LOW BINDING ENERGY PARTICLE IN CROSSED ELECTRIC AND MAGNETIC FIELDS

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Submitted March 17, 1971

Zh. Eksp. Teor. Fiz. 61, 956–967 (September, 1971)

The energy and decay probability of a particle in a short-range force field and in crossed electric and magnetic fields are calculated. The electric field is considered to be small compared with the atomic field. With increase of the magnetic field strength, the decay probability decreases and vanishes at a certain value of the magnetic field strength. The pre-exponential factor, which describes how the decay probability vanishes, is calculated. Ionization in the case of an alternating magnetic field is also considered. In weak electric and magnetic fields the energy level shift in a δ well is additive with respect to the fields. The electric field decreases the particle binding energy.

INTRODUCTION

THE present paper is devoted to consideration of a particle with low binding energy in perpendicular and homogeneous electric and magnetic fields. Such a problem arises, for example, in the study of ionization of highly-excited atoms or negative ions moving in a strong magnetic field. In a coordinate system connected with the moving particle, there will exist besides the magnetic field also an electric field, perpendicular to the latter and smaller in magnitude. Under the influence of the electric field, the electron can become detached by tunneling through the potential barrier. This process has come to be known in the literature as Lorentz ionization. A review of earlier papers on Lorentz ionization and an indication of possible technical applications of this process can be found in[1]. In theoretical papers devoted to Lorentz ionization (see[2]), account was taken only of the electric field that appears in the coordinate frame connected with the particle; the magnetic field was not taken into account. Under certain conditions, however (for example, at low velocities), it is not permissible to neglect the magnetic field. As will be shown below, the magnetic field can lead to a complete stabilization of the particle when tunneling is impossible.

Another example of the problem in question is the processes of tunneling in semiconductors in crossed external and magnetic fields investigated by Aronov and Pikus[3]; these processes can be influenced by weakly-bound excitons or impurity states. Unlike the Lorentz ionization, a case in which the electric field exceeds the magnetic one can occur here.

The problems analyzed in the zero-radius-potential approximation, in which the successive consideration of the wave function in the potential well is replaced by the boundary condition

$$\Psi = \frac{1}{4\pi}\left(\frac{1}{r} - \frac{1}{c}\right) \text{ as } r \to 0, \tag{1}$$

where $a = 1/\alpha_0$ is the scattering length of the particle and determines its binding energy in the δ well in the absence of the electric and magnetic fields. For a particle with mass m this energy is equal to $E_0 = -\hbar^2\alpha_0^2/2m$.

We note that the condition (1) is valid in our case of a homogeneous magnetic field whose vector potential is equal to zero at the point where the δ potential is concentrated ($r = 0$). The general case of a vector potential that differs from zero at this point was considered by Adamov, et al.[4], who also gave the corresponding boundary condition.

In such an analysis, the problem actually reduces to determination of the Green's function of the corresponding equation. This method was employed earlier[5,6] in the study of a particle with small binding energy in an electric or magnetic field. The present paper is a continuation of[5,6]. In Sec. 1 we derive an equation for the determination of the particle energy. In Sec. 2 we consider the case of electric and magnetic fields when decay is possible and the state of the particle in the well is quasistationary, while the ratio of the fields is such that the system is far from complete stabilization. We calculate the level shift and its width as functions of the fields. The level shift and its width are calculated by us in a unified fashion as the real and imaginary parts of the integral in (4). The level shift is additive with respect to the fields. The ionization is considered for an electric field that is small compared with the atomic field. This case is important also because a weak electric field broadens the levels insignificantly and makes them amenable to spectroscopic observation in semiconductors[7]. We calculate the dependence of the ionization probability on the magnetic field as a consequence of its influence on the exponential factor as well as on the pre-exponential term. The magnetic field stabilizes the system and decreases the decay probability, since by bending the trajectory it increases by the same token its sub-barrier part, leading to a growth of the effective thickness of the barrier.

