathbf{r}, \tag{7}$$

where $\dot{N}_r(t)$ is the operator of the rate of change of the particle number in the right-hand semiconductor, written in the Heisenberg representation. The averaging in (7) is carried out over the equilibrium canonical Gibbs ensemble with Hamiltonian $H_0 = H_{r0} + H_{l0}$, which conserves the number of electrons in each semiconductor. The procedure of deriving the formula for the tunnel current is well known (see, for example,[7]), although it is somewhat tiring, since it is necessary to operate with cumbersome expressions. We shall therefore not present all the details of the calculations, and write out only the final results.

The expression for the tunnel current is

$$J(t) = I_1 \sin 2Vt + I_2 \cos 2Vt + I_3, \tag{8}$$

where $I_1$, $I_2$, and $I_3$ are given by

$$I_1 = R^{-1}\left\{\int_{-\infty}^{+\infty} \operatorname{th}\frac{\omega}{2T}(\operatorname{Im} F_l^+(\omega)\operatorname{Re} F_r(\omega_{1-}) + \operatorname{Re} F_l^+(\omega_{2+})\operatorname{Im} F_r(\omega))\,d\omega - \int_{-\infty}^{+\infty} \operatorname{th}\frac{\omega}{2T}(\operatorname{Im} F_l(\omega)\operatorname{Re} F_r^+(\omega_{1+}) + \operatorname{Re} F_l(\omega_{2-})\operatorname{Im} F_r^+(\omega))\,d\omega\right\}, \tag{9}$$

$$I_2 = R^{-1}\left\{\int_{-\infty}^{+\infty}\left(\operatorname{th}\frac{\omega}{2T} - \operatorname{th}\frac{\omega_{2-}}{2T}\right)\operatorname{Im} F_r^+(\omega)\operatorname{Im} F_l(\omega_{2-})\,d\omega + \int_{-\infty}^{+\infty}\left(\operatorname{th}\frac{\omega}{2T} - \operatorname{th}\frac{\omega_{2+}}{2T}\right)\operatorname{Im} F_r^+(\omega)\operatorname{Im} F_l^+(\omega_{2+})\,d\omega\right\}, \tag{10}$$

$$I_3 = R^{-1}\left\{\int_{-\infty}^{+\infty}\left(\operatorname{th}\frac{\omega}{2T} - \operatorname{th}\frac{\omega_{1-}}{2T}\right)\operatorname{Im} G_{1l}^R(\omega)\operatorname{Im} G_{1r}^R(\omega_{1-})\,d\omega + \int_{-\infty}^{+\infty}\left(\operatorname{th}\frac{\omega}{2T} - \operatorname{th}\frac{\omega_{1+}}{2T}\right)\operatorname{Im} G_{2l}^R(\omega)\operatorname{Im} G_{2r}^R(\omega_{1+})\,d\omega\right\}. \tag{11}$$

Here

$$G^R(\omega) = \int G^R(\omega\xi)\frac{d\xi}{2\pi},\qquad F(\omega) = \int F(\omega\xi)\frac{d\xi}{2\pi}$$

are the Green's functions of the system, integrated over the energy, with $G_1^R$ and $G_2^R$ the ordinary Green's functions of the electrons in the electron and hole bands, respectively, the functions F and $F^+$ are analogous to the anomalous Gor'kov functions in superconductivity theory and correspond to the electron-hole pairing existing in the system. $R = \{4e|W_{rl}|^2(mp_F/2\pi^2)^2\}^{-1}$ is the resistance of the insulating layer between the normal semiconductors, $\omega_{1\pm} = \omega \pm V - U$ and $\omega_{2\pm} = \omega \pm V + U$.

