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Many-Baryon Resonances in the Model of an Isobar-Nucleon Exchange Interaction

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An analysis of the interaction in Δ2N, Δ3N, and Δ4N systems is carried out on the basis of a model in which the interaction between the Δ-isobar and the nucleon is achieved by the exchange of a "decaying" π meson. It is shown that the isobar and the nucleons can be bound in a many-baryon resonance only in the Δ3N system in the state with quantum numbers T = 1, $J^P = 1^+$.

1. INTRODUCTION

THE interaction of the Δ-isobar (the Δ-isobar is the $N^*_{33}(1236)$) with systems consisting of two, three, and four nucleons is investigated in the present article. The goal of this investigation is to establish the quantum numbers of the states in which an attraction is realized in such systems, and to estimate the magnitude of the attraction. A sufficiently strong attraction in the system of baryon + nucleons may lead to the formation of a many-baryon resonance (see, for example,[1-3]). In[2-4] it was shown that if the Δ-isobar is regarded as a system consisting of a π meson and a nucleon, then the exchange forces associated with the existence of the decay Δ → πN are dominant in the interaction between the isobar and the nucleon. In this connection it was established that a very strong attraction is established between the isobar and the nucleon in states with the quantum numbers T = 2, $J^P = 1^+$ and T = 1, $J^P = 2^+$. However, this attraction is not sufficiently strong for one to be able to confidently talk about the presence of a resonance in the ΔN system (for a more detailed explanation, see Sec. 2). In this article it is shown that among the systems with baryon numbers 3, 4, and 5, the possibility of forming a many-baryon Δ-resonance exists only in the state with quantum numbers B = 4, T = 1, and $J^P = 1^+$.

In Sec. 2 of the present article the exchange potential introduced in[3-4] is considered in more detail, and the experimental situation with regard to the two-baryon resonance T = 1, $J^P = 2^+$[5] is discussed. The interaction in the Δ2N, Δ3N, and Δ4N systems is studied in Secs. 3–5.

2. THE ΔN EXCHANGE POTENTIAL

At first neglecting for simplicity the presence of spins and isotopic spins, let us consider the integral equation for the ΔN-scattering amplitude in the nonrelativistic approximation (see[2]):

$$f(\mathbf{k}',\mathbf{k}) = -\frac{m}{2\pi}V(\mathbf{k}',\mathbf{k}) + \frac{1}{(2\pi)^3}\int V(\mathbf{k}',\mathbf{k}'')\frac{f(\mathbf{k}'',\mathbf{k})\,d^3k''}{E+i\Gamma/2-\mathbf{k}''^2/2m}, \qquad (1)$$

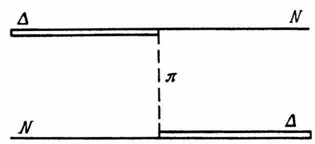

FIG. 1

where $m = m_N m_\Delta/(m_N + m_\Delta)$, and Γ is the width of the isobar.[1)] Equation (1) is the usual Lippmann-Schwinger equation in which the quantity $E' = E + i\Gamma/2$ plays the role of the energy. In general Γ depends on E and k″ in the following way:

$$\Gamma = \text{const}\,[2m(Q+E) - \mathbf{k}''^2]^{1/2},$$

where $Q = m_\Delta - m_N - \mu$ and μ is the mass of the π meson. However, if $|Q + E| \sim \mu$, then the dependence of Γ on k''^2 is weak and one can neglect it.[2,3]

The potential V(k′, k) corresponding to the exchange ΔN-interaction has the following form (see the diagram shown in Fig. 1):

$$V(\mathbf{k}',\mathbf{k}) = -\frac{g_1 g_2}{\mu^2+\mathbf{q}^2-(\Delta m+E-\mathbf{k}^2/2m_N-\mathbf{k}'^2/2m_N)},$$
$$\Delta m = m_\Delta - m_N, \quad \mathbf{q} = \mathbf{k}+\mathbf{k}', \quad g_1 \equiv g_1\left(\frac{m_N\mathbf{q}-\mu\mathbf{k}}{m_N+\mu}\right), \qquad (2)$$
$$g_2 \equiv g_2\left(\frac{m_N\mathbf{q}-\mu\mathbf{k}'}{m_N+\mu}\right)$$