In Sec. 3 we consider the opposite case of a stationary level far from decay possibility. We calculate its position in crossed fields. In particular, when the magnetic field is a perturbation relative to the field of the δ potential, the level shift turns out to be the same as in the presence of decay, and is also equal to the sum of corrections necessitated by each of the fields. As to the binding energy of the particle, the electric field causes it to decrease, compensating by the same token for the anti-diamagnetism effect considered in[6]. In Sec. 4 we consider intermediate ratios of the electric and magnetic fields, at which the quasistationary state approaches the threshold of complete stabilization; this should lead to a decrease of the particle ionization probability to zero, so as to leave the level stationary with further increase of the magnetic field. The analytic result confirming this statement is obtained by solving the Schrödinger equation by the quasiclassical method.

The concluding Sec. 5 is devoted to ionization in an alternating magnetic field in the adiabatic case, when its frequency is small compared with the Larmor frequency. By comparison with the results obtained in Sec. 2 for a constant field, it is shown that the oscillations of the magnetic field decrease its stabilizing action. The results are discussed in the conclusion.

1. EQUATION FOR THE DETERMINATION OF THE ENERGY

We consider a particle with mass m situated in the field of forces with a small effective radius, and also in homogeneous perpendicular electric and magnetic fields **F** and **H**; the electric field can be regarded as the sum of an external field and a Lorentz field. In the zero-radius-well approximation[5,6], the wave function Ψ of a particle with energy E in an electric field **F** directed along the y axis and in a magnetic field H directed along the z axis is a solution of the equation

$$\nabla^2\Psi + 2i\frac{eH}{\hbar c}y\frac{\partial\Psi}{\partial x} + \left(-\frac{e^2H^2}{\hbar^2c^2}y^2 + \frac{2meF}{\hbar^2}y + \frac{2mE}{\hbar^2}\right)\Psi = -\delta(\mathbf{r}). \quad (2)$$

The corresponding gauge is chosen from considerations of convenience in calculating the current, since one component of the vector potential differs from zero in such a gauge. Thus, the wave function of the problem coincides with the Green's function of a particle in crossed fields[1)].

Inasmuch as in condition (1), from which we shall subsequently determine the energy spectrum, we can approach the origin from any direction, particularly along the direction of the magnetic field, we confine ourselves in the function Ψ to its dependence on the z coordinate. We have obtained for equation (2) a solution that depends on one coordinate z in analogy with the case of one magnetic field[6] directly from the spectral expansion of the Green's function, in the form

$$\Psi(0,z) = -i(4\pi i)^{-3/2}\lambda^{-1}\int_0^\infty \frac{dx}{x^{1/2}\sin(x/2)}\exp i\left(-\frac{\alpha^2\lambda^2}{4}x - \frac{s^2}{\lambda^2}x + \frac{z^2}{\lambda^2 x} + \frac{s^2}{2\lambda^2}x^2\operatorname{ctg}\frac{x}{2}\right), \quad (3)$$

where $\alpha^2 = -2mE/\hbar^2$; $\lambda = (eH/2\hbar c)^{-1/2}$ is the Larmor radius; $s = mc^2F/eH^2$ is the distance from the center of the parabolic potential well characteristic of crossed fields to the origin.

Separating in the function (3) the part that diverges as $z \to 0$, and substituting this function into the condition (1), we arrive at an equation for the determination of the energy

$$-\alpha_0\lambda = -\alpha\lambda - (i\pi)^{-1/2}\int_0^\infty \frac{dx}{x^{3/2}}\left[\frac{\frac{x}{2}}{\sin(x/2)}\exp i\left(-\frac{\alpha^2\lambda^2}{4}x - \frac{s^2}{\lambda^2}x + \frac{s^2}{2\lambda^2}x^2\operatorname{ctg}\frac{x}{2}\right) - \exp\left(-i\frac{\alpha^2\lambda^2}{4}x\right)\right], \quad (4)$$

where $\alpha_0 = -2mE_0/\hbar^2$.

Before we proceed to solve this equation, let us discuss the characteristic features of the problem.

