Theory of Exciton-Phonon Complexes in a Strong Magnetic Field

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We construct a theory of a resonance interaction between excitons and optical phonons in a strong magnetic field for the case when the cyclotron frequencies are appreciably higher than the Coulomb energy I. The electron-phonon coupling constant α is assumed to be small and we consider changes in the spectrum which occur close to a resonance between the electron cyclotron frequency ω_e and the phonon frequency ω_0. Such an interaction has already been considered by several authors and the special feature of the present paper consists in a study of the influence of the Coulomb interaction on the magneto-phonon resonance. We apply the general formalism developed here to the single-particle branch of the spectrum; this corresponds for $\omega_e < \omega_0$ to an exciton formed near the electron magnetic band with n = 1. Near $\omega_e = \omega_0$ there is always a limit point T due to the Coulomb interaction. Near T the single-particle branch corresponds to a three-particle complex: an electron (in the n=0 band) + a hole + a phonon, which at the point T decays into an exciton and a phonon. The pinning pattern which arises was studied in detail in two limiting cases depending on the ratio of I and the electron-phonon interaction. We show that the Coulomb interaction significantly changes the pinning pattern even when it is small. We consider the absorption connected with the presence of (equilibrium or non-equilibrium) lattice phonons and accompanied by the formation of exciton-phonon complexes; we show that its intensity should be large.

INTRODUCTION

RECENTLY data have been published about the existence of different quasi-particle complexes in crystals. After some of the bands in the spectra of molecular crystals[1] and of semiconductors[2] had been interpreted in terms of exciton-phonon complexes (ExPC) a number of papers dealt with this problem.

At the same time the magneto-optic anomaly in interband absorption[3] was interpreted as the rearrangement of the electron energy spectrum when there is a resonance coupling with optical phonons; a more rigorous theory was developed in [4,5]. It was noted in [6] that this rearrangement was connected with the formation of electron-phonon complexes (ElPC). The ElPC spectrum has been obtained for the case of adiabatic coupling[6] and for the case of weak coupling in a magnetic field.[7]

In what follows we construct a theory of exciton spectra under conditions of magneto-phonon resonance (MPR) for the case when the electron cyclotron frequency ω_e is close to the phonon frequency ω_0. We emphasize that it is necessary to take the Coulomb interaction consistently into account when describing any structural interband absorption. The magnetic field H, first of all, allows us to establish the resonance conditions under which the exciton-phonon interaction is sharply enhanced and, secondly, generates a large parameter, the ratio of ω_e to the Coulomb binding energy I of the exciton. Because of that it is possible to construct an analytical theory.

We develop a general formalism (Secs. 1 and 2) and give a classification of diagrams, showing that (i) the problem can be reduced to the solution of an effective Schrödinger equation (Sec. 3) and (ii) one can restrict oneself to single-phonon diagrams with all possible Coulomb insertions. There is an important difference between the conditions for the formation of ElPC and of ExPC. This is connected with the fact that a diamagnetic exciton in contrast to a Landau electron has a finite transverse mass. This weakens the resonance singularity and leads to the consequence that ExPC can exist only for a narrow range of H. The final formulae are obtained in two limiting cases corresponding to different values of the ratio of I and the energy of the resonance electron-phonon interaction (Secs. 5 and 6). We consider the optical absorption corresponding to the formation of ExPC when thermal or non-equilibrium phonons are involved (Sec. 7).

1. GENERAL FORMULA FOR ABSORPTION

The conductivity tensor $\sigma_{\mu\nu}(\mathbf{K}, \Omega)$ can in a standard way be expressed in terms of current correlators. We restrict ourselves to simple electron and hold bands with masses m_e and m_h. The current operator can be expressed in terms of its interband matrix element j_{cv}:

$$\hat{j}_\mu(\mathbf{K}) = \sum_{\alpha,\alpha'} \langle \alpha | e^{-i\mathbf{K}\mathbf{r}} | \alpha' \rangle \{ j_{\mu vc}\, a_{v\alpha}^{+}\, a_{c\alpha'} + j_{\mu cv}\, a_{c\alpha}^{+}\, a_{v\alpha'} \}. \quad (1)$$

We choose as base of the smoothed functions ψ_α the Landau functions in the gauge $\mathbf{A}(\mathbf{r}) = \frac{1}{2}(\mathbf{H}\cdot\mathbf{r})$[8]

$$\psi_\alpha(\mathbf{r}) = \frac{e^{ipz}}{\sqrt{L_z}} \frac{1}{\sqrt{L_y}} \exp\left\{ \frac{iy(x+2k\lambda^2)}{2\lambda^2} \right\} \frac{1}{\lambda} \psi_n\left(\frac{x+2\lambda^2 k}{\lambda} \right), \quad \alpha = (n,k,p), \quad (2)$$

where $\psi_n(x)$ are oscillator functions and λ a magnetic length. They are also eigenfunctions of the operator $\hat{x}_0 = x - \lambda^2 \hat{\pi}_y$, the x-component of the center of the electron orbit; $\hat{\pi} = \hat{p} + e\mathbf{A}$. We change to the electron and hole functions and operators representation:

$$\psi_{nkp}^{e} = \psi_{nkp}, \quad \psi_{nkp}^{h} = \psi_{n,-k,-p}^{*}, \quad a(nkp) = a_{c,nkp}, \quad b(nkp) = a_{v,n,-k,-p}^{+}. \quad (3)$$

We also use the identity for the transverse parts of the functions (3) which is an expansion in terms of states with a well-determined momentum $\mathbf{K}_\perp$:

$$\psi_{n_1 k_1}^{e}(\mathbf{r}_{1\perp})\, \psi_{n_2 k_2}^{h}(\mathbf{r}_{2\perp}) = \frac{\lambda\sqrt{2\pi}}{L_x L_y} \sum_{K_x} e^{-i\lambda^2 k_2 K_x} e^{i[\mathbf{K}_\perp + e\mathbf{A}(\mathbf{r})]\mathbf{R}_\perp} \Phi_{n_1 n_2}^{\mathbf{K}_\perp}(\mathbf{r}_\perp), \quad (4)$$

where $\mathbf{r}$ and $\mathbf{R}$ are the relative coordinate and the center of mass coordinate,

$$\Phi^{\mathbf{K}_\perp}_{n_1 n_2}(\mathbf{r}_\perp) = \frac{1}{\lambda} J_{m, n_\rho}\left(\frac{\mathbf{r}_\perp - \mathbf{r}_{\mathbf{K}_\perp}}{\lambda}\right) \exp\left(-i\frac{m_e - m_h}{m_e + m_h}\mathbf{K}_\perp \mathbf{r}_\perp\right), \quad (5)$$

$$J_{m, n_\rho}(\zeta_\perp) = i^m (-1)^{(|m|-m)/2} \frac{e^{im\varphi}}{\sqrt{2\pi}} e^{-\zeta_\perp^2/4}\left(\frac{\zeta_\perp^2}{2}\right)^{|m|/2} L^{|m|}_{n_\rho}\left(\frac{\zeta_\perp^2}{2}\right)\sqrt{\frac{n_\rho!}{(n_\rho + |m|)!}}, \quad (6)$$

Here $m = n_1 - n_2$, $n_\rho = (n_1 + n_2 - |n_1 - n_2|)/2$, $\mathbf{r}_{\mathbf{K}\perp} = \lambda^2 \mathbf{H} \times \mathbf{K}/H$, $L^{|m|}_{n_\rho}$ is a Laguerre polynomial.