It should be noted that it would be possible to investigate the resonances in the πNN system on the basis of the Faddeev equations[6] (see, for example,[7]) or on

1)We start from the fact that, within the framework of the relativistic diagram technique, the description of an isobar differs from the description of stable particles only in the form of the propagator $\Pi_\Delta = i[p_\Delta^2 - m_\Delta^2 + im_\Delta\Gamma]^{-1}$. Of course, such a description makes sense only provided that $(p_\Delta^2 - m_\Delta^2)/m_\Delta \lesssim \Gamma$, which is satisfied in the case under consideration. In the indicated analysis Eq. (1) is derived from the Bethe-Salpeter equation for the ΔN-scattering amplitude, written down in the ladder approximation, by means of an integration over the energy variable. In the nonrelativistic approximation this integration reduces to a calculation of the residue at the pole of the nucleon or isobar propagator and corresponds to the substitution $-(p_N^2 - m_N^2 + i\epsilon)^{-1}(p_\Delta^2 - m_\Delta^2 + im_\Delta\Gamma)^{-1} \to 2\pi i\delta(2m_N E_N - p_N^2)(2m_\Delta E_\Delta - p_\Delta^2 + im_\Delta\Gamma)^{-1}$, $E_N + E_\Delta = E$.

the basis of Watson's theory of multiple scattering.[8] In order to make a comparison with these treatments, we note that the denominator of the potential (2) corresponds to the three-particle Green's function in these treatments, and the factors g_1 and g_2 in the numerator correspond to the parts of the πN-scattering amplitudes which are taken in factorized form[2)] in the case under consideration.

We shall be interested in the solution of Eq. (1) for small values of $E' = E + i\Gamma/2$, $|E'| \ll \Gamma/2$. Therefore in the denominator of expression (2) we set $E = -i\Gamma/2$, and we neglect the terms E', $k'^2/2m_N$ and $k^2/2m_N$ in comparison with Δm. If we recognize in addition that the dependence of the form factors on k and k′ is m_N/μ times weaker than their dependence on q, then we arrive at the conclusion that in order to determine the poles of the amplitude $f(\mathbf{k}', \mathbf{k})$ in E' for small values of E', one can approximate the potential $V(\mathbf{k}', \mathbf{k})$ by the local potential

$$V(\mathbf{q}) = -g_1(\mathbf{q})g_2(\mathbf{q}) / (\mu^2 + \mathbf{q}^2 - (\Delta m - i\Gamma/2)^2). \quad (3)$$

Thus, the problem of finding the poles of the ΔN-scattering amplitudes is equivalent to determining the eigenvalues of the Schrödinger equation with a local complex potential obtained by taking the Fourier transform of the potential (3), where the quantity $E' = E + i\Gamma/2$ plays the role of the energy in such an equation and E denotes the total kinetic energy of the isobar and of the nucleon.

If spins and isotopic spins are taken into consideration, then the operator for the ΔN exchange potential can be written in the following form:[4]

$$\hat{V}(\mathbf{r}) = (1/_6 + 2/_9 \boldsymbol{\tau}_\Delta \boldsymbol{\tau}_N)\{-(1/_2 + 2/_3 \mathbf{S}\boldsymbol{\sigma}_N)(V_0(r) + 5/_4 V_2(r)) \quad (4)^*$$
$$+ [2(\mathbf{Sn})^2 \mathbf{S}\boldsymbol{\sigma}_N + 2i(\mathbf{Sn})\mathbf{S}[\boldsymbol{\sigma}_N \mathbf{n}] + 1/_2 (\mathbf{Sn})^2 - 3(\mathbf{Sn})(\boldsymbol{\sigma}_N \mathbf{n})] V_2(r)\},$$

where $\mathbf{S}$, $\boldsymbol{\sigma}_N$ and $\boldsymbol{\tau}_\Delta$, $\boldsymbol{\tau}_N$ denote the spin and isotopic spin operators of the isobar and of the nucleon, $\mathbf{S}^2 = \boldsymbol{\tau}_\Delta^2 = (3/2)(3/2 + 1)$, $\boldsymbol{\sigma}_N^2 = \boldsymbol{\tau}_N^2 = (1/2)(1/2 + 1)$, $\mathbf{n} = \mathbf{r}/r$, and

$$V_\Lambda(r) = -\frac{\lambda^2}{2\pi^2\mu^2}\int \frac{q^4\, dq}{q^2 + \mu^2 - (\Delta m - i\Gamma/2)^2} i^\Lambda j_\Lambda(qr) g^2(q^2). \quad (5)$$