If the particle was in a bound state with energy $E_0 = -\hbar^2\alpha_0^2/2m$ in the absence of both fields, then when the fields are turned on the particle's behavior will vary with the ratio of the electric and magnetic field intensities. Outside the well, the particle moves only under the influence of the fields **F** and **H**, and its motion along the magnetic field remains free. As a result the energy spectrum of the particle outside the well is continuous, and in a magnetic field weak in comparison with the atomic field the spectrum, as will be shown subsequently, has an end point at $\epsilon \approx -mc^2F^2/2H^2$. The electric field causes the system to decay, and in our case, at sufficiently large values of this field ($|E_0| < |\epsilon|$), the level in the well is against the background of the continuous spectrum. The particle is then able to tunnel under the barrier from the δ well to the outside. Thus, in the case when the action of the magnetic field on the particle is weaker than the action of the electric field, it is necessary to consider both the shift of the level E_0 in the fields, and its width resulting from the ionization of the corresponding quasistationary state. The possibility of such an ionization was noted by Hiskes[2], and its probability in the quasiclassical case was calculated with exponential accuracy by Kotova, Perelomov, and Popov[8] by the method of Feynman integrals, generalized to the case of imaginary time.

The magnetic field stabilizes the particle, and therefore in our other case, when its action prevails over the action of the electric field and there is no possibility for a decay ($|E_0| > |\epsilon|$), one should speak of the shift of the stationary state of the particle in the magnetic field by the electric field.

We introduce the parameter

$$\gamma = |E_0/\varepsilon|^{1/2} = \hbar\alpha_0 H/mcF,$$

which characterizes different regimes of particle motion. When $\gamma < 1$ decay is possible and is the more probable the smaller γ. In the case of $\gamma \geq 1$ there is no possibility for decay, and large values of γ correspond to more stable states.

2. QUASISTATIONARY STATES $\gamma \ll 1$

Let us consider states whose quasistationary character is due to the possibility of system decay. The electric field is regarded as weak compared with the atomic field (the ionization has then a quasiclassical character) and sufficiently strong compared with the magnetic field so that the small stabilizing influence of the latter leaves a possibility for the decay of the system. The quasiclassical character of the process makes it possible to calculate the integral in (4) by the saddle-point method. The saddle points are determined from the equation

1)When the manuscript was being readied for the printer, the authors learned of a paper by Magarill and Savvinykh [8], in which the corresponding Green's function is given.

$$\left(\frac{x}{2}\right)^2\left[1+\left(\operatorname{ctg}\frac{x}{2}-\frac{2}{x}\right)^2\right]=-\gamma^2,$$
$$\gamma=\hbar\alpha H/mcF=\alpha\lambda^2/2s. \tag{5}$$

Equation (5) coincides with the equation for the instant of the emergence of the particle from under the barrier, which determines the exponential factor, considered in[9], of the decay probability[2)].

In order to retain the analytic form of the calculations that follow, we confine ourselves to small values of the roots of this equation. Taking the first two terms of the expansion of $\cot(x/2)$, we find a solution of (5) in the form

$$x_0=-2i\gamma(1+\gamma^2/18)$$

and valid under the condition

$$\gamma\ll 1. \tag{6}$$

As a result of the expansion of $\cot(x/2)$ near the saddle points $x_0 \ll 1$, the integral in (4) takes the form of an Airy integral, in which the role of the large parameter ensuring the possibility of using the saddle-point method is played by the quantity

$$2s^2\gamma^3/\lambda^2\gg 1. \tag{7}$$

An integral of this type was investigated in[10]. The integration contour consists in our case of a segment of the imaginary axis from the origin to the saddle point, and then of a branch of a hyperbola leading to an infinitely remote point. Calculation along the imaginary axis gives the real part of the integral, which determines the level shifts, and calculation along the hyperbola gives its imaginary part, which determines the level width or the particle ionization probability. Substituting the results of the integration into (4), we obtain an expression for the complex energy of the particle in the field of a well of small radius and in crossed electric and magnetic fields:

$$E=E_0-\frac{me^2F^2}{8\hbar^2\alpha_0^4}+\frac{e^2H^2}{24mc^2\alpha_0^2}+i\Gamma, \tag{8}$$

$$\Gamma(F,H)=|E_0|\frac{F}{2F_0}\left(1-\frac{\gamma^2}{6}\right)\exp\left[-\frac{2}{3}\frac{F_0}{F}\left(1+\frac{\gamma^2}{30}\right)\right], \tag{9}$$

$F_0 = \hbar^2\alpha_0^3/em$ is the atomic field.