If we use the conservation of momentum, we can write the two-particle Green function in the "momentum" representation,

$$G(\alpha_1, t_1, \alpha_2 t_2 | \alpha_1' t_1', \alpha_2' t_2') = \int d r_{121'2'} \psi_{\alpha_1}^{*e}(1)\, \psi_{\alpha_2}^{*h}(2)\, G(12|1'2')\, \psi_{\alpha_1'}^{e}(1')\, \psi_{\alpha_2'}^{h}(2') \quad (7)$$

in the form

$$G(\alpha_1\omega_1, \alpha_2\omega_2 | \alpha_1'\omega_1', \alpha_2'\omega_2') = \langle k_2 p_2 \omega_2 | G^{n_1 n_2}_{n_1' n_2'}(K_y K_z \Omega) | k_2' p_2' \omega_2' \rangle \cdot \delta(\omega_1 + \omega_2 - \omega_1' - \omega_2')\Delta(k_1 + k_2 - k_1' - k_2')\Delta(p_1 + p_2 - p_1' - p_2'),$$
$$K_y = k_1 + k_2, \quad K_z = p_1 + p_2, \quad \Omega = \omega_1 + \omega_2. \quad (8)$$

We classify the exciton states in a magnetic field according to their total momentum $\mathbf{K}$. It is therefore convenient to change from G to the functions $\mathcal{G}$ using the transformation:

$$\langle p_2\omega_2 | \mathcal{G}^{n_1 n_2}_{n_1' n_2'}(\mathbf{K}\Omega) | p_2'\omega_2' \rangle = \frac{2\pi\lambda^2}{L_x L_y} \sum_{k_2 k_2'} e^{-K_x(k_2 - k_2')\lambda^2} \langle k_2 p_2 \omega_2 | G^{n_1 n_2}_{n_1' n_2'}(K_y K_z \Omega) | k_2' p_2' \omega_2' \rangle \quad (9)$$

and to introduce the function

$$\langle p_2 | \mathcal{F}^{n_1 n_2}_{n_1' n_2'}(\mathbf{K}\Omega) | p_2' \rangle = \int \frac{d\omega_2}{2\pi}\frac{d\omega_2'}{2\pi} \langle p_2\omega_2 | \mathcal{G}^{n_1 n_2}_{n_1' n_2'}(\mathbf{K}\Omega) | p_2'\omega_2' \rangle. \quad (10)$$

In this notation $\sigma_{\mu\nu}$ has the form

$$\sigma_{\mu\nu}(\mathbf{K}\Omega) = -\frac{1}{V\Omega} j_{\mu cv} j_{\nu cv} \sum_{\substack{n_1 n_2 n_1' n_2' \\ p_2 p_2'}} [\sqrt{L_x L_y}\, \Phi^{\mathbf{K}_\perp}_{n_1 n_2}(0)] \cdot \langle p_2 | \mathcal{F}^{n_1 n_2}_{n_1' n_2'}(\mathbf{K}\Omega) | p_2' \rangle [\sqrt{L_x L_y}\, \Phi^{\mathbf{K}_\perp}_{n_1' n_2'}(0)]^*. \quad (11)$$

2. PROPERTIES OF THE TWO-PARTICLE GREEN FUNCTION. DIAGRAM TECHNIQUE

We consider in some detail the properties of G. If we use, as we did above, as the basis the general eigenfunctions of $\hat{x}_0$ and $\hat{\pi}_\perp^2$ which are independent of the gauge of the vector potential $\mathbf{A}$, then it is clear that G in the "momentum" representation is a gauge invariant.[1)] Indeed, the phase factors arising under a gauge transformation in the products of field operators in $G(12|1'2')$ and in the functions ψ_α in (7) are each other's complex conjugate and therefore cancel one another.

We show that G depends only on the difference $k_2 - k_2'$. To do this we perform the gauge transformation $\mathbf{A} \to \mathbf{A} + \nabla f$, $\psi \to \psi \exp(-ief)$ with the function

$$f = {}^1/_2[\mathbf{HB}]\mathbf{r}, \qquad \mathbf{B} \perp \mathbf{H}, \quad (12)^*$$

so that

$$G(12|1'2') \to G(12|1'2')\exp\{-ief(\mathbf{r}_1 - \mathbf{r}_2) + ief(\mathbf{r}_1' - \mathbf{r}_2')\}.$$

Noting that the transformation (12) is equivalent to the shift $\mathbf{r} \to \mathbf{r} + \mathbf{B}$ and changing from the coordinates $\mathbf{r}_1$, $\mathbf{r}_2$ to $\mathbf{R}$, $\mathbf{r}$, we get

$$G(\mathbf{R} + \mathbf{B}, \mathbf{r} | \mathbf{R}' + \mathbf{B}, \mathbf{r}') = G(\mathbf{R}, \mathbf{r} | \mathbf{R}', \mathbf{r}')\exp[ie\mathbf{BA}(\mathbf{r} - \mathbf{r}')]. \quad (13)$$

As (13) determines the way G transforms under translations, we have

$$G(\mathbf{R}, \mathbf{r} | \mathbf{R}', \mathbf{r}') = \bar{G}(\mathbf{R} - \mathbf{R}'; \mathbf{r}, \mathbf{r}')\exp\left[ie\frac{\mathbf{R} + \mathbf{R}'}{2}\mathbf{A}(\mathbf{r} - \mathbf{r}')\right]. \quad (14)$$

We use (4), (5), and (14) to transform (7) and change from $\mathbf{R}$, $\mathbf{R}'$ to $\mathfrak{R} = (\mathbf{R} + \mathbf{R}')/2$ and $\mathfrak{r} = \mathbf{R} - \mathbf{R}'$. If we also split off the factor depending on $\mathfrak{R}$ we verify easily that in it all terms containing the vector potential are cancelled. The subsequent integration over $\mathfrak{R}$ therefore produces a factor $\Delta(\mathbf{K} - \mathbf{K}')$ guaranteeing momentum conservation. As a result (7) is reduced to the form

$$\sum_{\mathbf{K}_x \mathbf{K}_x'} \exp\{i\lambda^2 k_2 K_x\}\exp\{-i\lambda^2 k_2' K_x'\}\Delta(\mathbf{K} - \mathbf{K}')\, Y_{\mathbf{K}\mathbf{K}'},$$

where $Y_{\mathbf{K}\mathbf{K}'}$ depends on all arguments bar k_2 and k_2'. Hence it follows that $\langle k_2 p_2 \omega_2 | G^{n_1 n_2}_{n_1' n_2'}(K_y K_z \Omega) | k_2' p_2' \omega_2' \rangle$ in (8) depends on k_2 and k_2' only through the difference $k_2 - k_2'$. For the case when there are no phonons this was noted in [10].

We emphasize that this property of the y-components of the momenta is a consequence of the conservation of the x-component of the total momentum. This is natural as k_2, apart from a coefficient is an eigenvalue of $\hat{x}_0$.

As G depends only on $k_2 - k_2'$ it follows that the transformation (9) is reciprocally unique. It allows us to get rid of the quantum number k_2 which is connected with the actual choice of wavefunctions when there is degeneracy and to change to the total exciton momentum representation.

We shall everywhere assume that the interaction between the carriers and the dispersionless polarization phonons is weak and in the perturbation theory series for the Green function we shall retain only resonance terms which are large when ω_e is close to ω_0. Assuming that there is no resonance between ω_0 and the hole cyclotron frequency we shall neglect the hole-phonon interaction.

The usual diagram technique is valid for G. It is, however, more convenient to consider diagrams for $\mathcal{G}$ the more so as the conductivity can, according to (10) and (11), be expressed directly in terms of it.