The derived formulas (9)–(11) are valid for both pure semiconductors and for semiconductors with impurities. We consider first semiconductors without impurities. In this case

$$G_1^R(\omega) = G_2^R(\omega) = -\frac{\omega}{(\Delta^2 - \omega^2)^{1/2}},\qquad F(\omega) = -F^+(\omega) = \frac{\Delta}{(\Delta^2 - \omega^2)^{1/2}}. \tag{12}$$

Substituting these expressions in (9)—(11), we can obtain the explicit form of the integrals $I_1$, $I_2$, and $I_3$. We shall not write them out, however, in view of the complexity of the resultant expressions. We discuss only the final results.

We consider the case T = 0. Then at $|V - U| < \Delta_l + \Delta_r$ and $|V + U| < \Delta_l + \Delta_r$ the integrals $I_2$ and $I_3$ are equal to zero, and the integral $I_1$ is equal to

$$I_1 = R^{-1} 2\Delta_l \Delta_r \left( \frac{K(x_{1-})}{[(\Delta_l + \Delta_r)^2 - (U - V)^2]^{1/2}} - \frac{K(x_{1+})}{[(\Delta_l + \Delta_r)^2 - (U + V)^2]^{1/2}} \right)$$

at $\quad |V - U| < |\Delta_l - \Delta_r|, \quad |V + U| < |\Delta_l - \Delta_r|,$

$$I_1 = R^{-1} \left\{ \frac{2\Delta_l \Delta_r K(x_{1-})}{[(\Delta_l + \Delta_r)^2 - (U - V)^2]^{1/2}} - (\Delta_l \Delta_r)^{1/2} K(x_{2+}) \right\}$$

at $\quad |V - U| < |\Delta_l - \Delta_r|, \quad |V + U| > |\Delta_l - \Delta_r|;$ и, (13)

and finally

$$I_1 = R^{-1} (\Delta_l \Delta_r)^{1/2} \{ K(x_{2-}) - K(x_{2+}) \}$$

at $\quad |U - V| > |\Delta_l - \Delta_r|, \quad |U + V| > |\Delta_l - \Delta_r|,$

where K(x) is a complete elliptic integral of the first kind,

$$x_{1\pm} = \left[ \frac{(\Delta_l - \Delta_r)^2 - (U \pm V)^2}{(\Delta_l - \Delta_r)^2 + (U \pm V)^2} \right]^{1/2}, \quad x_{2\pm} = \left[ \frac{(U \pm V)^2 - (\Delta_l - \Delta_r)^2}{4\Delta_l \Delta_r} \right]^{1/2}.$$

At

$$\max(|U - V|, \; |U + V|) = \Delta_l + \Delta_r$$

a normal current appears jumpwise

$$I_3(\Delta_l + \Delta_r) = -I_2(\Delta_l + \Delta_r) = {}^1/_2 R^{-1} \pi (\Delta_l \Delta_r)^{1/2}, \tag{14}$$

and the integral $I_1$ has a singularity. The singular part of the integral $I_1$ is

$$I_1 = R^{-1} \frac{\sqrt{\Delta_l \Delta_r}}{4} \ln \left( \frac{\Delta_l + \Delta_r}{|\beta - (\Delta_l + \Delta_r)|} \right), \tag{15}$$

where

$$\beta = \max(|U - V|, \quad |U + V|).$$

In the case U = V, $U + V \gg \Delta_l + \Delta_r$, we have

$$I_1 = R^{-1} \left[ \frac{2\Delta_l \Delta_r}{\Delta_l + \Delta_r} K\left( \frac{|\Delta_l - \Delta_r|}{\Delta_l + \Delta_r} \right) - \frac{\pi \Delta_l \Delta_r}{2U} \right], \tag{16}$$

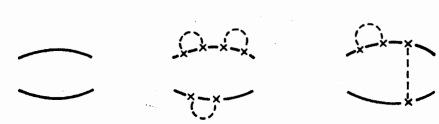
$$I_2 = -R^{-1} \frac{\Delta_l \Delta_r}{U} \ln \frac{2U}{(\Delta_l \Delta_r)^{1/2}}, \tag{17}$$

$$I_3 = R^{-1} \left( 2U - \frac{\Delta_l^2 + \Delta_r^2}{4U} \right). \tag{18}$$

We note that the tunnel current is equal to zero at U = 0.