Here $\Gamma = 120$ MeV and $\lambda = 2$. We select the form factor $g(q^2)$, allowing for the departure of the π meson and nucleon from the mass shell at the vertex $\Delta \to \pi N$, in the form

$$g(q^2) = (q_0^2 + c^2) / (q^2 + c^2),$$

where $q_0 = 231$ MeV/c. From data on the production of the Δ-isobar in πN and NN collisions, it follows that the constant c is close to 3μ.[9]

Graphs of the potentials $V_0(r)$ and $V_2(r)$ are given in[4]. It should be noted that formulas (4) and (5) for $\hat{V}(r)$ and $V_\Lambda(r)$ differ from the corresponding formulas (1) and (2) of[4] by numerical factors.[3)] The attraction due to the exchange interaction turns out to be strongest in the states $T = 2$, $J^P = 1^+$ and $T = 1$, $J^P = 2^+$. However, in these states the attraction is not sufficiently large for the formation of a resonance in the ΔN system.

The experimental data (see the review article[10]), which exists primarily for the two-baryon system with isospin 1, and the new data from the phase analysis of pp scattering[5] do not contradict the conjecture about the existence of a two-baryon resonance with $T = 1$, $J^P = 2^+$, and $M = 2160 \pm 60$ MeV. If such a resonance were to exist, then it would imply that there is a stronger attraction between the isobar and the nucleon than that which arises due to the exchange of a "decaying" π meson. A strong attraction between the Δ and N is obtained, for example, in the SU(6)-symmetry scheme,[11] which assumes a quark structure for all the particles, including the Δ-isobar. As yet it is not clear, however, whether it is actually necessary to assume the existence of the two-baryon resonance in order to explain the experimental data, since there is an alternative possibility of describing the peaks in the cross sections of the reactions $\pi d \to NN$ and $\pi d \to \pi d$ in the energy range 2100 to 2200 MeV as being due to the effect of the threshold for the production of the Δ-isobar (see[12-14]). It may turn out that this effect can be explained by the Argand diagram for the $^2d_1$ partial amplitude of pp-scattering,[5] all the more since it is still impossible to regard the presence of a loop on this diagram as certain.

3. THE Δ2N SYSTEM

In considering systems with B = 3, 4, and 5, in addition to the ΔN exchange potential determined by formula (4), we shall also take the nucleon-nucleon interaction into account. Bearing in mind the theoretical uncertainties in the exchange potential (the value of c) and in the induced non-exchange potential (see[2,3]), one can hardly claim an exact determination of the position of the poles with respect to E′ in the amplitudes for the interaction of the Δ with nucleons. All of the following estimates of the attraction in the system consisting of isobar + nucleons will basically be qualitative in nature. Nevertheless, as will be evident from what follows, such a consideration enables us to reach quite definite conclusions about the quantum numbers of the Δ-resonance with baryon numbers $3 \le B \le 5$. In determining the position of the level, we shall also neglect the imaginary part of the exchange potential, since it is appreciably smaller than the real part and can have a noticeable effect only on the width of the level. In this connection, the width of a many-baryon resonance will differ from the width of the free isobar by an amount of the order of the average value of the imaginary part of the potential. In such an approach the problem reduces to finding the eigenvalues

[2)]We also note that the transition from the πNN system to the ΔN system only implies the neglect of the non-resonance terms in the πN-scattering amplitude. In this connection the nucleon-nucleon interaction can be taken into consideration with the aid of the induced non-exchange potential (see[2,3]), which describes the nucleon-nucleon scattering between decay, $\Delta \to \pi N$, and "fusion", $\pi N \to \Delta$, events. However, as is shown in [2,3], the induced non-exchange potential is small in comparison with the exchange potential.