The result is valid when conditions (6) and (7) are satisfied; these can be written respectively in the form

$$\gamma=\omega_L/\omega_t\ll 1,\quad F/F_0=\omega_t/2\omega_0\ll 1, \tag{10}$$

where $\omega_0 = \hbar\alpha_0^2/2m$ is the atomic frequency, $\omega_t = eF/\hbar\alpha_0$ is the frequency of tunneling through the barrier, and $\omega_L = eH/mc$ is the Larmor frequency.

The level shift in (8) turns out to be equal to the sum of the corrections arising respectively in the electric[5] and magnetic[6] fields.

It is seen from (9) that the magnetic field, by decreasing both the pre-exponential and the exponential factors, leads to a decrease of the level width Γ, i.e., it exerts a stabilizing action on the system. The stabilization is due to the bending of the sub-barrier trajectory of the particle, and consequently to the increase of the effective thickness of the barrier by the magnetic field.

[2)] This circumstance was made clear as a result of a discussion with Yu. N. Demkov.

3. STATIONARY ENERGY LEVELS $\gamma \gg 1$

We consider another case, when the stabilizing action of the magnetic field on the particle is stronger than the ionizing influence of the electric field. It is meaningful here to speak of stationary states of the system. When separating the diverging part of the wave function $\Psi(0, z)$ (3), we use the Euler formulas for $\operatorname{cosec}(x/2)$ and combine the terms linear in x in the argument of the exponential. Assuming the ratio s^2/λ^2 to be small, we take into account the first two terms of the expansion of the exponential containing $\cot(x/2)$ in powers of this ratio. Following[6], we can express the result in terms of the generalized Riemann ζ functions. Using the recurrence relations given for the latter in[6], we reduce (4) to the form

$$-\alpha_0\lambda=\zeta\left(\frac{1}{2},\frac{\tilde{\alpha}^2\lambda^2}{4}\right)+\frac{3s^2}{8\lambda^2}\left[2\zeta\left(\frac{3}{2},\frac{\tilde{\alpha}^2\lambda^2}{4}\right)+\left(1-\frac{\tilde{\alpha}^2\lambda^2}{2}\right)\zeta\left(\frac{5}{2},\frac{\tilde{\alpha}^2\lambda^2}{4}\right)\right], \tag{11}$$

where

$$\tilde{\alpha}^2=\alpha^2+4\frac{s^2}{\lambda^4}+\frac{2}{\lambda^2},\qquad \alpha^2=-\frac{2mE}{\hbar^2}.$$

As shown in[6], the case $\tilde{\alpha}^2\lambda^2 \gg 1$ is realized for $\alpha_0 > 0$ when there is a bound state in the well in the absence of a magnetic field; the case $\tilde{\alpha}^2\lambda^2 \ll 1$ is realized for $\alpha_0 < 0$, when there are no bound states without a magnetic field. In either case $\alpha_0\lambda \gg 1$, i.e., the Larmor radius is much larger than the dimensions of the electron cloud in the absence of the field. We shall call such a field weak compared with the atomic field, and strong if it satisfies the opposite criterion $\alpha_0\lambda \ll 1$. Let us consider different limiting cases.

1) $\alpha_0 > 0$, $\alpha_0\lambda \gg 1$.

Using in (11) the expansion of the ζ functions in the region of large values of the argument[5], we obtain

$$\alpha_0^2\lambda^2=\alpha^2\lambda^2+\frac{1}{3\lambda^2(\alpha^2+4s^2/\lambda^4+2/\lambda^2)}-\frac{8s^2}{3\alpha^4\lambda^6},$$

or, changing over to ordinary units,

$$E=E_0-\frac{me^2F^2}{8\hbar^2\alpha_0^4}+\frac{e^2H^2}{24mc^2\alpha_0^2}. \tag{12}$$

In addition to the inequality $\alpha_0\lambda \gg 1$, expression (12) calls for the satisfaction of a condition inverse to (6):

$$\gamma\gg 1. \tag{13}$$

Generally speaking, the condition $\gamma \gg 1$ can be expanded also to $\gamma > 1$ by taking into account terms of higher order in s^2/λ^2 or by numerically calculating the integral in (4).

We note that expression (12) coincides with the real part of the energy (8) in the case of a quasistationary state.

2) $\alpha_0 < 0$, $|\alpha_0|\lambda \gg 1$.