We see thus that when a potential difference is applied to a semiconductor-insulator-semiconductor tunnel structure, in which a large concentration of nonequilibrium carriers is produced by an external field source, an oscillating electric current begins to flow through the structure. The magnitude of this current is described by formulas (8)—(10). At low voltages, only the integral $I_1$ in (8) differs from zero. When the applied voltage is increased, a singularity appears in the current at the point $\max(|U - V|, |U + V|) = \Delta_l + \Delta_r$; this singularity is connected with the fact that breaking of the electron-hole pair by the field and the penetration of one electron or one hole through the barrier become possible. At this point, a normal current appears jumpwise. In very strong fields, when $U \gg V, \Delta_l + \Delta_r$, the integrals $I_1$ and $I_2$ decrease and tend to zero, while the integral $I_3$ is proportional to U, corresponding to the usual Ohm's law. An unusual fact in our problem is that at U = V and $U \gg \Delta_l + \Delta_r$ we always have an oscillating component of the current through the junction. Thus, by appropriate choice of the semiconductors for the tunnel structure, we can obtain an oscillating current by applying strong fields to the structure. We note that the ratio of the amplitude of the oscillating current to the amplitude of the normal current is in this case of the order of $\Delta/U$ and not $(\Delta/U)^2$ as in the Josephson effect.

We note finally that the derived formulas (8)—(11) can be used also for semiconductors with charged impurities. Let us call attention to the singularity encountered when the voltage for the tunnel current (8) is averaged over the positions of the impurities. Usually in the calculation of the current in homogeneous crystals[8] one takes into account diagrams of the type



a b c

The solid lines correspond here to Green's functions, the crosses to impurity potentials, and the dashed lines joining two crosses mean that scattering occurs twice on the same impurity. An examination of formulas (8)—(11) for the tunnel current shows that the Green's functions in the integrands belong to different semiconductors, and therefore the diagrams of type (c) are of higher order of smallness with respect to the tunnel matrix element. Thus, in the calculation of the tunnel current we need not take into account the ladder diagrams of the type (c), which make an appreciable contribution in a homogeneous crystal. On the other hand, the result of taking the diagrams of type (a) and (b) into account is that the functions (12) are more complicated and can be written in parametric form

$$G^R(\omega) = -\frac{u}{(1 - u^2)^{1/2}}, \quad F(\omega) = \frac{1}{(1 - u^2)^{1/2}},$$
$$\omega/\Delta = u[1 - 1/\tau\Delta(1 - u^2)^{1/2}], \tag{19}$$

where $\tau$ is the free-path time.

Substitution of the expressions for the Green's functions (19) in the integrals $I_1$, $I_2$, and $I_3$, we can show that the results do not differ qualitatively from the impurity-free case at an impurity density not close to the critical value at which the gap in the spectrum vanishes.

**4.** We emphasize that the effect considered by us consists in the flow of an oscillating electric current through a tunnel structure and differs, generally speaking, from the Josephson effect, which in the case of electron-hole pairing should be represented as coherent tunneling of electron-hole pairs which transfer energy in the non-equilibrium system under consideration. The electric current should naturally be equal to zero in this case, since the charge of the pair is zero. The derivation of the formulas describing the coherent tunneling of electron-hole pairs entails difficulty. As a result, we obtain for the quantity $J'(t)$, which we define as the rate of change of the number of electrons in the upper band and of the holes in the lower band as a result of tunneling from semiconductor to semiconductor, the formula