Electron, hole, and phonon lines correspond to the usual Green functions $G_e^0(n_1 p_1 \omega_1)$, $G_h^0(n_2 p_2 \omega_2)$, and $iD(\mathbf{q}, \omega)$. The renormalized electron function G_e, as well as G_e^0, is diagonal in n_1 and independent of k_2.[11] The vertex of Fig. 1a corresponds to

$$\gamma_{n_1 n_2}(\mathbf{q}) = \left(\frac{4\pi\alpha\omega_0^2}{q^2 V\sqrt{2m_e\omega_0}}\right)^{1/2} \sqrt{2\pi} J_{m n_\rho}(\lambda q_\perp), \quad (15)$$

and the vertex of Fig. 1b to $\gamma^*_{n_2 n_1}(\mathbf{q})$. Here α is the electron-phonon coupling constant and the numbers m, n_ρ are connected with n_1, n_2 through Eq. (6). When changing from G to $\mathcal{G}$ there appear extra phase factors in the vertices. Since, however, these factors arising

[1)] This property is, of course, not connected with the actual choice of operators in terms of which the eigenfunctions are classified.

*$[\mathbf{HB}] \equiv \mathbf{H} \times \mathbf{B}$.

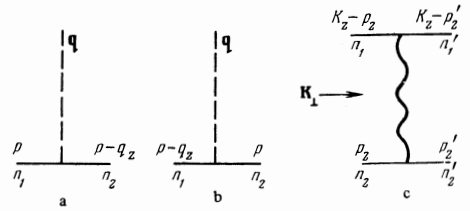

FIG. 1

from the ends of each phonon line cancel one another in pairs, we drop them. The Coulomb line (Fig. 1c) corresponds to

$$iu^{n_1n_2}_{K_\perp n_1'n_2'}(p_2-p_2') = \frac{-i}{L_z}\int dz\,\exp\{i(p_2-p_2')z\} \times \left\langle \Phi^{K_\perp}_{n_1n_2}(\mathbf{r}_\perp)\left|\frac{e^2}{\varepsilon_\infty\sqrt{z^2+r_\perp^2}}\right|\Phi^{K_\perp}_{n_1'n_2'}(\mathbf{r}_\perp)\right\rangle. \quad (16)$$

It is clear from (16) that the contribution from the Coulomb line depends on the total incoming momentum $\mathbf{K}_\perp$. There is a summation over all internal momenta and an integration over frequencies with weight $d\omega/2\pi$.

3. BETHE-SALPETER EQUATION

The Bethe-Salpeter equation for $\mathcal{G}$ is given in Fig. 2b while the connection between $\mathcal{G}$ and the complete vertex $\mathcal{K}$ is determined by Fig. 2a. Subsequent terms in the perturbation-theory series for the irreducible vertex $\mathcal{T}$ are given in Fig. 3. By analogy with (9) $\mathcal{T}$ is considered as function of Ω and the hole frequencies.

Consideration of the perturbation theory series shows that the vertices $\langle\omega_2'|\mathcal{T}|\omega_2''\rangle$ and $\langle\omega_2'|\mathcal{K}|\omega_2''\rangle$ have no singularities in the lower half-plane, neither in ω_2' nor in ω_2''; the condition for this is the absence of hole-phonon interactions.

In the last term of the equation of Fig. 2b the only factor which has a singularity in ω_2'' in the lower half-plane is thus $G^0_h(n_2''p_2''\omega_2'')$. The integration over ω_2'' is thus performed by closing the contour in the lower half-plane and evaluating the residue at the pole of G^0_h. As a result we get the values of $\mathcal{T}$ and $\mathcal{K}$ at the point $\omega_2'' = \epsilon_h(n_2''p_2'')$.

Starting from the equation of Fig. 2a we can verify that $\mathcal{T}$ can also be expressed in terms of the value of $\mathcal{K}$ on the mass surface. This enables us by integrating the equation of Fig. 2b over the external hole frequencies to change it into an equation for $\mathcal{T}$:

$$\langle p_2|\mathcal{T}^{n_1n_2}_{n_1'n_2'}(\mathbf{K}\Omega)|p_2'\rangle = -iB^{-1}_{n_1n_2}(K_zp_2\Omega)\left\{\Delta(p_2-p_2')\Delta(n_1-n_1') \times \Delta(n_2-n_2') + \sum_{p_2''n_1''n_2''}\langle p_2\varepsilon_h(n_2p_2)|\mathcal{T}^{n_1n_2}_{n_1''n_2''}(\mathbf{K}\Omega)|p_2''\varepsilon_h(n_2''p_2'')\rangle \times\langle p_2''|\mathcal{T}^{n_1''n_2''}_{n_1'n_2'}(\mathbf{K}\Omega)|p_2'\rangle\right\}, \quad (17)$$

where

$$B_{n_1n_2}(K_zp_2\Omega) = G_e^{-1}(n_1, K_z-p_2, \Omega-\omega_2)|_{\omega_2=\varepsilon_h(n_2p_2)} = B^0_{n_1n_2}(K_zp_2\Omega) - \Sigma_e(n_1, K_z-p_2, \Omega-\varepsilon_h(n_2p_2)),$$
$$B^0_{n_1n_2}(K_zp_2\Omega) = \Omega-\varepsilon_e(n_1, K_z-p_2)-\varepsilon_h(n_2p_2)-\varepsilon_g,$$
$$\varepsilon_e(np) = \omega_e(n+1/2)+p^2/2m_e, \qquad \varepsilon_h(np) = \omega_h(n+1/2)+p^2/2m_h, \quad (18)$$

Σ_e is the electron self-energy part and ϵ_g the width of the forbidden band.

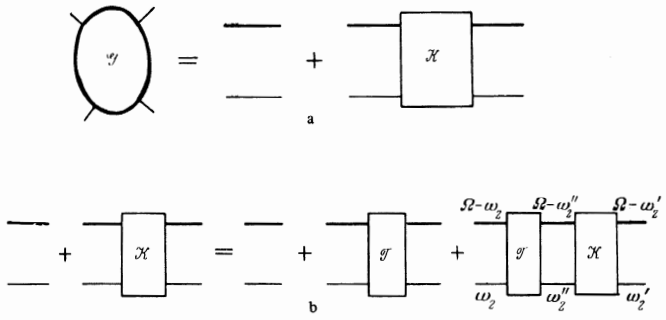

FIG. 2

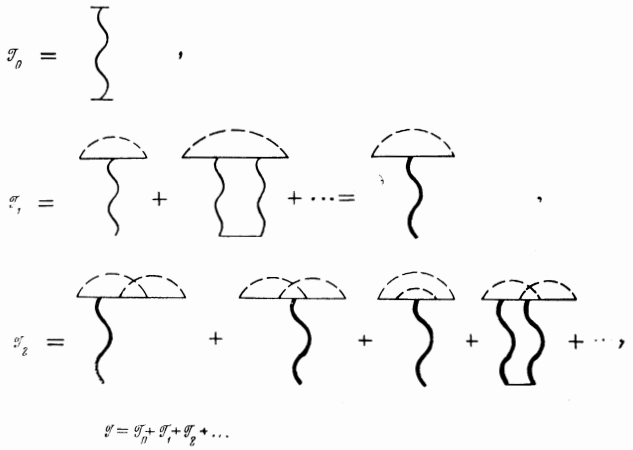

FIG. 3

We have thus been able to change from the usual Bethe-Salpeter equation for $\mathcal{G}$ containing three frequencies (Ω, ω_2, and ω_2') to an equation for $\mathcal{T}$ depending on Ω only.

For small α we can limit ourselves to the terms $\mathcal{T}_0$ and $\mathcal{T}_1$ in the series for $\mathcal{T}$ (Fig. 3) and to the single-phonon contribution to Σ_e. The Coulomb binding energy I in the exciton we shall everywhere in what follows assume to be small compared to ω_e; when $I \ll \omega_e$ the Coulomb interaction weakly destroys the MPR. We put $n_1 = n_2 = 1$ in the external lines, which corresponds to the lowest allowed transition for which MPR occurs. We put $n_1 = 0$ in the internal electron lines in $\mathcal{T}_1$, i.e., we retain only terms with small resonance denominators.

The thick line corresponding to an unretarded Coulomb interaction is independent of the hole frequencies ω_2 and ω_2'. It is connected with the Coulomb Green function $\mathcal{G}_C$ through the equation of Fig. 4.

