DEFORMATION MECHANISM OF ELECTROMAGNETIC EXCITATION OF SOUND IN SEMIMETALS

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The deformation mechanism of electromagnetic excitation of sound is investigated under normal skin effect conditions of redistribution of carriers between the valleys. Cases corresponding to various causes of the redistribution arising are considered, e.g., anisotropy of the conductivity tensors of Hall drift in crossed electric and magnetic fields. It is demonstrated that such a mechanism of sound excitation in semimetals should be very effective at low temperatures and, in contrast to the mechanism connected with action of a ponderomotive force, it possesses a strong temperature dependence. An experimental investigation of sound resonances may yield information on the intervalley relaxation times and the recombination rates at the surface.

INTRODUCTION

A large number of recent papers are devoted to direct excitation of acoustic oscillations in metals by an electromagnetic wave incident on the sample surface. In most experiments the sample was in a constant magnetic field $\mathbf{H}_0$ perpendicular to the direction of the skin current $\mathbf{j}$.

In the presence of a field $\mathbf{H}_0$, the effect of sound excitation usually is connected with the action of the ponderomotive force $\mathbf{F}_p = \mathbf{j} \times \mathbf{H}_0/c$. Let us note a characteristic feature of this generation mechanism. If the acoustic wavelength λ is much larger than the depth of the skin layer δ under the conditions of the normal skin effect, then the amplitude of the sound excited by $\mathbf{F}_p$ is determined completely by the skin layer, i.e., by the given value of the magnetic field of the incident wave on the surface H(0), and is independent of the conductivity of the sample. In this case the amplitude of the acoustic resonance, corresponding to establishment of standing acoustic waves in the interior of the plate, is proportional to $H_0^2 Q$, where Q, the acoustic Q of the sample, is the only parameter (other than the elastic moduli) sensitive to the properties of the material.

According to the general theory[1], sound can be excited also by the so-called deformation force $\mathbf{F}_d = \nabla \int d\tau_p f\hat{\Lambda}$ (f is the electron distribution function, $\hat{\Lambda}$ the tensor of the deformation potential, and $d\tau_p = 2h^{-3}d^3p$). The effect due to $\mathbf{F}_d$ depends significantly on the kinetic characteristics of the sample and on the structure of its Fermi surface. Let us consider the case of a Fermi surface consisting of isolated "valleys," which is characteristic of semimetals. The individual valleys will be described by associated distribution functions f^α and deformation potentials $\hat{\Lambda}^\alpha$, with

$$\mathbf{F}_d = \sum_\alpha \nabla \int d\tau_p f^\alpha \hat{\Lambda}^\alpha.$$

Each quantity $\hat{\Lambda}^\alpha$ can be represented in the form of a sum of a part Λ^{α_0} independent of p ($\Lambda_{ik}^{\alpha_0} u_{ik}$ characterizes the deformation shift of the bands whose weak overlap has led to the occurrence of the semimetallic state, Λ^0 is of the order of the width of the band), and a p-dependent part $\hat{\Lambda}^{\alpha_1}(\mathbf{p})$, which characterizes the deformation change of the valley ($\Lambda^{\alpha_1} \sim \epsilon_F \ll \Lambda^{\alpha_0}$). As a result, the deformation force consists of the terms

$$\mathbf{F}_d^0 = \sum_\alpha \Lambda^{\alpha 0} \nabla n^\alpha \qquad \left(n^\alpha = \int d\tau_p f^\alpha\right),$$

$$\mathbf{F}_d^1 = \sum_\alpha \nabla \int d\tau_p f^\alpha \hat{\Lambda}^{\alpha 1}(\mathbf{p}).$$

The force $\mathbf{F}_d^0$ can appear only when loss of equilibrium leads to a change in the concentrations of the electrons in the valleys (without change of the electroneutrality, which calls for conservation of the total concentration[1)]). As is well known[2], in the presence of carrier groups that are weakly coupled by mutual transitions the loss of equilibrium in the system leads to a change in the concentrations in the case when the electromagnetic field forces the carriers to drift normally to the surface of the plate. The reason for the drift may be either the orientation of the valleys or the presence of crossed electric and magnetic fields. The carrier drift leads to a change in their concentrations n^α at the boundaries, a process hindered by the diffusion flux and by the direct intervalley transitions; the condition that these processes must balance each other, expressed by the continuity equation for each type of carrier, makes it possible to determine n^α[2)].

