

ELECTRON-INERTIAL EXPERIMENTS

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The role of metal deformation in electron-inertial experiments is discussed. The role of deformations should have been insignificant in those experiments which have been carried out hitherto, and this seems to be borne out by the experiments. Under certain conditions, however, it may be important to take deformations into account.

IN the usual electron-inertial experiments one either measures the current produced in the circuit when a conductor constituting part of this circuit is accelerated (the Tolman-Stewart effect) or determines the acceleration of the conductor when the current flowing through it is varied (see the review<sup>[4]</sup>, and also<sup>[2]</sup>). It is convenient to express the results of the experiments in terms of the "extraneous" field  $E^{el}$  connected with the acceleration  $a$  of the conductor. From very general considerations it follows that, regardless of the effective mass of the carriers in the conductor and of the sign of the Hall effect (i.e., whether the conductivity is produced by electrons or holes) etc., the field is given by

$$E^{el} = ma/e, \tag{1}$$

where  $m$  is the mass and  $-e$  is the charge of the free electron (see<sup>[3]</sup> and the literature cited therein). This conclusion is fully confirmed by measurements<sup>[4]</sup> within the limits of the accuracy attained in them, which unfortunately is only of the order of 1%.

The result (1) is obtained, however, without any allowance for the influence of the acceleration on the lattice of the conductor. Yet the acceleration of the conductor causes it to become deformed, just as it becomes deformed under the influence of the force of gravity. Recently a number of authors have shown that deformation of a metal in a gravitational field produces in the metal an electric field whose absolute magnitude is of the order of  $Mg/e$  ( $M$  is the mass of the nucleus of the metal atom and  $g$  is the free-fall acceleration), i.e., larger by 4-5 orders of magnitude than the field  $mg/e$  that would exist in the absence of deformation (see<sup>[5,6]</sup>). This means that when a conductor is accelerated there should appear in it, generally speaking, fields  $\sim Ma/e$  exceeding  $E^{el}$  by many times. This raises the question of the degree to which formula (1) is valid for the description of electron-inertial experiments.

To answer this question, let us analyze the expression for the current density in a normal (nonsuperconducting) metal with allowance for its acceleration and deformation. This expression can easily be derived by starting from the kinetic equation obtained by many authors for electrons (see<sup>[7,8]</sup> and the literature cited there):

$$j_i = \sigma_{ij} \left( E_j + \frac{1}{e} \frac{\partial}{\partial x_j} \bar{\lambda}_{kl} u_{kl} + E_j^{el} \right) + \Gamma_{i\mu l} \frac{\partial \dot{u}_{kl}}{\partial x_j}. \tag{2}$$

Here  $E$  is the electric field,  $u$  the lattice displacement vector,  $u_{kl}$  the deformation tensor, and  $E^{el} = m\ddot{u}/e \equiv ma/e$  the extraneous Tolman-Stewart field; the superior dot denotes differentiation with respect to time,  $\sigma_{ijk}$  is the electric conductivity tensor, and  $\bar{\lambda}_{kl}$  is the value of the deformation potential  $\lambda_{kl}(\mathbf{p})$ , which describes the interaction of the electron with the deformation, averaged over the Fermi surface. In addition,

$$\Gamma_{i\mu l} = e \int \frac{dS_p}{4\pi^3 \hbar} \frac{v_i v_j}{v} \tau_p^2 \Lambda_{kl}(\mathbf{p}), \tag{3}$$

where  $\mathbf{v}(\mathbf{p})$  is the velocity of an electron with quasi-momentum  $\hbar\mathbf{p}$ ,  $\tau_p$  is the momentum scattering time,  $\Lambda_{kl}(\mathbf{p}) = \lambda_{kl}(\mathbf{p}) - \bar{\lambda}_{kl}$ , and the integration in (3) is carried out over the Fermi surface. The tensor  $\Gamma_{ijkl}$  is symmetrical against permutation of the first two indices, and also against permutation of the last two indices. In an isotropic medium there are two independent components of the tensor,  $\Gamma_{1111}$  and  $\Gamma_{1212}$ .