*$[\sigma_N \mathbf{n}] \equiv \sigma_N \times \mathbf{n}$.

[3)]The potential $\hat{V}(\mathbf{r})$ in formula (4) of [4] contains a numerical mistake made in connection with writing down and is therefore overestimated by a factor of three. In addition, there is a misprint in formula (5).

of the Schrödinger equation for the system consisting of Δ + nucleons.

The ΔN and NN potentials are such that one would expect a level in the systems under investigation only in the absence of centrifugal effects. Therefore, in what follows we shall confine our attention to only states in which all the relative orbital momenta are equal to zero. In this case the term $V_2(r)$ in the exchange potential defined according to formula (4) does not give any contribution.

Let us consider the $\Delta 2N$ system in the state with spin J and isotopic spin T. When all the internal orbital momenta are equal to zero, the Schrödinger equation for the radial wave function can be written in the following form:

$$[\hat{T}_3 + \delta_{S_{12}S_{12'}}\delta_{T_{12}T_{12'}}V_{NN}^{S_{12}T_{12}}(\mathbf{r}_{12}) + V_{S_{12'}T_{12'}}^{S_{12}T_{12}}(\mathbf{r}_{13})$$
$$+ V_{S_{12'}T_{12'}}^{S_{12}T_{12}}(\mathbf{r}_{23})]R_{S_{12'}T_{12'}}^{JT}(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3) = E'R_{S_{12}T_{12}}^{JT}(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3). \tag{6}$$

Here $\hat{T}_3$ is the kinetic energy operator, the subscripts 1 and 2 refer to the nucleons, the subscript 3 refers to the isobar, S_{12} and T_{12} denote the spin and isotopic spin of the nucleon pair (in accordance with the Pauli exclusion principle $S_{12} = 1 - T_{12}$), and

$$\mathbf{r}_{ik} = \mathbf{r}_i - \mathbf{r}_k, \qquad V_{S_{12'}T_{12'}}^{S_{12}T_{12}}(\mathbf{r}_{ik}) = -\alpha_{S_{12}S_{12'}}^{JT}V_0(r_{ik}). \tag{7}$$

The numerical values of the coefficients α are as follows:

$$\alpha_{11}^{5/2\,3/2} = \alpha_{00}^{3/2\,5/2} = -\frac{1}{6}, \qquad \alpha_{11}^{3/2\,3/2} = \alpha_{00}^{3/2\,3/2} = -\frac{1}{36},$$
$$\alpha_{10}^{3/2\,3/2} = \alpha_{01}^{3/2\,3/2} = -\frac{5}{36}, \qquad \alpha_{11}^{1/2\,3/2} = \alpha_{00}^{3/2\,1/2} = \frac{1}{18}. \tag{8}$$

An attraction between the isobar and the 2N system exists only in the states $J^P = 1/2^+$, $T = 3/2$ and $J^P = 3/2^+$, $T = 1/2$. In these states Eq. (6) is diagonal with respect to the spin and isotopic spin indices.

The nucleon-nucleon attraction is maximal in the state $T = 3/2$, $J^P = 1/2^+$, which corresponds to $S_{12} = 1$, $T_{12} = 0$. In order to understand whether a level with these quantum numbers exists in the $\Delta 2N$ system, let us consider two possible configurations of the particles.

A. Let the average distances between all of the particles in the system be of the same order of magnitude, $\bar{r}_{12} \sim \bar{r}_{23} \sim \bar{r}_{13}$. In this case one can solve the problem by the method of K-harmonics[15]. In the $K = K_{min}$ approximation the Schrödinger equation for A particles reduces to the one-dimensional equation

$$\left\{\frac{d^2}{d\rho^2} - \frac{\mathcal{L}_K^{(A)}(\mathcal{L}_K^{(A)}+1)}{\rho^2} + \frac{2m_N}{\hbar^2} \times [E' - W_K^{(A)}(\rho)]\right\}R_{K_{min}}(\rho) = 0, \tag{9}$$

where

$$\mathcal{L}_K^{(A)} = K_{min} + 3/2(A-2),$$
$$W_K^{(A)}(\rho) = \int d\Omega\, U_{K\gamma}^+\Big(\sum_{i>j} V_{ij}\Big)U_{K\gamma} \tag{10}$$