$$J'(t) = I_1' \sin 2Vt + I_2' \cos 2Vt + I_3', \tag{20}$$

$$I_1' = R^{-1}\left\{\int_{-\infty}^{+\infty} \operatorname{th}\frac{\omega}{2T}(\operatorname{Im} F_l^+(\omega)\operatorname{Re} F_r(\omega_{1-}) + \operatorname{Im} F_r(\omega)\operatorname{Re} F_l^+(\omega_{2+}))\,d\omega + \int_{-\infty}^{+\infty}\operatorname{th}\frac{\omega}{2T}(\operatorname{Im} F_l(\omega)\operatorname{Re} F_r^+(\omega_{1+}) + \operatorname{Re} F_l(\omega_{2-})\operatorname{Im} F_r^+(\omega))\,d\omega\right\}, \tag{21}$$

$$I_2' = R^{-1}\left\{\int_{-\infty}^{+\infty}\left(\operatorname{th}\frac{\omega}{2T} - \operatorname{th}\frac{\omega_{2-}}{2T}\right)\operatorname{Im} F_r^+(\omega)\operatorname{Im} F_l(\omega_{2-})\,d\omega - \int_{-\infty}^{+\infty}\left(\operatorname{th}\frac{\omega}{2T} - \operatorname{th}\frac{\omega_{1+}}{2T}\right)\operatorname{Im} F_r(\omega)\operatorname{Im} F_l^+(\omega_{2+})\,d\omega\right\}, \tag{22}$$

$$I_3' = R^{-1}\left\{\int_{-\infty}^{+\infty}\left(\operatorname{th}\frac{\omega}{2T} - \operatorname{th}\frac{\omega_{1-}}{2T}\right)\operatorname{Im} G_{1l}^R(\omega)\operatorname{Im} G_{1r}^R(\omega_{1-})\,d\omega - \int_{-\infty}^{+\infty}\left(\operatorname{th}\frac{\omega}{2T} - \operatorname{th}\frac{\omega_{1+}}{2T}\right)\operatorname{Im} G_{2l}^R(\omega)\operatorname{Im} G_{2r}^R(\omega_{1+})\,d\omega\right\}. \tag{23}$$

Substituting further in (21)—(23) the expressions for the Green's function from (12), we can easily obtain explicit expressions for the integrals $I_1'$, $I_2'$, and $I_3'$, which are similar in form to the formulas for $I_1$, $I_2$ and $I_3$, and differ only in the plus and minus signs in the corresponding places. We therefore do not write out the formulas for $I_1'$, $I_2'$, and $I_3'$, and discuss only the results of the calculations.

At T = 0 and $|U - V|$, $|U + V| < |\Delta_l - \Delta_r|$, the integrals $I_2'$ and $I_3'$ are equal to zero. The only nonzero integral is

$$I_1' = R^{-1} 2\Delta_l\Delta_r\left(\frac{K(x_{1-})}{[(\Delta_l + \Delta_r)^2 - (V - U)^2]^{1/2}} + \frac{K(x_{1+})}{[(\Delta_l + \Delta_r)^2 - (V + U)^2]^{1/2}}\right). \tag{24}$$

We note that, unlike (13), the integral $I_1'$ differs from zero at U = 0 (we recall that U is the external field applied to the structure). Further, as seen from (24), $I_1'$ is not equal to zero in the case when U = 0 and V = 0, i.e., we arrive at the conclusion that the static Josephson effect for electron-hole pairs exists in the system under consideration. Indeed, an accurate allowance for the initial phases of the anomalous Green's function yields the following result:

$$J'(t) = 2R^{-1}\frac{\Delta_l\Delta_r}{\Delta_l + \Delta_r} K\left(\frac{|\Delta_l - \Delta_r|}{\Delta_l + \Delta_r}\right)\sin(\varphi_r - \varphi_l). \tag{25}$$