The presence of ∇n^α, as shown in[2-4], strongly influences the effective conductivity of the sample; we shall show below that the force $\mathbf{F}_d^0$ connected with the change of the concentration plays in a number of cases a decisive role in the sound-generation effect. As to the force $\mathbf{F}_d^1$, its contribution is analogous to the contribution of the deformation force in ordinary metals, where no con-

[1)]We disregard insignificant effects connected with the deviation from the electroneutrality conditon.

[2)]We recall that in the usual case of a singly-connected Fermi surface the drift is stopped by the electric field produced in the sample and normal to the surface, whereas in the present situation such a field cannot stop the flow of carriers of all groups.

centration gradients are produced; in a strong magnetic field H_0 (i.e., when $\Omega\tau \gg 1$, where Ω is the cyclotron frequency and τ is the time of the intravalley relaxation), under conditions of the normal skin effect, the role of F_d^1 in the excitation of the sound is negligibly small compared with F_p[5]. We shall consider henceforth only the deformation-force component $F_d^0 = \sum_\alpha \hat{\Lambda}^{\alpha 0}\nabla n^\alpha$ which is typical of semimetals (see also[6]).

We note immediately one obvious qualitative difference between the generation effect due to F_d^0 and the indicated singularity of generation under the influence of F_p when $\lambda \gg \delta$. When the temperature is increased, short-wave phonons capable of greatly decreasing the time of intervalley relaxation T and bring it closer to τ begin to take part in the relaxation processes (for bismuth, e.g., this temperature is $\sim 40°K$). Intense intervalley transitions easily equalize n^α, so that the effectiveness of the force F_d^0 should be greatly reduced. This is one of the reasons why the deformation excitation of sound is temperature dependent, and may permit identification of the deformation mechanism. Other reasons of the temperature dependence will be revealed in the detailed calculation.

Gantmakher and Dolgopolov[7], in experiments on bismuth samples in a field $H_0 \sim 100$ Oe parallel to the surface, at a frequency $\omega \sim 10^6$ Hz (i.e., at $\lambda \gg \delta$), observed for the first time a strong temperature dependence of the amplitude of acoustic resonances (independent measurements have shown that the Q remains constant). Since in ordinary metals no excitation of sound was observed at all under such conditions, this fact calls attention to the deformation mechanism described above, which is peculiar to semimetals. We consider below such a problem for cases corresponding to different causes of the drift, such as Hall drift, and in particular the case of a field H_0 normal to the surface and generation of longitudinal sound (at such a geometry, F_p produces only transverse sound), and drift due to the anisotropy of the conductivity tensors at $H_0 = 0$ and $H_0 \parallel j$.

We consider the case of normal skin effect (the diffusion approximation for the currents). To simplify the problem we disregard the additional change of the distribution functions under the influence of the deformations, i.e., the electronic damping of the sound is disregarded (the damping of the sound will be introduced phenomenologically without specifying the mechanism). This means that the force exciting this sound is determined only by the equations of electrodynamics. Such an approximation makes it possible to find the energy lost by the electromagnetic wave to sound excitation; it can be shown that in an elementary calculation these losses produce resonant increments (when standing waves are established) to the surface impedance, which are usually observed experimentally.

FUNDAMENTAL EQUATIONS

An electromagnetic wave E_x is incident on a semimetal plate $0 < z < d$ normally to the surfaces and causes a change in the electron concentration n^α in the valleys and displacements u in the lattice. The equation for the displacements due to the deformation and ponderomotive forces is written in the form

$$\rho s^2\left(\frac{d^2}{dz^2}+k_s^2\right)u_i = -\sum_\alpha \Lambda_{iz}^\alpha \frac{dn^\alpha}{dz} - \frac{1}{c}[\mathbf{j}\mathbf{H}_0]_i. \qquad (1)^*$$