($W_K^{(A)}(\rho)$ denotes the potential in ρ-space, $U_{K\gamma}$ are the hyperspherical functions, and $d\Omega$ is the volume

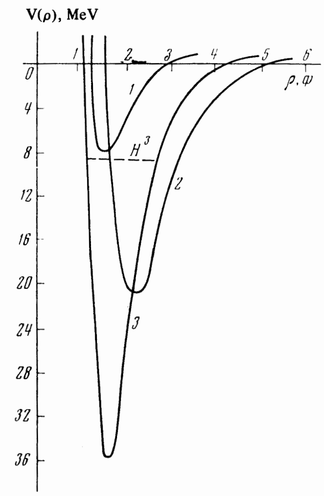

FIG. 2. Plots of the effective potentials for the $\Delta 2N$ (curve 1), $\Delta 3N$ (curve 2), and 3N systems (curve 3). The nucleon-nucleon potential is assumed to have the shape of a rectangular well with the same parameters as used in article [17].

element in the space of $(3A - 4)$ angular variables. For the three-particle system $\rho^2 = \xi^2 + \eta^2$, where

$$\xi = \frac{1}{\sqrt{2}}(\mathbf{r}_1 - \mathbf{r}_2),$$
$$\eta = \sqrt{\frac{2m_\Delta}{2m_N + m_\Delta}}\left(\frac{\mathbf{r}_1+\mathbf{r}_2}{2} - \mathbf{r}_3\right).$$

The effective potential $V_{eff}^{(3)} = W_K^{(3)}(\rho) + V_{\mathcal{L}}^{(3)}(\rho)$, where $V_{\mathcal{L}}^{(A)}(\rho) = \hbar^2\mathcal{L}_K^{(A)}(\mathcal{L}_K^{(A)}+1)/2m_N\rho^2$ is shown in Fig. 2 (curve 1) for the state $T = 3/2$, $J^P = 1/2^+$. By introducing, instead of $V_{eff}^{(3)}(\rho)$, the potential $\bar{V}(\rho)$ which coincides with $V_{eff}^{(3)}(\rho)$ for $\rho \geq \rho_0$ and vanishes for $\rho < \rho_0$ (ρ_0 denotes the point nearest to the origin of coordinates, at which $V_{eff}^{(3)}(\rho_0) = 0$), we only increase the attraction in the system. For the potential $\bar{V}(\rho)$ one can find an upper limit on the number of levels from Bargman's formula:[16]

$$n < I, \text{ where } I = \int_0^\infty r|2m_N\bar{V}(r)/\hbar^2|\,dr = 0.6. \tag{11}$$

Since there is no level in the potential $\bar{V}(\rho)$, there should also not be one in the potential $V_{eff}^{(3)}(\rho)$.

Thus, in the $K = K_{min}$ approximation there is no level in the $\Delta 2N$ system, that is, in this approximation the average potential energy turns out to be smaller than the average kinetic energy $\bar{V} + E_{kin} > 0$. In general, allowance for the next approximations might lead to the appearance of a level; however, for this to happen it would be necessary to increase the effective attraction in the system by at least several times. Thus, for example, if we write down the average potential energy of the system in the form $\bar{V} = \bar{V}_0 + \bar{V}'$, where

$$\bar{V} = \int d^3\mathbf{r}_1\, d^3\mathbf{r}_2\, d^3\mathbf{r}_3\, \delta\left(\frac{\mathbf{r}_1+\mathbf{r}_2+\mathbf{r}_3}{3}\right)\psi^*(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)\,V(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)\,\psi(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)$$

denotes the exact value of the average potential energy,

$$\bar{V}_0 = \int d^3\rho\, \psi^*(\rho)\, V_{eff}^{(3)}(\rho)\,\psi(\rho)$$

denotes the value of the potential energy in the approximation $K = K_{min}$, and $\bar{V}' = \bar{V} - \bar{V}_0$, then in order for a level to appear in the system under consideration, it is necessary that $\overline{V}'$ should exceed $\overline{V}_0$ by approximately a factor of two. But the method of K-harmonics converges very well for configurations in which clustering is not present. A large value of $\overline{V}'$, and consequently a large contribution from the terms with $K > K_{min}$, would imply that a cluster configuration is realized in the system.