In it we change from $\mathcal{G}_C$ to $\mathcal{F}_C$ by analogy to (10) and we introduce the function

$$\langle p|\overline{\mathcal{F}}_c(\mathbf{K}\Omega)|p'\rangle = i\left\langle p+\frac{m_h}{M}K_z|\mathcal{F}_c(\mathbf{K}\Omega)|p'+\frac{m_h}{M}K_z\right\rangle; \quad (19)$$

we define $\overline{\mathcal{T}}$ similarly. This transformation corresponds to a transition to the exciton center of mass system.

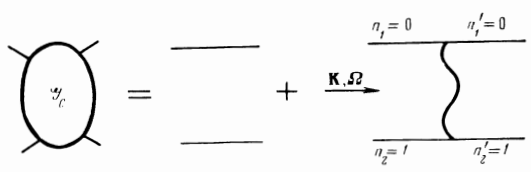

FIG. 4

The function $\overline{\mathscr{F}}_C$ is the usual exciton Green function in a strong magnetic field (when one can neglect the Coulomb mixing of the different Landau zones):

$$\langle p|\overline{\mathscr{F}}_c^{01}(\mathbf{K}\Omega)|p'\rangle = \langle p|(\Omega - H_{\mathbf{K}}^{01})^{-1}|p'\rangle. \quad (20)$$

The frequency Ω is reckoned from $\epsilon_g + \omega_0 + \frac{1}{2}\omega_e + \frac{3}{2}\omega_h$. The Hamiltonian of the longitudinal internal motion is

$$H_{\mathbf{K}}^{n_1n_2} = \frac{\hat{p}^2}{2\mu} + \frac{\mathbf{K}_z^2}{2M} + U_{\mathbf{K}}^{n_1n_2}, \quad (21)$$

where the operator U is determined by Eq. (16) (with $n_1 = n_1'$, $n_2 = n_2'$) and μ and M are the reduced and total masses.

We find the thick Coulomb line from the equation of Fig. 4 and substitute it into the vertex $\mathscr{T}_1$ in (17). In the z-representation we then get, using (20),

$$[\Omega - \varkappa - H_{\mathbf{K}}^{11}(z)]\overline{\mathscr{F}}(zz') - \int \mathscr{M}(\mathbf{K}\Omega; z, z'')\overline{\mathscr{F}}(z''z')\,dz'' = \delta(z - z'), \quad (22)$$

$$\mathscr{M}(\mathbf{K}, \Omega; z, z') = \sum_{\mathbf{q}} w_{\mathbf{q}}(z)\langle z|(\Omega - H_{\mathbf{K}-\mathbf{q}}^{01})^{-1}|z'\rangle w_{\mathbf{q}}^*(z'), \quad (23)$$

$$w_{\mathbf{q}}(z) = |\gamma_{10}(\mathbf{q})|\exp(-icq_z z), \quad c = m_h/M, \quad \varkappa = \omega_e - \omega_0.$$

Introducing the eigenfunctions of the operator (21)

$$H_{\mathbf{K}}^{01}\chi_m^{\mathbf{K}\perp}(z) = E_m^{01}(\mathbf{K})\,\chi_m^{\mathbf{K}\perp}(z) \quad (24)$$

we can write $\mathscr{M}$ down explicitly:

$$\mathscr{M}(\mathbf{K}\Omega; z, z') = \sum_{m\mathbf{q}} |\gamma_{10}(\mathbf{q})|^2 \times \frac{\exp(-icq_z z)\,\chi_m^{\mathbf{K}\perp-\mathbf{q}\perp}(z)\,\chi_m^{*\mathbf{K}\perp-\mathbf{q}\perp}(z')\exp(icq_z z')}{\Omega - E_m^{01}(\mathbf{K}-\mathbf{q}) + i\delta}. \quad (25)$$

In [12] a set of equations equivalent to (22) was obtained. We can write Eq. (22) symbolically in the form

$$[\Omega - \varkappa - \hat{L}(\mathbf{K}\Omega)]\overline{\mathscr{F}}(\mathbf{K}\Omega) = 1. \quad (26)$$

Its solution can be written in the form[13]

$$\overline{\mathscr{F}}(\mathbf{K}\Omega) = \sum_s |\Psi_s(\mathbf{K}\Omega)\rangle\langle\overline{\Psi}_s(\mathbf{K}\Omega)|/(\Omega - \varkappa - W_s(\mathbf{K}\Omega) + i\delta), \quad (27)$$

where $|\Psi_S(\mathbf{K}\Omega)\rangle$ and $W_S(\mathbf{K}\Omega)$ satisfy the equation

$$\hat{L}(\mathbf{K}\Omega)|\Psi_s(\mathbf{K}\Omega)\rangle = W_s(\mathbf{K}\Omega)|\Psi_s(\mathbf{K}\Omega)\rangle. \quad (28)$$

The energy spectrum of the system is determined by the poles of $\overline{\mathscr{F}}$, i.e., by the equation

$$\mathscr{E}(\mathbf{K}, \varkappa) = W_s(\mathbf{K}, \mathscr{E}(\mathbf{K}, \varkappa)) + \varkappa. \quad (29)$$

4. GENERAL FORM OF THE SPECTRUM. LIMIT POINT

We discuss the general structure of the energy spectrum for K = 0. If we neglect the Coulomb interaction the energy spectrum is determined by the spectrum of the electron which interacts resonantly with the phonons. It is shown in Fig. 5 by curve 3.[3,4] Curve 3 describes in its right-hand part ElPC.[6] Curves 1 and 2 correspond to an electron with $n_1 = 1$ and $n_1 = 0$ for $\alpha = 0$.

If we neglect the electron-phonon coupling but take the Coulomb attraction into account two sets of Coulomb levels appear which are connected with the electron bands $n_1 = 1$ and $n_1 = 0$ and the hole band $n_2 = 1$. The lowest Coulomb levels 1′ and 2′ (Fig. 5) are shifted with respect to 1 and 2 by the binding energy of the corresponding excitons (I_{11} and I_{01}).

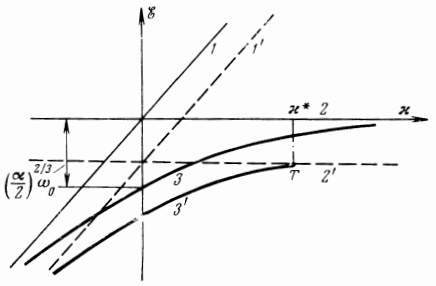

FIG. 5

When both interactions are taken into account the lowest state corresponds to a single-particle branch of the spectrum shown by curve 3′. Its right-hand part corresponds to a complex containing an electron (in the band $n_1 = 0$), a hole, and a phonon.

It is clear from (25) that the limit point T of the single-particle spectrum is the threshold for the decay of the complex into an exciton and a phonon. The line 2′ limits from below the region of the two-particle exciton + phonon excitations.

There is a qualitative difference between the ExPC and ElPC spectra: the first one has a limit point, but the second does not have one. This is a consequence of the fact that an exciton has a transverse mass[14] while the energy of a Landau electron is independent of the transverse momentum. As a result the self-energy part diverges[4,7] in the second case when we approach the straight line 2. In contrast, in the first case $\mathscr{M}$ (see (25)) remains finite, even though it has a singularity on the line 2′.