Expanding in (11) the ζ functions in the region of small values of the argument[6], we obtain

$$\alpha^2+\frac{2}{\lambda^2}=\frac{4}{\alpha_0^2\lambda^4}\left[1+\frac{\zeta(1/2)}{\alpha_0\lambda}+\frac{3}{16}s^2\alpha_0^4\lambda^2\right]^{-2}.$$

The left-hand side gives us the binding energy of the particle $\mathscr{E} = -E + \hbar eH/2mc$, which is thus equal to

$$\mathscr{E} = \frac{e^2H^2}{2mc^2\alpha_0^2}\left[1 + \frac{1{,}46}{|\alpha_0|\lambda} + \frac{3}{16}s^2\alpha_0^4\lambda^2\right]^{-2}. \tag{14}$$

The obtained expression is valid under the conditions $|\alpha_0|\lambda \gg 1$ and $s^2\alpha_0^5\lambda^3 \ll 1$. The inequality (13) is a consequence of these two. The electric field decreases the binding energy of the particle, and prevents the antidiamagnetic energy shift considered in[6].

3) $\alpha_0 = 0$.

The left-hand side of (11) is equal to zero, and it is necessary to find the root of the right-hand side. This is easily done by using the approximate formulas for the ζ functions[5], as a result of which we get

$$\frac{1}{\tilde{\alpha}^2\lambda^2} = \frac{\tilde{\alpha}^2\lambda^2}{4} + \frac{1}{2} - \frac{s^2}{\lambda^2},$$

or

$$\mathscr{E} = -E + \frac{\hbar eH}{2mc} = 0.29\frac{\hbar eH}{mc} - 0.70\frac{mc^2F^2}{2H^2}. \tag{15}$$

Formula (15) is valid for $s^2/\lambda^2 \ll 1$.

4) α_0--any strong magnetic field.

In this case the right-hand side of (11) must be expanded in the vicinity of its root. Calculations analogous to those in[6] give for the binding energy the expression

$$\mathscr{E} = \frac{\hbar eH}{mc}(0.29 + 0.24\alpha_0\lambda + 0.11\alpha_0^2\lambda^2) - \frac{mc^2F^2}{2H^2}(0.70 - 0.26\alpha_0\lambda - 0.11\alpha_0^2\lambda^2), \tag{16}$$

which is valid for $\alpha_0\lambda \ll 1$ and $s^2/\lambda^2 \ll 1$.

In the last two cases of a strong magnetic field, the electric field also decreases the binding energy. The same takes place when the electric field acts on a Coulomb particle in a strong magnetic field[11].

4. TRANSITIONAL CASE $\gamma \approx 1$ ($E_0 \approx \epsilon$)

We now consider the case when the end point of the continuous spectrum $\epsilon = -mc^2F^2/2H^2$ lies close to and somewhat below the level E_0 in the δ well. The parameter $\gamma = |E_0/\epsilon|^{1/2}$ is then somewhat smaller than unity. With increasing ratio H^2/F^2, the end point of the continuous spectrum approaches the level E_0, and when the two coincide one should expect the width of this level, due to the tunneling of the particle into the region of infinite motion, to decrease to zero. The vanishing of the width should occur at a value $\gamma = 1$. At larger values of $H^2/F^2 (\gamma > 1)$ the end point of the continuous spectrum turns out to be higher than the level in the δ well, there is no possibility for its decay, and it remains stationary.

Unlike for the limiting values of γ, for the values $\gamma \approx 1$ of interest to us it is impossible to obtain a solution of the general equation in analytic form. To calculate the decay probability of the level in the δ well in crossed fields we therefore use the quasiclassical method of solving the Schrödinger equation. This method, used to calculate the ionization in a weak electric field $F \ll F_0$[12], is applicable also in the presence of a magnetic field, since the latter increases the thickness of the barrier, leaving the tunneling quasiclassical.