The elastic moduli are taken for simplicity in the isotropic approximation (s is the speed of sound corresponding to polarization of the oscillations), the time dependence is expressed by the factor $e^{-i\omega t}$, and $k_s = \omega/s$. According to the statements made in the introduction, when determining the right-hand side of (1) we use the electrodynamics equations and the diffusion approximation for the currents[4,8]:

$$\frac{d^2E_x}{dz^2} = -\frac{4\pi i\omega}{c^2}j_x, \qquad \sum_\alpha j_z^\alpha = \sum_\alpha n^\alpha = 0, \qquad (2)$$

$$\frac{1}{e}\frac{dj_z^\alpha}{dz} = \frac{n^\alpha}{T_\alpha'} - \sum_\beta \frac{n^\beta}{T_{\beta\alpha}}, \qquad \frac{1}{T_\alpha'} = -i\omega + \sum_\beta \frac{1}{T_{\alpha\beta}}, \qquad (3)$$

$$j_i^\alpha = \sigma_{ik}^\alpha E_k + eD_{iz}^\alpha \frac{dn^\alpha}{dz}, \qquad D_{iz}^\alpha = \frac{\sigma_{iz}^\alpha}{e^2}\left(\frac{d\mu}{dn_0}\right)_\alpha. \qquad (4)$$

Here $T_{\alpha\beta}$ is the time of relaxation between valleys α and β, σ_{ik}^α is the conductivity tensor of the α-th valley, D_{ik}^α is the diffusion tensor, μ is the Fermi level, and n_0 is the equilibrium concentration. For the sake of uniformity, the continuity equations (3) and the expression for the currents (4) are written for the case of electrons, and reformulation for the case of hole valleys entails no difficulty.

Equations (2) and (3) should be supplemented with boundary conditions spelling continuity of the tangential components of the electric and magnetic fields and with boundary current-balance conditions, which can be written in the form

$$j_z^\alpha = \pm e\sum_\beta S_{\alpha\beta}(n_\beta - n_\alpha), \qquad z = 0, d.$$

Here $S_{\alpha\beta}$ are the weights of surface recombination, and in accordance with[3,4]

$$S_{\alpha\beta} \sim v_F d_{\alpha\beta},$$

where $d_{\alpha\beta}$ is the probability of intervalley scattering in collision of an electron with the surface.

We consider the simplest case, but one retaining all the principal features of the problem, when the concentrations change in only two valleys, with $n_1 = -n_2 = n$. Leaving out the simple intermediate steps, we write down the dispersion equation that follows from (2)—(4)

$$k^4 + k^2(i\delta_0^{-2} - L^{-2}) - iL^{-2}\delta^{-2} = 0,$$

$$L^{-2} = \frac{1+q}{T'D_{zz}b_{zz}}, \qquad \delta^{-2} = \frac{4\pi\omega}{c^2}\tilde{\sigma}_{xx},$$

$$\delta_0^{-2} = \delta^{-2}\left[1 - \frac{\sigma_{zx}^{(1)}\sigma_{xz}}{\sigma_{zz}^{(1)}\tilde{\sigma}_{xx}}\left(1 - \frac{b_{xz}}{b_{zz}}\right)\right], \qquad \sigma = \sum_\alpha \sigma^\alpha, \qquad (5)$$

$$\tilde{\sigma}_{xx} = \sigma_{xx} - \frac{\sigma_{xz}\sigma_{zx}}{\sigma_{zz}}, \qquad q = \left(\frac{d\mu}{dn_0}\right)_1\left(\frac{d\mu}{dn_0}\right)_2^{-1},$$

$$b_{ik} = 1 - \frac{\sigma_{ik}^{(1)} - q\sigma_{ik}^{(2)}}{\sigma_{ik}}.$$

Solving the boundary-value problem, we can express the amplitudes of the fields and the concentrations in terms of the values of the alternating magnetic field on the

*$[\mathbf{j}\mathbf{H}_0] \equiv \mathbf{j} \times \mathbf{H}_0$.

surface of the sample. We confine ourselves to the case when $H(0) = H(d)$, with $E_x(0) = -E_x(d)$ and $n(0) = n(d)$. We present the subsequently-needed result for the concentration:

$$n(z)=\sum_{i=1}^{2} f_i \frac{\operatorname{ch} k_i(z-d/2)}{\operatorname{ch}(k_i d/2)}$$
$$f_{1,2}=\mp\frac{i\omega e}{c}\left(\frac{dn_0}{d\mu}\right)_1 \frac{\sigma_{zx}^{(1)}L^2P_{1,2}(1+P_{2,1})H(0)}{\sigma_{zz}^{(1)}b_{zz}g}, \quad (6)$$
$$g=(k_1^2L^2-1)(1+P_2)P_1-(k_2^2L^2-1)(1+P_1)P_2,$$
$$P_{1,2}=\frac{2TSk_{1,2}}{1+q}\operatorname{cth}\frac{k_{1,2}d}{2},$$

k_i are the roots of the dispersion equation (5).