We establish the behavior of the single-particle excitation $\mathscr{E}(\kappa) \equiv \mathscr{E}(0, \kappa)$ close to T. It is determined by (28) while $\partial W/\partial\Omega = \langle\partial\hat{L}/\partial\Omega\rangle = \langle\partial\mathscr{M}/\partial\Omega\rangle$. As $\gamma_{10} \to$ constant as $\mathbf{q} \to 0$, the decisive contribution to $\partial\mathscr{M}/\partial\Omega$ near 2′ (i.e., for small $I + \Omega$) is according to (25) given by the integration over small $\mathbf{q}$ in the term with the smallest value of m and then $\partial\mathscr{M}/\partial\Omega \approx |I + \Omega|^{-1/2}$. Hence $W(\Omega) \approx a + b|I + \Omega|^{1/2}$ and from (29)

$$\mathscr{E}(\varkappa) + I \sim (\varkappa - \varkappa^*)^2.$$

Let us compare the different quantities with the dimensions of an energy which occur in the theory. We always assume that ω_0 is much larger than all interaction energies. I is a characteristic energy for the electron-hole interaction. The magnitude of the electron-phonon interaction at the resonance point $\kappa = 0$ is $(\alpha/2)^{2/3}\omega_0$.[3,4] Yet another energy $\alpha^2\omega_0$ occurs when we take into account the contribution from two-phonon diagrams of the kind $\mathscr{T}_2$. On the whole of the single-particle branch $\mathscr{T}_2 \ll \mathscr{T}_1$ provided $(\alpha^2\omega_0/I)^{1/2} \ll 1$; we shall assume that that inequality is satisfied. It means that the range of energies $\sim\alpha^2\omega_0$ where the complex spectrum of the ElPC is formed[7] is small compared to I.

The competing energies are thus $(\alpha/2)^{2/3}\omega_0$ and I. We shall in what follows call the cases $(\alpha/2)^{2/3}\omega_0 \ll I$ and $(\alpha/2)^{2/3}\omega_0 \gg I$ the cases of strong and weak Coulomb interaction.

The exciton spectrum for $I \ll \omega_e \approx \omega_0$ was studied in [14,15,16]. In the logarithmic approximation $L \equiv \ln(a/2\lambda)^2 \gg 1$, the binding energy $I \approx Ry\, L^2$ and the longitudinal size of the exciton $a \approx a_0/L$. Here $Ry = (2\mu a_0^2)^{-1}$ and a_0 is the exciton Bohr radius for H = 0. In the more exact theory I depends on the numbers n_1 and n_2 of the magnetic bands. However, for the two bands $n_1 = 1$, $n_2 = 1$, and $n_1 = 0$, $n_2 = 1$ one can show that up to terms $\sim(\lambda/a_0)^2 Ry$ we have $I_{11} = I_{10} = I = Ry/m_0^2$. Here m_0 is the root of the equation

$$\frac{1}{m_0}+\ln\frac{1}{m_0^2}-2[\psi(1)-\psi(1-m_0)]=\ln\left(\frac{a_0}{2\lambda}\right)^2-2\gamma+2\alpha_{11},$$
$$\alpha_{11}=\alpha_{01}={}^1/_2(\ln 2-1+\gamma). \qquad (30)$$

Therefore the intersection of the lines 1′ and 2′ in Fig. 5 starts at practically the same value of κ as the intersection of 1 and 2.

5. CASE OF STRONG COULOMB INTERACTION

The transitions to the exciton ground state dominate in the exciton spectra in a strong magnetic field.[15] We therefore consider the part of the exciton-phonon spectrum corresponding to it. We retain in (25) in the sum over m only the resonance term with lowest $m = m_0$. Using the exciton dispersion relation[14] and (15) we can show that the actual integration in (25) covers $q_\perp \lesssim \lambda^{-1}$ and $q_z \lesssim L^{1/2}a_0^{-1}$. Since the functions $\chi_{m_0}^{\mathbf{K}_\perp-\mathbf{q}_\perp}$ for $|\mathbf{K}_\perp-\mathbf{q}_\perp| \lesssim \lambda^{-1}$ are concentrated in the region $\sim a_0/L$, we can put $\exp(icq_z z) \approx 1$ for $L \gg 1$. With the same accuracy, $\chi_{m_0}^{\mathbf{K}_\perp-\mathbf{q}_\perp} \approx \chi_{m_0}^{0}$. After integrating over $\mathbf{q}$ in (25), Eq. (28) reduces to the form

$$H_K^{11}(z)\Psi(K\Omega;z)+\sum_q \frac{|\gamma_{10}(\mathbf{q})|^2}{\Omega-E_{m_0}^{01}(\mathbf{K}-\mathbf{q})}\chi_{m_0}^0(z)\cdot\int\chi_{m_0}^{*0}(z')\Psi(K\Omega;z')dz'=W(K\Omega)\Psi(K\Omega;z). \qquad (31)$$

In the same approximation its solution is

$$\Psi(K\Omega;z)\approx\chi_{m_0}^0(z),\quad W(K\Omega)=E_{m_0}^{11}(K)+\sum_q\frac{|\gamma_{10}(q)|^2}{\Omega-E_{m_0}^{01}(K-q)}. \qquad (32)$$

The spectrum of the single-particle excitations $\mathscr{E}_{11}(\mathbf{K},\kappa)$ is determined by (32) and (29). We introduce the binding energy of the complex $\epsilon(\mathbf{K},\kappa) = |I + \mathscr{E}_{11}(\mathbf{K},\kappa)|$. When we put $\epsilon(\kappa) \equiv \epsilon(0,\kappa) = 0$ for $\mathbf{K} = 0$, we find the limit point of the spectrum:

$$\varkappa^*=\frac{b\alpha^*}{M_\perp\lambda^2},\quad\text{where }\alpha^*=\frac{\alpha}{2L^{1/2}}\left(\frac{a_0}{\lambda}\right)^3\left(\frac{m_h^3}{m_e M^2}\right)^{1/2},$$
$$b=\int_0^\infty e^{-x}\left(\int_0^x e^{-t}\ln\frac{x}{t}\,dt\right)^{-1/2}dx\approx 1.80, \qquad (33)$$

$M_\perp = \mu^0 L^{-1}(a_0/\lambda)^2$ is the transverse mass of the exciton. The quantity α^* is the effective coupling constant. One can see from (33) that $\alpha^* \gg \alpha$.

Close to the resonance point $\epsilon(0) \approx \alpha^* b/M_\perp\lambda^2$ and close to T when $|\epsilon(\kappa)| \ll \alpha^{*2}/M_\perp\lambda^2$

$$\varepsilon(\varkappa)=M_\perp\lambda^2\left(\frac{\varkappa-\varkappa^*}{2\alpha^*}\right)^2 \qquad (34)$$

From (32) and (29) we find for the effective mass of the ExPC near κ^*:

$$\frac{M_{z\,\mathrm{eff}}}{M}\approx\frac{M_{\perp\,\mathrm{eff}}}{M_\perp}\approx\frac{2(\alpha^*)^2}{M_\perp\lambda^2|\varkappa-\varkappa^*|}. \qquad (35)$$

The one-particle branch describes for $\kappa < 0$ a perturbed exciton with $n_1 = n_2 = 1$. Near κ^* the contribution from the states with $n_1 = 0$ increases and the state arising is an ExPC with binding energy (34). Since the phonon is then nearly real, it is natural that M_{eff} increases without bound as $\kappa \to \kappa^*$.