B. In the presence of clustering in the $\Delta 2N$ system, two types of configurations are possible: $\overline{r}_{12} \ll \overline{r}_{13}, \overline{r}_{23}$ and $\overline{r}_{12} \gg \overline{r}_{13}$ (or $\overline{r}_{12} \gg \overline{r}_{23}$). The configuration $\overline{r}_{12} \ll \overline{r}_{13}, \overline{r}_{23}$, when the distance between the nucleons is small in comparison with the distance between the isobar and the 2N system, is clearly unfavorable due to the exchange nature of the ΔN interaction. Using the properties of the two-baryon system ΔN considered in Sec. 2, one can easily verify that in the configurations $\overline{r}_{12} \gg \overline{r}_{13}$ or $\overline{r}_{12} \gg \overline{r}_{23}$ it is also impossible to obtain in the $\Delta 2N$ system a binding energy larger than the binding energy of the deuteron. One can understand this from the fact that in any arbitrary state of the ΔN system its average kinetic energy exceeds the average potential energy.

Thus, within the framework of the model under consideration, the attraction in the $\Delta 2N$ system is not sufficiently strong for the formation of a three-baryon resonance.

4. THE Δ3N SYSTEM

When all the internal orbital momenta are equal to zero, the wave function of this system has the form

$$\psi_{JM,Tt}(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4) = \sum \Phi_{1/2\mu_N, 1/2 t_N}(3N)\,\chi_{9/2\mu_\Delta}\, C^{JM}_{1/2\mu_N\, 3/2\mu_\Delta}\, C^{Tt}_{t_N, 3/2 t_\Delta}\, R^{JT}(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4). \quad (12)$$

Here the subscripts 1, 2, 3 pertain to the nucleons, the subscript 4 pertains to the isobar, $\Phi(3N)$ denotes the normalized spin-isospin function of the 3N system with total spin $1/2$ and isotopic spin $1/2$, the summation runs over μ_N and t_N, and

$$\Phi(3N) = \frac{1}{2\sqrt{6}}[(\chi_2\sigma_y\boldsymbol{\sigma}\chi_1)(\boldsymbol{\sigma}\chi_3)(\varphi_2\tau_y\varphi_1)\varphi_3 - (\chi_2\sigma_y\chi_1)\chi_3(\varphi_2\tau_y\boldsymbol{\tau}\varphi_1)(\boldsymbol{\tau}\varphi_3)]. \quad (13)$$

Here φ and χ are two-component spinors, and $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$ are the Pauli matrices.

Using formulas (12) and (13) one can obtain the following equation for the radial wave function:

$$\{\hat{T}_4 + V_{3N}{}^{JT}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) + {}^1/_3\beta_{JT}(V_0(r_{14}) + V_0(r_{24}) + V_0(r_{34}))\} \times R^{JT}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4) = E'R^{JT}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4), \quad (14)$$

where $\beta_{22} = \beta_{12} = \beta_{21} = -1/3$, and $\beta_{11} = 7/27$. From here it follows that an attraction is realized between the isobar and the 3N system only in the state with quantum numbers $T = 1$, $J^P = 1^+$.

Again, just as in Sec. 3, let us first consider that configuration of the particles in which the average distance between all of the particles is of the same order of magnitude, $\overline{\xi}_1 \sim \overline{\xi}_2 \sim \overline{\xi}_3$, where $\xi_1 = (\mathbf{r}_1 - \mathbf{r}_2)/\sqrt{2}$,

$$\xi_2 = \left(\frac{\mathbf{r}_1+\mathbf{r}_2}{2} - \mathbf{r}_3\right)\sqrt{2/3} \qquad \xi_3 = \left(\frac{\mathbf{r}_1+\mathbf{r}_2+\mathbf{r}_3}{3} - \mathbf{r}_4\right)\sqrt{3m_\Delta/(3m_N + m_\Delta)}.$$