In the derivation of formula (2) for the current it was assumed that the deformation of the conductor has, first, a low frequency (the characteristic frequency  $\omega$  is much lower than the electron collision frequency  $\tau_p^{-1}$ ) and, second, a large wavelength (the characteristic scale of variation in space  $k^{-1}$  is much larger than the mean free path  $l = v_F \tau_p$ , where  $v_F$  is the velocity on the Fermi surface). Therefore the use of expression (2), generally speaking, does not suffice when one considers such an interesting case as the appearance of a current when a shock wave passes through a metal. In addition, only quantities linear in the deformation have been retained in (2). Finally, we confine ourselves to media having a symmetry center.

In the electron-inertial experiments of interest to us, the deformation is produced, in final analysis, by acceleration of the conductor, and must be determined from the elasticity-theory equations. The field  $e^{-1} \nabla(\bar{\lambda}_{kl} u_{kl})$  in (2) can readily be estimated to be of the order of  $Ma/e$ , i.e., much larger than the field  $E^{el}$ . However, the field  $e^{-1} \nabla(\bar{\lambda}_{kl} u_{kl})$  does not make any contribution to the current in the sense that, unlike the Tolman-Stewart field, it produces no emf in the circuit, owing to its potential character: it is cancelled out in the metal by the potential electric field.

Before we estimate the last term in (2)<sup>11</sup>, we recall that in the original Tolman-Stewart experiment they measured the total charge, i.e.,  $\int_{-\infty}^{+\infty} dt I(t)$ , flowing through the circuit during the deceleration of a circular ring. In other electron-inertial experiments one measures the alternating current produced in a ring (coil) executing torsional oscillations or, conversely, one observes the buildup of torsional oscillations of a coil when the current flowing through it is periodically reversed (see<sup>[1]</sup>). Thus, in all the experiments performed to date, one observes an effect of uneven rotation of the ring. When a circular ring is unevenly rotated, there is produced in it, besides the radial deformation  $u_{\rho\rho}$  proportional to the square of the angular velocity  $\dot{\Omega}$ , also a deformation  $u_{\rho\varphi}$ , since the elastic displacement  $u_{\varphi}$  in the direction perpendicular to the radius depends nonlinearly on the distance  $\rho$  to the axis. This deformation is proportional to  $\dot{\Omega}$ , and one should therefore expect the current density produced by it to be proportional to  $\dot{\Omega}$ . Indeed,

$$\left( \Gamma_{i\varphi i} \frac{\partial \dot{u}_{\varphi i}}{\partial x_j} \right)_{\varphi} = \Gamma_{i2i2} \frac{\partial}{\partial \rho} \frac{1}{\rho} \frac{\partial (\rho \dot{u}_{\varphi})}{\partial \rho}.$$

From the equation of the corresponding elasticity-theory problem it follows that for not too thick a ring

$$\frac{\partial}{\partial \rho} \frac{1}{\rho} \frac{\partial (\rho u_{\varphi})}{\partial \rho} = \frac{\rho \dot{\Omega}}{s_t^2}$$

where  $s_t$  is the velocity of the transverse sound. The ring current of interest to us, neglecting self-induction, is equal to (the first term is the Tolman-Stewart current and  $V$  is the volume of the ring)

$$I = \frac{\sigma V}{2\pi} \left[ \frac{m}{e} \dot{\Omega} + \frac{\Gamma_{i2i2}}{\sigma s_t^2} \ddot{\Omega} \right]. \quad (4)$$

Obviously, the deformation makes no contribution whatever to the total charge flowing through the circuit during the entire deceleration time, i.e., it does not influence the result of the Tolman-Stewart experiment. Since the nonzero components of the tensor  $\Gamma_{ijkl}$  are of the order of  $\sigma \tau_p \Lambda / e \sim \sigma \tau_p m v_F^2 / e$ , the ratio of the instantaneous values of the current produced by the deformation and the Tolman-Stewart current is of the order of

$$\mu = \left| \frac{e \Gamma_{i2i2} \ddot{\Omega}}{m \sigma s_t^2 \dot{\Omega}} \right| \sim \omega \tau_p \frac{v_F^2}{s_t^2} \sim \omega \tau_p \frac{M}{m},$$

where  $\omega$  is the characteristic frequency of variation of  $\dot{\Omega}$  and  $\ddot{\Omega}$ . In spite of the very large value of  $M/m$ , the entire ratio  $\mu$  is in practice always very small, owing to the smallness of  $\omega \tau_p$ .