The absorption spectrum (when K = 0) is according to (11), (19), and (27)

$$\mathscr{I}(\Omega)\sim|\Phi_{11}^0(0)\Psi(\Omega,z=0)|^2\times\mathrm{Im}[\Omega-\varkappa-W(\Omega)]^{-1}. \qquad (36)$$

Near κ^* absorption in the single-particle band

$$\mathscr{I}(\Omega)\sim\left[1+\frac{2\alpha^{*2}}{M_\perp\lambda^2|\varkappa-\varkappa^*|}\right]^{-1}\delta(\Omega-\mathscr{E}_{11}(\varkappa)) \qquad (37)$$

decreases as $\kappa \to \kappa^*$. In the two-particle region ($\Omega + I > 0$) the imaginary part of the sum over q in (32) is nonvanishing and the absorption is continuous. Its spectral distribution is for $(\Omega + I)M_\perp\lambda^2 \ll 1$:

$$\mathscr{I}(\Omega)\sim\frac{1}{\pi}\frac{2\alpha^*\sqrt{(\Omega+I)M_\perp\lambda^2}}{M_\perp\lambda^2[\Omega+I-\varkappa+\varkappa^*]^2+(2\alpha^*)^2(\Omega+I)}. \qquad (38)$$

It is given by the curves 4 in Fig. 6. The absorption curves are steeply asymmetric; the decrease from the maximum by a factor two at the short-wavelength side is about 15 times slower than at the long-wavelength side. The distance of the maximum from the threshold is determined by (34) for both signs of $\kappa - \kappa^*$: the half-width is also approximately symmetric with respect to κ^*. The steep long-wavelength edge of the two-particle band and the weak dependence of the position of the maximum on κ near κ^* must, apparently, lead to a pinning pattern.

The criterion for the applicability of perturbation theory is $\epsilon(0) \ll I$ which is equivalent to $\alpha^* \ll L$. However, even for small α^* the relative contribution of terms with $m > m_0$ in (25) is of order $L^{-1/2}$ since the region over which one actually integrates over $q_\perp \lesssim \lambda^{-1}$ is independent of α^*.

6. CASE OF WEAK COULOMB INTERACTION

We expand $(\Omega - H_{K-q}^{01})^{-1}$ in (23) in powers of $U_{K_\perp-q_\perp}^{01}$ restricting ourselves to the first two terms:

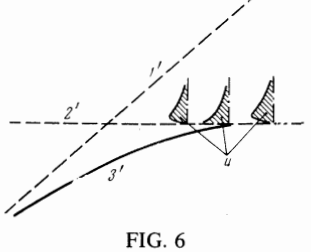

FIG. 6

$$\mathcal{M} = \Sigma + \mathcal{V} = \sum_{\mathbf{q}} |w_{\mathbf{q}}|^2 \langle z|\hat{N}(K_z\Omega q_z)|z'\rangle + \sum_{\mathbf{q}} |w_{\mathbf{q}}|^2 \langle z|\hat{N}(K_z\Omega q_z) U^{01}_{\mathbf{K}_\perp - \mathbf{q}_\perp} \hat{N}(K_z\ \Omega\ q_z)|z'\rangle, \quad (39)$$

where

$$\hat{N}(K_z\Omega q_z) = \left(\Omega - \frac{(\hat{p}+cq_z)^2}{2\mu} - \frac{(K_z-q_z)^2}{2M}\right)^{-1} \quad (40)$$

This approximation corresponds to retaining in $\mathcal{T}_1$ only the first diagram. We put $\mathbf{K} = 0$. Integration in Σ gives

$$\Sigma = \Sigma_e(\Omega)\mathcal{G}(\Omega, z-z'),\ \Sigma_e(\Omega) = -\frac{\alpha\omega_0}{2}\left|\frac{\omega_0}{\Omega}\right|^{1/2},$$
$$\mathcal{G}(\Omega, z-z') = \frac{1}{\pi}|2m_h\Omega|^{1/2}K_0(|2m_h\Omega|^{1/2}|z-z'|), \quad (41)$$

where K_0 is a Macdonald function[9] and $\mathcal{G}$ is normalized to unity. For large Ω we have $\mathcal{G} \to \delta(z-z')$ so that when there is no Coulomb interaction Σ gives a contribution Σ_e to the energy in accordance with [4].

If in the logarithmic approximation we neglect the dependence of $U^{01}_{\mathbf{K}_\perp-\mathbf{q}_\perp}$ on the lower index, we have

$$\mathcal{V}(\Omega; z, z') = \mathcal{V}_e(\Omega)\mathcal{U}(\Omega; z, z'),$$
$$\mathcal{V}_e(\Omega) = -\partial\Sigma_e/\partial\Omega = {}^1/_4\alpha|\omega_0/\Omega|^{3/2},$$
$$\mathcal{U}(\Omega; z, z,) = \frac{1}{\pi}|\Omega|\sqrt{\mu m_h}\int_{-\infty}^{+\infty} d\zeta U_0^{01}(\zeta)\int_{-\infty}^{+\infty}\frac{dt}{\operatorname{ch} t}\exp(-\eta\operatorname{ch} t - i\xi\operatorname{sh} t), \quad (42)$$

where

$$\xi = |2\Omega\mu m_h/m_e|^{1/2}(z-z'), \qquad \eta = |2\mu\Omega|^{1/2}(|\zeta - z| + |\zeta - z'|). \quad (43)$$

It is clear from (42) that owing to the interaction with the phonons there arises instead of the local potential U_0^{01} an effective non-local potential $\mathcal{U}(\Omega; z, z')$. According to (42) and (43) the region of the "smearing out" of the non-local potential, i.e., the region of values of $|z-z'|$ in which $\mathcal{U}$ is large, is of order $|2\mu\Omega|^{-1/2}$.[2)] The potential $\mathcal{U}$ has an important property: if we substitute in (42) instead of U_0^{01} any integrable potential U, we have

$$\iint_{-\infty}^{+\infty}\mathcal{U}(\Omega; z, z')\,dz\,dz' = \int_{-\infty}^{+\infty} U(\zeta)\,d\zeta. \quad (44)$$

The Schrödinger equation is obtained by substituting (41) and (42) into (28). We shall first of all solve it in the shallow well approximation which is valid in the problem with a Coulomb potential in the logarithmic approximation. The change to a non-local potential changes the situation. In the region $|z|, |z'| \lesssim |2\mu\Omega|^{-1/2}$ the potential $\mathcal{U}$ changes slowly and is large: $\mathcal{U}(z, z') \approx \mathcal{U}(0, 0) = -(e^2/\epsilon_\infty)2|\Omega|\sqrt{\mu m_h})\,L_2(\Omega)$, $L_2(\Omega) = \ln|\omega_0/\Omega| \gg 1$. For large $|z|$ and $|z'|$ it is close to $-\delta(z-z')e^2/\epsilon_\infty|z|$. The shallow well approximation is valid if the extension of the wave function $a \gg |2\mu\Omega|^{-1/2}$. The energy is then determined by integrating $\Sigma(z-z')$ and $\mathcal{V}(z, z')$ over the region $|z|, |z'| \lesssim a$. Owing to (44)[3)] and the normalization of $\mathcal{G}(z-z')$ the non-local character has practically no effect on the magnitude of these integrals and its effect completely drops out. The non-local potential can thus be replaced by a local one which in the logarithmic approximation has the same wave function and energy of the ground state. As a result (28) reduces to

$$\left[\frac{\hat{p}^2}{2\mu} + (1+\mathcal{V}_e(\Omega))U(z)\right]\Psi(\Omega, z) = [W(\Omega) - \Sigma_e(\Omega)]\Psi(\Omega, z), \quad (45)$$

where $U(z) \approx U_0^{11}(z) \approx U_0^{01}(z)$. This equation describes a free diamagnetic exciton with the square of the charge renormalized by a factor $1 + \mathcal{V}_e$. The radius of the state and its eigenvalue are

$$a(\Omega) = \frac{a_0}{L_1(\Omega)[1+\mathcal{V}_e(\Omega)]},\ W(\Omega) = \Sigma_e(\Omega) - L_1^2(\Omega)\operatorname{Ry}[1+\mathcal{V}_e(\Omega)]^2$$
$$L_1(\Omega) = \ln\left(\frac{a(\Omega)}{2\lambda}\right)^2 \quad (46)$$

In zeroth order in the Coulomb interaction the energy is determined from the equation $\mathcal{E}_0(\kappa) = \Sigma_e(\mathcal{E}_0(\kappa)) + \kappa$ in agreement with [4].