We now proceed to the problem of sound generation. The amplitude of the acoustic wave is given by the solution of Eq. (1) with boundary conditions that follow from the continuity of the total momentum flux density (for an unconstrained sample). Using the expression given in[1] for the momentum flux density, we obtain

$$du/dz=\Phi \quad \text{for} \quad z=0,d,$$
$$\Phi(z)=-\frac{1}{\rho s^2}\sum_{i\alpha}\Lambda^\alpha n^\alpha(z). \quad (7)$$

We note that the inhomogeneous boundary condition holds only in the presence of a deformation force. The expression that follows from (1) and (7) for the displacement u_d excited only by the deformation force is

$$u_d(z)=\frac{\cos k_s z}{\sin k_s d}\int_z^d dt\,\Phi(t)\sin k_s(t-d)+\frac{\cos k_s(z-d)}{\sin k_s d}\int_0^z dt\,\Phi(t)\sin k_s t. \quad (8)$$

Let us find the energy lost by the electromagnetic wave to excitation of the oscillations u_d. To this end it is necessary to calculate the change of the elastic energy of the plate per unit time:

$$\dot w_r=\frac{d}{dt}\int_0^d dz\frac{\rho}{2}\left[\dot u_n^2+s^2\left(\frac{du}{dz}\right)^2\right]=\frac{\rho s^2}{2}\operatorname{Re}\int_0^d dz\frac{du_d}{dz}\Phi^*. \quad (9)$$

The term $\sin k_s d$ in the denominator of (8) describes the resonances when standing waves are established. Under the excitation conditions described here, only resonances with odd numbers of acoustic half-waves remain (after the calculations the denominator is left with $\cos(k_s d/2)$).

As already noted in the introduction, we shall take the sound damping into account phenomenologically, i.e., we represent k in the form

$$k_s=k_s'+ik_s''=\omega s^{-1}(1+iQ^{-1}),$$

where Q is the acoustic quality factor. Performing the integration in (9) with the aid of (6) and (8), we obtain the energy loss under the resonance conditions, i.e., at $k_s'd = (2N + 1)\pi$ (N are integers), in the form

$$\dot w_d=\frac{Q}{(N+1/2)\pi}s\frac{w_0}{4\pi\rho s^2}\left|\frac{(\Lambda_1-\Lambda_2)cT'}{e(1+q)}k_s^2\frac{\sigma_{zx}^{(1)}}{\bar\sigma_{xx}}\cdot\frac{k_1^2k_2^2(1-G_1)}{(k_1^2+k_s^2)(k_2^2+k_s^2)}\right|^2$$
$$w_0=\frac{|H(0)|^2}{4\pi},\quad G_1=(P_1-P_2)L^2(k_s^2-i\delta_0^{-2})g^{-1}. \quad (10)$$

We present an expression for the energy loss $\dot w_p$ when sound is excited only by the ponderomotive force; the calculations are analogous to those given above (the boundary condition for du/dz is homogeneous):

$$\dot w_p=\frac{Q}{(N+1/2)\pi}sw_0\alpha^2|1-G_2|^2,\quad \alpha^2=\frac{H_0^2}{4\pi\rho s^2},$$
$$G_2=\frac{k_s^2}{(k_1^2+k_s^2)(k_2^2+k_s^2)g}[(k_s^2+k_2^2)(k_1^2L^2-1)P_1(1+P_2)-(k_s^2+k_1^2)(k_2^2L^2-1)P_2(1+P_1)]. \quad (11)$$

PARTICULAR CASES

We proceed to consider concrete examples of the application of the results.