To calculate the ionization probability in the quasiclassical case it is necessary to find the solution of the homogeneous equation (2) near the direction of the electric field parallel to the y axis[13]. We use for this purpose the parabolic coordinates

$$y = (\xi - \eta)/2, \quad z = (\xi\eta)^{1/2}\cos\varphi, \quad x = (\xi\eta)^{1/2}\sin\varphi,$$

in which the condition for proximity to the y axis takes the form $\eta/\xi \ll 1$. Changing over in (2) to the parabolic coordinates, we can easily show that the condition $\eta/\xi \ll 1$, and also the fact that we are considering an s-state in a δ well, enable us to neglect the paramagnetic term in this equation. Then the variables separate and the solution of the homogeneous equation (2) at $\eta/\xi \ll 1$ can be represented in the form

$$\Psi = (2\pi\xi\eta)^{1/2} f_1(\xi) f_2(\eta), \tag{17}$$

with the functions f_1 and f_2 satisfying the equations

$$f_1'' + \left(-\frac{e^2H^2}{16\hbar^2c^2}\xi^2 + \frac{meF}{4\hbar^2}\xi - \frac{A}{\xi} + \frac{1}{4\xi^2} - \frac{\alpha^2}{4}\right) f_1(\xi) = 0, \tag{18}$$

$$f_2'' + \left(\frac{A}{\eta} + \frac{1}{4\eta^2} - \frac{\alpha^2}{4}\right) f_2(\eta) = 0, \tag{19}$$

where A is the separation constant.

The wave function of a particle in a small-radius well

$$\Psi_0 = \left(\frac{\alpha_0}{2\pi}\right)^{1/2} \frac{e^{-\alpha_0 r}}{r}$$

has in the region $\eta \ll \xi$ the form

$$\Psi_0 = \frac{1}{(2\pi\xi\eta)^{1/2}} f_{10}(\xi) f_{20}(\eta), \tag{20}$$

where

$$f_{10}(\xi) = (4\alpha_0)^{1/2} e^{-\alpha_0\xi/2}\xi^{-1/2}, \tag{21}$$

$$f_{20}(\eta) = \eta^{1/2} e^{-\alpha_0\eta/2}. \tag{22}$$

Since Eq. (19) does not contain an electric or a magnetic field, its solution $f_2(\eta)$ can be the unperturbed function $f_{20}(\eta)$, substitution of which in (19) makes it possible to determine the meaning of the separation constant $A = \alpha_0/2$.

The potential energy in (18) is a parabolic well with center at a sufficiently remote point

$$\xi_0 = 2mc^2F/eH^2,$$

perturbed by the Coulomb term A/ξ. The particle tunnels into this well, the minimum of which lies in the region of negative energy, from a δ well localized at the origin and separated from the parabolic well by a broad barrier. During the tunneling process the magnetic field causes the particle to twist and then tunnel further in a direction perpendicular to the directions of the electric and magnetic fields, in our case in the x-axis direction. Thus, if the particle has tunneled into a parabolic well near the ξ axis, i.e., along directions for which the barrier thickness is minimal, then after twisting the particle again returns to the barrier in a region much farther away from the ξ axis, and will be forced to tunnel back into the δ well along trajectories for which the barrier thickness is much larger than in the case of a straight-line motion. Consequently, the probability that the particle will return to the δ well can be neglected, making it possible to assume that the ionization probability is proportional to the probability for the appearance of the particle in the center of the parabolic well at the point $\xi = \xi_0$.

The wave function of the particle in the region of the parabolic potential should be chosen to be a solution of (18) in the region of large ξ; this solution increases towards the origin:

$$f_1(\xi) = \frac{C}{(\alpha/2)^{1/2}} \exp\left\{-\frac{(\xi-\xi_0)^2}{4\lambda^2}\right\}. \quad (23)$$

The ionization probability is determined by the formula

$$W = \int v\Psi^2(\varphi, \eta, \xi = \xi_0)\, dS,$$

in which v is the particle velocity and the integration is carried out over a plane perpendicular to the ξ axis, the element of which is $dS = (1/2)\xi d\eta d\varphi$. Using the explicit form of the function (17) expressed in terms of $f_{20}(\eta)$ (22) and $f_1(\xi = \xi_0)$ (23), we obtain for the ionization probability

$$W = vC^2 / \alpha^2. \quad (24)$$

We determine the coefficient C from the condition that the function of the particle in the δ well (21) coincide inside the barrier with the function (23) continued into this region. Such a continuation, satisfying Eq. (18), must now be carried out with the Coulomb term taken as the perturbation. The continuation of the function (23) to the interior of the barrier can be represented in the form of the quasiclassical solution of (18):

$$f_1(\xi) = -\frac{iC}{|p|^{1/2}} \exp\left(\left|\int_{\xi_1}^{\xi} p(x)\,dx\right|\right), \quad (25)$$

$$p(x) = \left(-\frac{\alpha^2}{4} - \frac{e^2H^2}{16\hbar^2c^2}x^2 + \frac{meF}{4\hbar^2}x - \frac{A}{x}\right)^{1/2}, \quad A = \frac{\alpha_0}{2},$$

and ξ_1 is the nearest turning point.