The Coulomb interaction shifts the single-particle level downwards relative to this energy. Assuming the shift to be small compared to $\mathcal{E}_0(\kappa)$ we get from (29), (42), and (46)

$$\varepsilon(\varkappa) \equiv \mathcal{E}(\varkappa) - \mathcal{E}_0(\varkappa) = -L_1^2(\mathcal{E}_0)\operatorname{Ry}[1+\mathcal{V}_e(\mathcal{E}_0)]. \quad (47)$$

At resonance $\kappa = 0$ and $\mathcal{E}_0(0) = -(\alpha/2)^{2/3}\omega_0$ and $\mathcal{V}_e = {}^1/_2$. When $\kappa \gg ({}^1/_2\alpha)^{2/3}\omega_0$ the energy $\mathcal{E}_0(\kappa)$ decreases in absolute magnitude $\mathcal{E}_0(\kappa) \approx -\omega_0(\alpha\omega_0/2\kappa)^2$, and $\mathcal{V}_e$ increases, $\mathcal{V}_e = {}^1/_4\alpha|\omega_0/\mathcal{E}_0(\kappa)|^{3/2}$; therefore $\mathcal{V}_e \gg 1$. The Coulomb binding energy then increases as $|\epsilon(\kappa)| \sim |\mathcal{E}_0(\kappa)|^{-3/2} \sim \kappa^3$. The inequality $\mathcal{V}_e \gg 1$ means that the first diagram in $\mathcal{T}_1$ which is appreciably larger than $\mathcal{T}_0$ is decisive.

The absorption intensity is according to (36) equal to

$$\mathcal{I}(\Omega) \sim \frac{\pi/a(\Omega)}{1 - \partial\Sigma_e/\partial\Omega}|\Phi_{11}^0(0)|^2\delta[\Omega - \mathcal{E}_0(\varkappa) - \varepsilon(\varkappa)]. \quad (48)$$

According to (42) and (46) the coefficient in front of the δ-function in (48) is independent of κ, i.e., the intensity is constant.

The shallow well approximation is valid when $a \gg |2\mu\mathcal{E}_0(\kappa)|^{-1/2}$, i.e., up to κ_S such that $\mathcal{E}_S \equiv \mathcal{E}_0(\kappa_S)$ and $\mathcal{V}_e(\kappa_S)$ are equal to

$$|\mathcal{E}_s| \approx \left[\frac{(\alpha/4)^{2/3}\omega_0}{\operatorname{Ry} L_1^2(\mathcal{E}_s)}\right]^{3/4}\operatorname{Ry} L_1^2(\mathcal{E}_s),$$
$$\mathcal{V}_e(\mathcal{E}_s) = \left[\frac{(\alpha/4)^{2/3}\omega_0}{\operatorname{Ry} L_1^2(\mathcal{E}_s)}\right]^{3/8} \gg 1. \quad (49)$$

When $\kappa > \kappa_S$ the analysis of the equation with the non-local potential $\mathcal{M}$ shows that the radius of the state $a(\mathcal{E}_0) \sim |2\mu\mathcal{E}_0(\kappa)|^{-1/2}$, i.e., of the order of the well dimensions. An estimate of the contribution to the energy from $\mathcal{V}(z, z')$ through functions of such a radius gives $\mathcal{V}_e\mathcal{U}(0, 0)a(\mathcal{E}_0)$. Neglecting U_0^{11} and as before estimating the contribution Σ to be Σ_e we get Eq. (29) in the form

$$\varkappa - \beta\mathcal{V}_e L_2(\mathcal{E}_0)|\mathcal{E}_0\operatorname{Ry}|^{1/2} - \frac{\alpha\omega_0}{2}\left|\frac{\omega_0}{\mathcal{E}_0(\varkappa)+\varepsilon(\varkappa)}\right|^{1/2} = 0,\ \beta \sim 1, \quad (50)$$

and hence

$$\varepsilon(\varkappa) = -\beta L_2(\mathcal{E}_0)|\mathcal{E}_0(\varkappa)\operatorname{Ry}|^{1/2}. \quad (51)$$

[2)] Provided all masses are of the same order of magnitude, as we always assume.

[3)] The integral of $U_0^{01}(\zeta)$ on the right-hand side of (44) diverges for large $|\zeta|$. For us, however, it is important that a relation analogous to (44) is valid also when we integrate over a finite domain with dimensions appreciably larger than $|2\mu\Omega|^{-1/2}$.

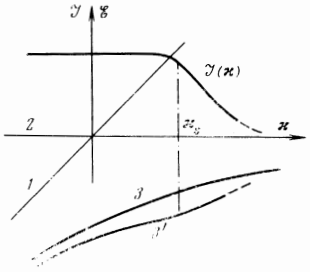

FIG. 7

The exciton binding energy thus decreases as $|\mathscr{E}_0(\kappa)|^{1/2}$.

The intensity of the absorption is determined by (48) but now a $(\mathscr{E}_0) \sim \mathscr{E}_0^{-1/2}(\kappa)$. Using (42) we get $\mathscr{I} \sim \mathscr{E}_0^2(\kappa)$, i.e., $\mathscr{I}$ decreases rapidly.

$\mathscr{I}$ is thus large and changes little in the region where the binding energy $|\epsilon(\kappa)|$ begins to decrease. The κ-dependence of $\mathscr{E}$, ϵ, and $\mathscr{I}$ is illustrated in Fig. 7.

The criterion for the resolvent in (23) to be expandable is that the second term in (39) be small compared to the first one, i.e., $|\epsilon(\kappa)| \ll |\mathscr{E}_0(\kappa)|$. It is always satisfied when $|\mathscr{E}_0(\kappa)| > |\mathscr{E}_S|$, and when $|\mathscr{E}_0(\kappa)| < |\mathscr{E}_S|$ it reduces according to (51) to $L_0^2(\mathscr{E}_0)\,\mathrm{Ry} \ll |\mathscr{E}_0(\kappa)|$.

7. INFLUENCE OF FREE PHONONS ON THE ABSORPTION SPECTRUM

When there are free phonons present (either thermal or non-equilibrium[17]) there must appear a new absorption spectrum connected with ExPC. Namely, if the optical transition, corresponding to the formation of an exciton with $n_1 = n_2 = 0$, occurs near a free phonon, an ExPC may appear in the final state. This absorption must be shifted to the low-frequency side as compared to the exciton band with $n_1 = n_2 = 0$ and its intensity must increase in a resonance fashion.[18]

The picture of the energy levels of the system is similar to the one given in Fig. 5. However, the exciton binding energies I_{10} and I_{00} are different: $I_{10} = I$ while I_{00} is determined from (30) through replacing α_{11} by $\alpha_{00} = \frac{1}{2}(\ln 2 + \gamma)$. Since $I_{00} > I_{10}$ the intersection of the exciton levels occurs at smaller values of $\omega_e \equiv \omega_{10}$ than that of the electron levels.

The intensity of this absorption is in the linear approximation in the phonon occupation number $n_{\mathbf{q}}$ determined by the contribution from the diagram of Fig. 8 to the two-particle function. The double dashed line indicates that part of the phonon Green function which is proportional to $n_{\mathbf{q}}$. The corresponding contribution to $\mathscr{F}$ (see (10)) is equal to

$$-\sum_{\mathbf{q}} |\gamma_{10}(\mathbf{q})|^2 n_{\mathbf{q}} \langle p_2 | \mathscr{F}_{c00}(\mathbf{K}\Omega)\mathscr{F}_{10}(\mathbf{K}+\mathbf{q}, \Omega+\omega_0)\mathscr{F}_{c00}(\mathbf{K}\Omega)|p'_2\rangle. \tag{52}$$

If the Coulomb interaction is strong and if there are in the initial state only phonons of momentum $\mathbf{q}$ the absorption intensity is

$$\mathscr{I}_{\mathbf{q}}(\Omega) \sim \frac{n_{\mathbf{q}}|\gamma_{10}(\mathbf{q})|^2 |\Phi_{00}^0(0)|^2 \pi a^{-1}\delta[\Omega - \mathscr{E}_{10}(\mathbf{q},\varkappa)]}{[\Omega - E_{m_0}^{00}(0)]^2\left[1 + \sum_{\mathbf{q}'}(\Omega - E_{m_0}^{00}(\mathbf{q}-\mathbf{q}'))^{-1}\right]}. \tag{53}$$

The energy is reckoned from $\epsilon_g + \frac{1}{2}(\omega_e + \omega_h)$.