1. Strong Magnetic Field Parallel to the Surface

A. We analyze first the case $\mathbf{H}_0 \perp \mathbf{j}$, when both F_p and F_d act. In such a geometry, a sufficiently good approximation is obtained by taking into account the conductivity anisotropy introduced only by the field H_0, neglecting the weaker effects connected with the orientation of the valleys. We can therefore use for simplicity the model of a semimetal with two spherical valleys—electron (n) and hole (p). The parameters in (5) are

$$L^2=\frac{l_n^2T'}{3\tau_n}\frac{\gamma_n^2\gamma_p}{\gamma_n+\gamma_p},\quad \delta_0^{-2}=\frac{4\pi\omega}{c^2}(\sigma_{0n}+\sigma_{0p}),$$
$$\delta^{-2}=\frac{4\pi\omega}{c^2}\sigma_{0n}\gamma_n^2\left(1+\frac{\gamma_p}{\gamma_n}\right)\ll\delta_0^{-2},$$

l_n is the electron mean free path, σ_0 the conductivity at $H_0 = 0$, and it is assumed that

$$\gamma_{n,p}=\Omega_{n,p}^{-1}\tau_{n,p}^{-1}\ll 1.$$

Taking the inequality $\delta \gg \delta_0$ into account, the roots of the dispersion equation (5) can be written in the form

$$k_1^2=L^{-2}-i\delta_0^{-2},$$
$$k_2^2=-i\delta^{-2}(1-iL^2\delta_0^{-2})^{-1}.$$

We present first the results for acoustic waves whose length exceeds the damping length of the fields and the concentrations, i.e., at $k_s \ll k_{1,2}$. The energy loss $\dot w_d$ is in this case

$$\dot w_d=\dot w_pA^2,$$
$$A=\frac{(\Lambda_n-\Lambda_p)T'l_n^2k_s^2|1-G_1|}{2\mu_n(1+\gamma_p\gamma_n^{-1})(1+m_nm_p^{-1})\tau_n}. \quad (12)$$

Here $\dot w_p$ in the energy lost to excitation of sound as a result of the ponderomotive force; in this case we have in (11) $G_2 \ll 1$ regardless of the values of the parameters $P_{1,2}$. Since in semimetals $\Lambda \gg \mu$ and at low temperatures $T \gg \tau$ (in Bi we have $\Lambda/\mu \sim 10^2$[9] and $T/\tau \sim 10^2$ at helium temperatures[10]), a large value $A^2 \gg 1$ (i.e., a predominant role of the deformation force in the sound excitation) can be ensured with a large margin. The temperature and field dependences of the effect are contained in the factor $H_0^2T^2\tau_n^2|1-G_1|^2$; they are different for strong ($P_{1,2} \gg 1$) and weak ($P_{1,2} \ll 1$) intervalley scattering from the boundary: when $P \gg 1$ we have $G_1 \ll 1$; when $P \ll 1$ we have

$$1-G_1=\frac{\delta_0^2-iL^2}{\delta_0^2-iL^2k_1k_2^{-1}}\quad (k_2<\gamma k_1\ll k_1).$$

It should be noted that the values of $\mathbf{P}_{1,2}$, in accordance with (6), are determined both by the velocity of the intervalley surface recombination S, which is connected with the properties of the surface, and by the values of T and $k_{1,2}$, which depend on the temperature and H_0. Therefore at a finite value of S the change of the field and of the temperature can, in principle, be accomplished by changing over from one limiting case to the other.

Definite interest attaches also to the case of short acoustic waves, when the opposite inequality $k_s \gg k_{1,2}$ is realized. This can occur at large numbers, $N \gg 1$, of the acoustic resonance. According to (11), the effectiveness of the ponderomotive force then decreases sharply. We present the result for $\dot{w}_d$, corresponding to weak scattering by the boundaries ($P_{1,2} \ll 1$):

$$\dot{w}_d = \frac{Q}{(N+{}^1/_2)\pi} s w_0 \alpha^2 \left[\frac{3}{2}\frac{\Lambda_n - \Lambda_p}{\mu_n(1+m_n/m_p)}\right]^2, L\delta \gg \delta_0^2. \qquad (13)$$

It follows therefore that a decrease in the sound wavelength to values that are small compared with the thickness of the skin layer can be accompanied by a very appreciable absorption proportional to H_0^2 and independent of the temperature. In the case of effective intervalley scattering, when $P_{1,2} \gg 1$, the value of $\dot{w}_d$ decreases significantly in comparison with (13).

B. We turn now to the case $H_0 \parallel j$, when $F_p = 0$. The drift fluxes normal to the boundary appear in such a geometry only as a result of the anisotropy of the valley conductivity tensors. An analysis, which will not be prepresented here, shows that the deformation mechanism considered by us has little effect: $\dot{w}_d$ decreases in comparison with (12) and (13) by a factor $\gamma^{-4} \gg 1$.