When the parameters U and V increase a normal "current" of electrons and holes from semiconductor to semiconductor appears jumpwise at the points $\max(|U - V|, |U + V|) = \Delta_l + \Delta_r$. In the limiting case when $(|U - V|, |U + V|) \gg \Delta_l + \Delta_r$ we obtain

$$I_1' = R^{-1}\pi\Delta_l\Delta_r\left(\frac{1}{U + V} + \frac{1}{U - V}\right), \tag{26}$$

$$I_2' = -2R^{-1}\Delta_l\Delta_r\left(\frac{1}{U + V}\ln\frac{|U + V|}{(\Delta_l\Delta_r)^{1/2}} - \frac{1}{U - V}\ln\frac{|U - V|}{(\Delta_l\Delta_r)^{1/2}}\right), \tag{27}$$

$$I_3' = R^{-1}\left\{(U + V)\left(1 - \frac{\Delta_l^2 + \Delta_r^2}{2(U + V)^2}\right) - (U - V)\left(1 - \frac{\Delta_l^2 + \Delta_r^2}{2(U - V)^2}\right)\right\}. \tag{28}$$

The obtained formulas (20)—(23) are valid also for semiconductors with charged impurities. In this case, everything said above in the preceding section concerning averaging over impurities in the expressions for J(t) remains in force. As a result, at an impurity concentration not close to the critical value at which the gap in the spectrum disappears, the results do not differ qualitatively from the impurity-free case.

**5.** Thus, when considering quantum phenomena connected with the coherence of the phase of the wave function in semiconductors with electron-hole pairing, we encounter a unique situation wherein the tunnel current from semiconductor to semiconductor depends on two parameters, namely V, determined by the rate of carrier generation and by the difference between their lifetimes in the semiconductors making up the tunnel structure, and U, which is the external voltage applied to the structure. At different ratios of U and V, oscillating electric current with frequency $\omega = 2V$ can flow through a tunnel junction of two semiconductors with electron-hole pairing.

The result can be interpreted as follows. First, the absence of a dc component of the current at small biases is a symptom of dielectric pairing. As to the oscillating component, it is useful to recall here the situation that arises in the Josephson effect. As is well known, the cause of the Josephson current is the nonconservation of the number of particles in the superconducting state (the deviation from zero of the anomalous mean values $\langle\psi_1^+\psi_1^+\rangle$, the phase difference of which on the left and on the right determines the oscillating frequency). In our case of electron-hole pairing from different bands, the total number of particles on the left and on the right is conserved individually (without allowance for tunneling), but the number of particles in each individual band is not conserved, since the anomalous mean values $\langle\psi_1^+\psi_2^+\rangle$ differ from zero. A tunnel structure with a constant bias is asymmetrical for electrons and holes, and is therefore sensitive to nonconservation of the number of particles in each band. On the other hand, the oscillation frequency is determined by the phase difference between the anomalous mean values $\langle\psi_1^+\psi_2\rangle$ on the left and on the right this quantity is V and not U as in the case of superconducting pairing. The value of U, on the other hand, determines the magnitude of the oscillating current, since the degree of asymmetry of the tunnel junction for electrons and holes depends on U.

The result is a nonzero oscillating current described by formulas (8)—(18). In the presence of interaction with a radiation field, the oscillations of the electric current can be accompanied by emission of real photons with frequency $\omega = 2V$, which can be lower by several orders of magnitude than the frequency of the interband recombination radiation. We note that registration of such a radiation from a tunnel structure may turn out to be the most convenient method of observing the discussed effect experimentally. The radiation line width will be determined by the reciprocal lifetime of the non-equilibrium carriers and at the lifetimes $10^{-3}$—$10^{-6}$ sec which are typical for certain semiconductors it can be very small compared with the radiation frequency, which can be of the order of $10^{12}$—$10^{14}$ $sec^{-1}$.

Although the final expressions for the current were obtained for the case $na_0^3 \gg 1$, these results remain in force qualitatively also for an arbitrary carrier density, provided the conditions for the condensation of the electron-hole pairs are satisfied.

Thus, the effects connected with the coherence of the phase of the wave function in semiconductors with electron-hole pairing have a number of interesting properties, the investigation of which may prove useful in the study of the nature of the exciton phase in non-equilibrium semiconductors.

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