Changing over to ρ-space $\rho^2 = \xi_1^2 + \xi_2^2 + \xi_3^2$, we obtain in the $K = K_{min}$ approximation Eq. (10) with $\mathscr{L}_K^{(4)} = 3$ and with the potential energy

$$W_K^{(4)}(\rho) = {}^3/_2(V_{31}(\rho) + V_{13}(\rho)) + \beta_{JT}V_0(\rho), \quad (15)$$

where $V_{31}(\rho)$ and $V_{13}(\rho)$ are the nucleon-nucleon potentials, and $V_0(\rho)$ denotes the exchange potential averaged over the hyperspherical functions $U_{K\gamma}$.

The effective potential $V_{eff}^{(4)}(\rho) = W_K^{(4)}(\rho) + V_{\mathscr{L}}^{(4)}(\rho)$ for the state $T = 1$, $J^P = 1^+$ is shown in Fig. 2 (curve 2). Comparison of this potential with the potential that arises in the problem of the binding energy of the ground state of tritium[17] (see curve 3 in Fig. 2) indicates that the binding energy for the level in the potential $V_{eff}^{(4)}(\rho)$ is smaller than the binding energy of tritium. This solution may correspond to the solution for tritium + a free isobar. However, in contrast to the three-baryon system considered in Sec. 3, in this case a comparatively small change of the effective potential is required for the appearance of a level with a binding energy exceeding the binding energy of tritium, and in general this small change may occur upon taking the higher harmonics into account.

It is clear that owing to the uncertainties indicated at the beginning of Sec. 3, it is still impossible to conclude that the four-baryon resonance $\Delta 3N$ certainly must exist. From the cited estimates one can only conclude that the existence of such a resonance is quite likely.

5. THE Δ4N SYSTEM

For the $\Delta 4N$ system in the S-state, the wave function has the form

$$\psi(\mathbf{r}_1, \ldots, \mathbf{r}_5) = \Phi(4N)\chi_{3/2\mu_\Delta} \quad \varphi_{3/2 t_\Delta} R(\mathbf{r}_1, \ldots, \mathbf{r}_5), \quad (16)$$

where $\Phi(4N)$ denotes the spin-isospin wave function of four nucleons with zero spin and zero isospin:

$$\Phi(4N) = \frac{1}{4\sqrt{6}}[(\chi_2\sigma_y\boldsymbol{\sigma}\chi_1)(\chi_3\sigma_y\boldsymbol{\sigma}\chi_4)(\varphi_2\tau_y\varphi_1)(\varphi_3\tau_y\varphi_4) - (\chi_2\sigma_y\chi_1)(\chi_3\sigma_y\chi_4)(\varphi_2\tau_y\boldsymbol{\tau}\varphi_1)(\varphi_3\tau_y\boldsymbol{\tau}\varphi_4)]. \quad (17)$$

Separating the spin-isospin variables, we obtain an equation of the type (14) for the radial wave function with the potential

$$V_{4N}(\mathbf{r}_1, \ldots \mathbf{r}_4) - \frac{1}{12}\sum_{i=1}^{4} V_0(\mathbf{r}_5 - \mathbf{r}_i),$$

from which it immediately follows that the isobar is repelled from the 4N system.

6. CONCLUSION

Thus, within the framework of the model in which the isobar is regarded as a system consisting of a π meson and a nucleon, the only one, out of all the states with $2 \le B \le 5$, in which the Δ-isobar and the nucleons can be bound in a many-baryon resonance, is the state with quantum numbers $B = 4$, $T = 1$, $J^P = 1^+$ (isohelium according to the terminology proposed in[18]). As already indicated above, the existing experimental data[10] primarily refers to the two-baryon ΔN-system with isotopic spin 1. A detailed experimental investigation of many-baryon systems with other quantum numbers in the region of the Δ-resonance would certainly be of interest with regard to understanding the nature of the interaction of the Δ-isobar with nucleons and nuclei. It is of especial interest, in particular, to investigate the quasi-elastic peaks associated with the scattering of high-energy particles by nuclei, when a small amount of momentum can be transferred to the target nucleus at 300 MeV excitation (see[1]).

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