2. STRONG MAGNETIC FIELD NORMAL TO THE SURFACE

Under these conditions one usually considers excitation of transverse sound by the force F_p. We shall discuss excitation of longitudinal sound by the force F_d. We consider the simplest ellipsoidal model describing the electronic structure of bismuth. Let the axis C_3 be directed along x (parallel to the applied field E_x), and C_2 along y. Calculation shows that the influence of the resultant Hall field E_y can be neglected; in this case the dispersion law is given by (5) and formula (10) is valid. Using the results of[11] for the conductivity tensor of the electron ellipsoid in a magnetic field, we obtain the values of the parameters in (5) and (10):

$$L^2 = \frac{Tl_n^2}{6\tau_n}\frac{\varepsilon_1\varepsilon_3}{\varepsilon},$$

$$\delta^{-2} = \frac{4\pi\omega}{c^2}\sigma_{xx}, \quad \delta_0^{-2} = \delta^{-2}\left[1 + \frac{3}{8}\frac{\sigma_n\gamma_n^2(\varepsilon_1-\varepsilon_3)^2}{\sigma_{xx}\varepsilon\varepsilon_1\varepsilon_3}\right],$$

$$\sigma_{xx} = \tilde{\sigma}_{xx} = \sigma_n\gamma_n^2\left(\frac{2}{\varepsilon} + \frac{1}{\varepsilon_1}\right) + \frac{\sigma_p\gamma_p^2}{\varepsilon_{1p}},$$

$$\sigma_{zx}^{(1)} = \frac{\sqrt{3}}{4}\frac{\varepsilon_1 - \varepsilon_3}{\varepsilon}\sigma_n\gamma_n.$$

Here $l_n^2 = 2\mu_n\tau_n^2/m$ and $\epsilon_i = m/m_i$, where m is the mass of the free electron and m_i are the principal values of the effective-mass tensor (we assume that $m_1 = m_2$),

$$\varepsilon = {}^1/_4\varepsilon_1 + {}^3/_4\varepsilon_3,$$

$$\sigma_n = \frac{e^2 n_0 \tau_n}{m}, \quad \sigma_p = \frac{3n_0e^2\tau_p}{m}, \quad \gamma_{n,p} = \frac{cm}{eH_0\tau_{n,p}}.$$

We note that, unlike in Sec. 1, here L does not depend on the magnetic field but δ_0 does ($\sim H_0$). Using the presented values of the parameters in (10), we obtain for $\dot{w}_d$ values that practically coincide with (12) and (13) in the corresponding cases.

Analogous results are obtained also for excitation of transverse sound in the deformation mechanism.

3. The Case $H_0 = 0$

The greatest deviation from the examples considered above is due to the ratio $\sigma_{zx}^{(1)}/\tilde{\sigma}_{xx}$ in (10): in a strong magnetic field under conditions of Secs. 1A and 2, this ratio is $\sim \gamma^{-1} \gg 1$, whereas for $H_0 = 0$ it is determined entirely by the anisotropy of the conductivity tensor. It is therefore clear that excitation of sound at $H_0 = 0$ is as a rule a much smaller effect than in a strong field (with the exception of the case $\mathbf{H}_0 \parallel \mathbf{j}$ considered in Sec. 1B). A characteristic feature here is the appreciable dependence on the orientation of the crystallographic axes.

By way of an example, we consider the simplest model of a Fermi surface[8], namely two mutually-perpendicular electron ellipsoids lying in the xz plane, with one of them inclined at an angle ψ to the normal. Then

$$L^2 = \frac{Tl_n^2}{3\tau}\frac{\varepsilon_1\varepsilon_3}{\varepsilon_1+\varepsilon_3}\varepsilon_0^2,$$

$$\delta^{-2} = \frac{4\pi\omega}{c^2}\sigma_{xx}, \quad \delta_0 = \delta\varepsilon_0,$$

$$\sigma_{xx} = \tilde{\sigma}_{xx} = \sigma_n(\varepsilon_1 + \varepsilon_3),$$

$$\sigma_{zx}^{(1)} = -\sigma_n(\varepsilon_1 - \varepsilon_3)\sin\psi\cos\psi.$$

Here

$$\varepsilon_0^2 = 1 + \frac{(\varepsilon_1-\varepsilon_3)^2}{\varepsilon_1\varepsilon_3}\sin^2\psi\cos^2\psi$$

and the remaining symbols are the same as in Sec. 2 above.