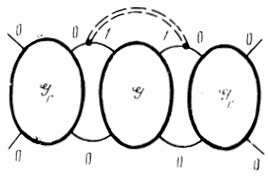

FIG. 8

We verify that (53) contains a large factor due to which the absorption considered can apparently be observed experimentally even for low phonon concentrations. The absorption (53) per phonon must of course be compared with the exciton absorption per elementary cell (with volume v) when there are no phonons. At the resonance point $\omega_e = \omega_{10}$ the ratio of their intensities is for a phonon with $q = 0$ approximately equal to

$$\left|\frac{V}{v}\frac{\gamma_{10}(0)}{\Omega - E_{m_0}^{00}(0)}\right|^2 = \frac{2\sqrt{2}\pi}{b^2 L^{1/2}}\left(\frac{\mu}{M}\right)^{1/2}\frac{1}{\alpha^*}\frac{\lambda^2\alpha}{v}. \tag{54}$$

It is very large ($\sim 10^5$) due to the last factor.

The cause of the high intensity is the same as for impurity absorption.[19] However, if the initial phonon distribution is not very steep there must occur, instead of a peak, continuous absorption with a threshold corresponding to the complex energy with $\mathbf{K} = 0$. Near κ^* in the immediate vicinity of the threshold

$$\mathscr{I}(\Omega) \sim \frac{2}{a}\bar{n}\alpha^{*3/2}(2L\,\mathrm{Ry})^{3/4}\frac{\sqrt{\Omega - \Omega_{\mathrm{thr}}}}{\varepsilon^{3/4}(\varkappa)}|\Phi_{00}^0(0)|^2, \tag{55}$$

where $\Omega_{\mathrm{thr}} = -I_{00} - \epsilon(\kappa)$. For smaller κ we have

$$\mathscr{I}(\Omega) \sim \frac{2}{a}\bar{n}\alpha^*(2L\,\mathrm{Ry})^{1/2}\frac{\sqrt{\Omega - \Omega_{\mathrm{thr}}}}{\varepsilon^2(\varkappa)}|\Phi_{00}^0(0)|^2. \tag{56}$$

When Ω approaches $E_{m_0}^{00}(0)$, i.e., the purely exciton band, the absorption is strongly magnified as compared with (55) and (56) because the factor $\Omega - E_{m_0}^{00}(0)$ in (53) decreases; we assume that $n_{\mathbf{q}}$ changes sufficiently slowly.

When the Coulomb interaction is weak and for $q_z \ll \pi/a$ we get from (52)

$$\mathscr{I}_{\mathbf{q}}(\Omega) \sim \frac{n_{\mathbf{q}}|\gamma_{10}(\mathbf{q})|^2}{\mathscr{E}_0^2(\varkappa)}\frac{\pi}{a}|\Phi_{00}^0(0)|^2\frac{\delta[\Omega - \mathscr{E}_{10}(\mathbf{q},\varkappa)]}{1 - \partial\Sigma_c/\partial\Omega}. \tag{57}$$

The H-dependences in (48) and (57) differ from one another by a fast changing factor $\mathscr{E}_0^2(\kappa)$ which also ensures the high intensity.

CONCLUSION

We summarize some of the results obtained and emphasize the important restrictions in the theory.

We considered an exciton in a strong magnetic field ($\omega_e \gg I$) interacting with optical phonons. There is always a single-particle branch in the energy spectrum of this system. The region where this branch exists can roughly be determined by the condition $\omega_e < \omega_0$. This branch ends at some value $H = H^*$ of the field and the limit point H^* corresponds to the decay of the single-particle state into an exciton and a phonon (we assume that $I \gg \alpha^2\omega_0$). Near the point H^* the single-particle branch corresponds to a stable complex including an

electron, a hole, and an optical phonon. From this it is clear that the problem is essentially a three-particle one and this entails that one can obtain solutions only in limiting cases.

The presence of pinning is considered to be the most characteristic MPR phenomenon in experimental works.

In the strong Coulomb interaction approximation (Sec. 5) apart from absorption in the single-particle band the above threshold absorption corresponding to the formation of dissociated states of the complex must influence the pinning pattern significantly. The pinning pattern must to a large extent arise just when one takes into account the progressive approach of the maxima of the single-particle and above-threshold absorption as $H \to H^*$ (Fig. 6). An experimental study of the spectral distribution of the absorption near threshold is therefore of interest.

We emphasize that in the weak Coulomb interaction approximation the pinning pattern obtained by us (Sec. 6) also significantly differs from the one obtained when the Coulomb interaction is ignored. The decisive role is here played by the fact that as $H \to H^*$ the effect of the Coulomb interaction is amplified in a resonance fashion. The first of the vertices $\mathcal{T}_1$ then acquires a decisive value and becomes appreciably larger than $\mathcal{T}_0$ (Fig. 3). As a result in a well-defined range of H-values already in the pinning region the intensity of the single-particle band is constant and after that begins to decrease rapidly (Fig. 7). The frequency at which pinning is observed may thus turn out to be slightly smaller than the threshold frequency corresponding to the formation of free quasi-particles (an exciton and a phonon).

At the edge of the purely electron spectrum there must be bands corresponding to the formation of ExPC with the participation of (equilibrium or non-equilibrium) lattice phonons and their intensity must be anomalously high (Sec. 7).

A comparison of the results about the formation of localized vibrations near impurity centers for a resonance electron-phonon coupling[20] and those about ElPC [3-5] with our results for ExPC shows that in that order the magnitude of the electron-phonon interaction effect progressively decreases. The reason for this lies in the fact that in the first case the electron spectrum is discrete, in the second case it depends only on the longitudinal momentum, and in the third case on all three components of the momentum.

All final formulae are obtained in a completely lucid form as a result of a rigorous expansion in well-defined parameters. This is achieved by making rigid limitations: weakness of the coupling, closeness to resonance, etc. If we drop the restrictions we can obtain only approximate formulae of the intermediate coupling theory type as in the theory of ExPC for $H = 0$.[21]

At the same time the quantitative accuracy of the final formulae when they are applied to real crystals is unfortunately apparently small. This is primarily connected with the application of the logarithmic approximation $L \gg 1$. For instance, for InSb with $m_e = 0.014 m_0 \ll m_h$, $\epsilon_\infty = 17.0$, $\omega_0 = 22 \times 10^{-3}$, we get $(a_0/2\lambda)^2 \approx 4.6$ and $L \approx 1.5$. For the same crystal with $\alpha = 0.015$ we get $(\alpha/2)^{2/3}\omega_0 \approx 0.85 \times 10^{-3}$ eV, $I \approx 0.6 \times 10^{-3}$ eV, i.e., these quantities lie close to one another. InSb presents thus a case intermediate between strong and weak Coulomb coupling.

All results obtained refer to the low-frequency part of the spectrum; going into the high-frequency region requires a more thorough investigation of the kernel $\mathcal{M}$ (Eq. (28)).

We note in conclusion that there exists yet another large parameter for many crystals: the mass ratio m_h/m_e. An analysis of our results shows that in both limiting cases (Secs. 5 and 6) an increase in this parameter leads to a magnification of the exciton-phonon interaction. It is tempting to use this parameter to weaken some of the limitations in the theory.

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