Carrying out the integration in (25), we obtain for f_1 inside the barrier

$$f_1(\xi) = \frac{C}{(\alpha/2)^{1/2}} e^{-\alpha\xi/2}\xi^{-1/2}\left(\frac{2\xi_0\gamma^2}{\sqrt{1-\gamma^2}}\right)^{1/2} \times \exp\frac{\alpha\xi_0}{4}\left(1 + \frac{1-\gamma^2}{2\gamma}\ln\frac{1-\gamma}{1+\gamma}\right).$$

A comparison of the result with (21) enables us to determine the coefficient C. Then substituting this coefficient in (24), in which we must put $v = \hbar\alpha_0/m$, we obtain a final expression for the ionization probability of a particle in the field of small-radius potential forces in crossed electric and magnetic fields:

$$W = \frac{|E_0|}{\hbar}\frac{F}{F_0}(1-\gamma^2)^{1/2}\exp\left(-\frac{2}{3}\frac{F_0}{F}G(\gamma)\right), \quad (26)$$

$$G(\gamma) = \frac{3}{2\gamma^2}\left(1 + \frac{1-\gamma^2}{2\gamma}\ln\frac{1-\gamma}{1+\gamma}\right). \quad (27)$$

For the integration in (25) we used the condition $\alpha/\xi \ll \alpha^2/4$, which makes it possible to regard the Coulomb term as a perturbation, and also the inequalities $\xi/\gamma^2\xi_0 \ll 1$ and $\xi/\gamma\xi_0 \ll 1$. These inequalities determine the conditions for the applicability of formula (26), one of which is contained in (10):

$$F / F_0 \ll 1,$$

while the other

$$\omega_L / \omega_0 \ll 1$$

is a consequence of the inequalities in (10). In particular, in Eq. (2), written in parabolic coordinates, the last parameter is the proportionality coefficient preceding the first derivatives in the paramagnetic term. The smallness of this parameter makes it possible to neglect these derivatives in comparison with the first derivatives in the Laplace operator, which have unity as a coefficient.

The expressions for the decay probability were obtained by us under the condition $\omega_L \ll \omega_0$, i.e., for the case when the energy of the zero-point oscillations in the magnetic field $\hbar\omega_L/2$ is small compared with the binding energy of the particle in the δ well $|E_0| = \hbar\omega_0$. Since the latter is, in turn, always smaller in the case of decay than the shift of the end point of the continuous spectrum, we can neglect in the latter also the quantity $\hbar\omega_L/2$ and assume, as we have done, that the continuous spectrum begins the values of $\epsilon \approx -mc^2F^2/2H^2$.

As to the parameter γ, expression (26) is meaningful for all values $0 \le \gamma \le 1$. At $\gamma \ll 1$, the form of $G(\gamma)$ (27) is $G \approx 1 + \gamma^2/5$, as a result of which the exponential factor, which determines, in the main, the dependence on the magnetic field in formula (26), differs somewhat from the factor obtained in formula (9) as a result of direct integration in (4). The stronger dependence on the magnetic field in (26) is connected with the fact that in its derivation we have neglected, in the diamagnetic term of the Schrödinger equation, the compensating terms of order η/ξ and $(\eta/\xi)^2$, which had signs opposite to those included, but which prevented us from separating the variables.

According to formula (26), the ionization probability in crossed fields decreases monotonically with increasing parameter γ and vanishes at the critical value $\gamma = 1$. The vanishing of the probability occurs in accordance with the law

$$W_{\Delta\epsilon\to 0} \approx \frac{|E_0|}{\hbar}\frac{F}{2F_0}\left(\frac{\Delta\epsilon}{\epsilon}\right)^{1/2} e^{-F_0/F},$$

where $\Delta\epsilon = E_0 - \epsilon$ is the energy deficit of the particle in the δ well relative to the end point of the continuous spectrum.