The analysis becomes simpler in the case of sharp anisotropy of the effective-mass tensor, as is the case, for example, in bismuth, where $\epsilon_1/\epsilon_3 \sim 10^2$. Then, at angles $\psi \sim \pi/4$ the anisotropy factor is $\epsilon_0^2 \gg 1$. The roots of Eq. (5) are then

$$k_1^2 = \begin{cases} L^{-2} + i\delta^{-2} \\ \varepsilon_0^2 L^{-2} \end{cases}, \quad k_2^2 = \begin{cases} -i\delta^{-2}, & L \ll \delta \\ -i\delta_0^{-2}, & L \gg \delta_0\varepsilon_0 \end{cases}.$$

Calculations lead to the following expression for $\dot{w}_d$ in the limiting case of long sound waves ($k_s \ll k_{1,2}$):

$$\dot{w}_д = \frac{Q}{(N+{}^1/_2)\pi} s w_0 \frac{nm_1c^2}{\rho s^2\omega_1^2\tau^2}A_0^2,$$

$$A_0 = \frac{3}{4}\frac{\Lambda_1 - \Lambda_2}{\mu}L^2k_s^2\frac{(\varepsilon_1-\varepsilon_3)\sin\psi\cos\psi}{\varepsilon_3\varepsilon_0^2}|1 - G_1| \qquad (14)$$

Here $\omega_1 = (4\pi ne^2m_1^{-1})^{1/2}$ is the plasma frequency pertaining to the mass m_1; for $P_{1,2} \gg 1$ we have $G_1 \ll 1$, for $P_{1,2} \ll 1$ with $L \ll \delta$ we have $G_1 \ll 1$, and at $L \gg \delta_0\epsilon_0$

$$1 - G_1 = \frac{\delta_0^2\varepsilon_0^2 + iL^2}{\delta_0^2\varepsilon_0^2 + iL^2k_2k_1^{-1}}.$$

For resonances with $N \gg 1$, for which $k_s \gg k_{1,2}$, we have in (14)

$$A_0 = \frac{3}{4}\frac{\Lambda_1 - \Lambda_2}{\mu}\frac{(\varepsilon_1-\varepsilon_3)\sin\psi\cos\psi}{\varepsilon_3\varepsilon_0^2}\cdot\begin{cases} L\delta^{-1}, & L \ll \delta \\ \varepsilon_0^2, & L \gg \delta_0\varepsilon_0, \end{cases}$$

which is valid for weak intervalley scattering by the boundary ($P_{1,2} \ll 1$), and at $P_{1,2} \gg 1$ the effect decreases sharply.

CONCLUSION

The results obtained above allow us to assume that the deformation mechanism of sound excitation in semimetals is quite effective. In this connection, experimental investigations of acoustic resonances can be a new method for measuring such parameters as the times of intervalley relaxation, the rate of surface recombination, etc. Data on these parameters can be obtained by measuring the temperature dependence of the effect. For example, an analysis of the results of [7] shows that in bismuth, apparently, the case of strong intervalley scattering on the surface is realized, when according to (12) $\dot{w}_d \sim T^2\tau^2$; an analysis of the temperature dependence of T and τ allows us to expect a change in the effect by an approximate factor of 10 in the temperature interval from 2 to 4°K, which is close to the value measured in [7].

We stop in conclusion to consider the "usual" deformation mechanism connected with the force discussed in the introduction:

$$\mathbf{F}_d{}^1 = \nabla \int d\tau_p f \hat{\Lambda}_1(\mathbf{p}).$$

If we use the expression for the deformation potential of free electrons[12] $\Lambda_{ik} = -mv_iv_k$, then, multiplying the kinetic equation by v_i and integrating with respect to the momenta, we find that in the absence of a magnetic field

$$F_{di}{}^1 = en\left(E_i - \frac{j_i}{\sigma_0}\right), \quad \sigma_0 = \frac{e^2n\tau}{m}$$

(the collision integral is taken in the τ-approximation). Such an expression for the force was obtained in[13] in a less formal manner and was used in the analysis of the sound-generation effect. The results for w_d, obtained in Sec. 3 above, differ from the losses due to the force F_d^1, particularly in the large dimensionless factor $(\Lambda T/\mu\tau)^2$, which is typical of metals and determines actually the absolute magnitude of the effect.

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