One can conceive, however, of another formulation of the electron-inertial experiment: a conductor moving linearly with a certain velocity  $v_0$  is sharply decelerated (experiences impact), and the current produced thereby is short-circuited by the immobile part of the circuit. What is measured is the instantaneous value of the current (and not only the integral of the current with respect to time).

As is well known, a deformation wave with a steep front is produced in a rod striking a partition and moves away from the point of impact. For simplicity we assume the deformation to be one-dimensional. Then, as seen from (2), the last term makes no contribution to the emf (since the deformation does not vary in time far from the deformation front) if the electric conductivity and the tensor  $\Gamma_{ijkl}$  do not depend on the coordinates in the region where the deformation changes, i.e., if the conductor is homogeneous. The deformation emf is produced when the wave passes through the region of inhomogeneity of the material. Let us assume that the dimension of the deformation wave front is smaller than the length  $L$  of this region. Then, if the change of the quantity  $\Gamma_{1111}/\sigma$  is of the order of the quantity itself, the ratio of the deformation emf to the emf produced by the field  $\mathbf{E}^{e1}$  is of the order of

$$\left| \frac{\Gamma_{1111}}{\sigma L} \right| \frac{e}{m s} \sim \frac{v_F}{s} \frac{l}{L},$$

since  $e \Gamma_{1111} \sim \sigma \tau_p m v_F^2$  and  $|\dot{u}_{ij}/\dot{u}_i| \sim k/\omega \sim s^{-1}$ , where  $s$  is the speed of sound. This ratio may turn out to be not small and consequently, in the general case, electron-inertial experiments should not be regarded as trivial in the sense that they do not always lead to a determination (which is unnecessary in this case) of the ratio  $e/m$  for the free electron. At the same time it is understandable why in the already performed electron-inertial experiments<sup>[1,4]</sup>, within the limits of the attained accuracy, everything reduces to the need for taking the Tolman-Stewart extraneous field into account. It is curious that inside a metallic tube, under the influence of the force of gravity (with acceleration  $g$ ), there is likewise observed<sup>[10]</sup> only the field  $\mathbf{E} = -\mathbf{E}^{e1} = mg/e$ , although the reason for the cancellation of the electric field produced by the deformation of the metal in the gravitational field is still not sufficiently clear (see<sup>[11]</sup>).

<sup>1</sup>S. J. Barnett, Rev. Mod. Phys., 7, 129 (1935).

<sup>2</sup>L. D. Landau and E. M. Lifshitz, *Elektrodinamika sploshnykh sred* (Electrodynamics of Continuous Media), Gostekhizdat, 1957, Sec. 50 [Addison-Wesley, 1959].

<sup>3</sup>V. L. Ginzburg, *Sbornik statei pamyati A. A. Andronova* (Collection of Papers in Memory of A. A. Andronov), AN SSSR, 1955, p. 622.

<sup>4</sup>C. F. Kettering, and G. G. Scott, Phys. Rev., 66, 257 (1952), 1944. G. G. Scott, Phys. Rev., 83, 656, (1951). S. Brown and S. J. Barnett, Phys. Rev., 87, 601 (1952).

<sup>5</sup>A. J. Dessler, F. C. Michel, H. E. Rorschach, and G. T. Trammel, Phys. Rev., 168, 737 (1968).

<sup>6</sup>C. Herring, Phys. Rev., 171, 1361 (1968).

<sup>7</sup>V. M. Kontorovich, Zh. Eksp. Teor. Fiz. 45, 1638 (1963) [Sov. Phys.-JETP 18, 1125 (1964)].

<sup>8</sup>I. G. Lang and S. T. Pavlov, Fiz. Tverd. Tela 12, 2412 (1970) [Sov. Phys.-Solid State 12, 1926 (1971)].

<sup>9</sup>M. A. Leontovich and V. D. Khait, ZhETF Pis. Red. 13, 579 (1971) [JETP Lett. 13, 414 (1971)].

<sup>10</sup>F. C. Witteborn and W. M. Fairbank, Phys. Rev. Lett., 19, 1049, 1967; Nature, 220, 436 (1968).

<sup>11</sup>L. I. Schiff, Phys. Rev., B1, 4649 (1970).

<sup>1</sup>We note that this term in the current density was examined recently by Leontovich and Khait<sup>[9]</sup>. They drew attention to the possibility of measuring the electric and magnetic fields excited by this current in transverse-sound-propagation experiments.