5. IONIZATION IN AN ALTERNATING MAGNETIC FIELD $\gamma \ll 1$

We consider the ionization of a weakly bound particle in an electric field crossed with an alternating magnetic field under the condition that the frequency of the magnetic field Ω is small compared with the smallest Larmor frequency in this case, $\Omega \ll \omega_L$. In this adiabatic case the value of Γ_Ω can be obtained by averaging over the period of the magnetic field. As a result we obtain

$$\Gamma_\Omega(F,H)=|E_0|\frac{F}{2F_0}\exp\left(-\frac{2}{3}\frac{F_0}{F}\right)\Phi(F,H),$$
$$\Phi(F,H)=e^{-q}\left[I_0(q)-\frac{\gamma^2}{12}(I_0(q)-I_1(q))\right],$$
$$q=\omega_0\omega_L{}^2/45\omega_t{}^3=\hbar^4\alpha_0{}^5H^2/90em^3c^2F^3, \qquad (28)$$

and I_0 and I_1 are Bessel functions. In the case of the limiting values of q we have $\Phi_{q\to 0}\approx 1-q$; $\Phi_{q\to\infty}\approx(2\pi)^{-1/2}q^{-1/2}$. As expected, oscillations of the magnetic field decrease its stabilizing action, whereas at $q\gg 1$ the value of Γ (9) decreased with the magnetic field exponentially, like e^{-2q}, the expression Γ_Ω (28) decreases in this case in power-law fashion, like $q^{-1/2}$.

DISCUSSION

Besides the limitations on the parameters of the problem that were given for each of the approximate expressions derived here, let us note the general criteria for the applicability of the small-radius-well model. One such criterion is the condition that the dimension of the electron cloud of the bound state greatly exceed the radius of the potential well r_0. When $\alpha_0>0$ this condition takes the form $\alpha_0 r_0\ll 1$, and when $\alpha_0<0$ we have $r_0/|\alpha_0|\lambda^2\ll 1$. The last inequality signifies that when $\alpha_0\approx 10^7$ cm^{-1} the well dimensions are $r_0\approx 10^{-8}$ cm and $H\ll 10^8$ Oe. Another criterion is $r_0/\lambda\ll 1$, whereby the radius of the Larmor orbit is much larger than the well dimensions r_0. Putting $r_0\approx 10^{-7}$ cm, we obtain $H\ll 10^7$ Oe.

The decay probability calculated by the quasiclassical method in the transition region $E_0\approx\epsilon$, which vanishes when $\gamma=1$, agrees with the form of the spectrum of the quantum-mechanical problem and does not contradict qualitatively the results of calculations of the integral contained in Eq. (4). It should be noted that a similar preexponential factor, which vanishes at a certain critical ratio of the electric and magnetic fields, was obtained by Aronov and Pikus[3] in an analysis of interband tunneling in crossed fields in semiconductors with narrow forbidden bands.

The decrease of the level width as a result of the magnetic field, which was considered by us, was observed experimentally in the form of a narrowing of exciton peaks in a study of the photo Hall photoeffect in a Cu_2O crystal by Agekyan and Zakharchenya[14].

It follows from (9) that the ionization probability of a particle having the mass of the free electron and a binding energy $E_0\approx 0.03$ eV and situated in an electric field $F=180$ V/cm decreases by a factor of 20 when a magnetic field $H=360$ Oe is turned on. From the stabilization condition $\gamma>1$ we can find that there will be no decay of a particle having the mass of a free electron and an energy $E_0\approx 0.03$ eV ($\alpha_0=10^7$ cm^{-1}) in a field $F=10^4$ V/cm if the magnetic field exceeds $H=10^5$ Oe.

The results can be applied to the now intensely investigated doped semiconductors placed in external fields, where bound exciton or impurity states are present. The ionization considered by us in an alternating magnetic field can be taken into account, in particular, when plotting the differential magnetooptic spectra produced in the detection of signals at the frequency of the alternating magnetic field.

In conclusion, the authors thank Yu. N. Demkov for a number of valuable remarks and S. Yu. Slavyanov for interest in the work.

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Translated by J. G. Adashko
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