SCATTERING OF PARTICLES BY LONG-RANGE POTENTIALS

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The scattering of particles by long-range potentials is investigated. It is shown that a consistent analysis of the development in time of the collision process leads to results which differ significantly from the results of the stationary scattering theory. It turns out that for long-range potentials the quantity which plays the role of the scattering cross-section depends on the coordinates of the source and the detector. These anomalies are manifested in processes of depolarization of colliding particle beams, as well as in inelastic scattering processes.

IT is shown that the scattering amplitude for potentials, which fall off at large distances like or more slowly than r^{-3}, either diverges or is not unique when the scattering angle tends to zero[1,2]. It has been shown in[2] that the nonuniqueness, which exists at zero angle in the magnetic scattering amplitude, is connected with the fact that for a magnetic dipole-dipole interaction $\sim r^{-3}$ the standard asymptotic form of the wave function, $\psi_{sc} \sim fe^{ikr}/r$, does not exist. As a result, the magnetic scattering amplitude, calculated in the usual manner, does not describe the scattering process.

We must suppose that a similar situation arises for other long-range potentials, i.e., that the coefficient associated with the outgoing spherical wave does not completely describe the collision process and cannot be used to determine the scattering cross section.

Indeed, the divergence (nonuniqueness) of the small-angle scattering amplitude is at variance with the fact that a wave function that is a solution of the Schrödinger equation is unique and does not contain any divergences. Thus, the Coulomb scattering amplitude diverges at small angles, whereas the exact solution does not contain any divergences at all. Another example is connected with the well-known Schwinger scattering of a neutron by a nucleus. The corresponding interaction potential has the form[3] *

$$V(\mathbf{r}) = i\frac{Z\mu\hbar e}{mc}\boldsymbol{\sigma}\left[\frac{\mathbf{r}}{r^3}\nabla_r\right],$$

where $\mathbf{r}$ is the distance between the neutron and the nucleus, μ is the magnetic moment of the neutron, and the remaining designations are standard ones. The scattering amplitude for such an interaction, computed in the Born approximation, is described by the expression

$$f = i\frac{Ze\mu}{\hbar c}\frac{\boldsymbol{\sigma}[\mathbf{kn}]}{k-\mathbf{kn}}$$

where $\mathbf{n} = \mathbf{r}/r$, and $\mathbf{k}$ is the wave vector of the neutron. As we can see, at small angles $f \sim 1/\theta$, where θ is the scattering angle. The divergence of the small-angle Schwinger scattering amplitude points to the fact that the asymptotic expression for $\psi_{sc} \sim fe^{ikr}/r$ cannot, in the present case, be valid. Indeed, by performing the explicit integration in the expression for the wave function, we can, without difficulty, obtain in the Born approximation

$$\psi_{sc} = -\frac{m}{2\pi\hbar^2}\int\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}V(\mathbf{r}')e^{i\mathbf{kr}'}d^3r', \tag{1}$$

i.e.,

$$\psi_{sc} = i\frac{Ze\mu}{\hbar c}\frac{\boldsymbol{\sigma}[\mathbf{kn}]}{k-\mathbf{kn}}\frac{e^{ikr}-e^{i\mathbf{kr}}}{r}.$$

It is obvious that the wave function (1) is finite at any scattering angle and at large distances does not have the asymptotic form e^{ikr}/r. As will be seen below, the second term in the expression (1) not only removes the divergence in the scattered flux at small angles, it also makes contributions to the differential and total scattering cross sections. A similar situation obtains in the case of other long-range potentials.

Let us for the analysis of the resulting problem assume that the wave packet $\psi(\mathbf{r}, 0)$ describing the particle the scattering of which we want to study, was formed at the time $t = 0$ around the point $\mathbf{r}_0(0, 0, -z_0)$. The packet has a spatial spread $(\Delta x, \Delta y, \Delta z)$ and moves in the direction of the z-axis with velocity $\hbar k/m$. The expansion of the packet in terms of plane waves at the moment of time $t = 0$ has the form

$$\psi(\mathbf{r}, 0) = \int d^3q\, b(\mathbf{k}-\mathbf{q})e^{i\mathbf{qr}}, \tag{2}$$

where the factor $\exp[i\mathbf{q}\cdot\mathbf{r}]$ characterizing the initial position of the packet has been included in the amplitude $b(\mathbf{k}-\mathbf{q})$ of the expansion. The development in time of the packet is described by the equation

$$\psi(\mathbf{r}, t) = \exp\left\{-\frac{i}{\hbar}Ht\right\}\psi(\mathbf{r}, 0), \quad H = -\frac{\hbar^2}{2m}\Delta + V(\mathbf{r}). \tag{3}$$

Let us expand the function $\psi(\mathbf{r}, t)$ in terms of the eigenfunctions $\psi_p(\mathbf{r})$ of the operator H. We have

$$\psi(\mathbf{r}, t) = \int d^3p\, c(\mathbf{p})\psi_p(\mathbf{r})\mathscr{E}_p(t), \quad \mathscr{E}_p(t) = \exp\left\{-i\frac{\hbar}{2m}p^2t\right\},$$
$$c(\mathbf{p}) = \int d^3q\, b(\mathbf{k}-\mathbf{q})\int d^3r'\,\psi_p^*(\mathbf{r}')e^{i\mathbf{qr}'}. \tag{4}$$

The time evolution of the function $\psi(\mathbf{r}, t)$ will further be studied with the aid of the Born expansion in terms of the eigenfunctions $\psi_p(\mathbf{r})$ of the operator H. We obtain as a result

*$[\mathbf{r}\nabla] \equiv \mathbf{r} \times \nabla$.

$$\psi(\mathbf{r},t)=\int d^3p\, b(\mathbf{k}-\mathbf{p})\, e^{i\mathbf{pr}}\mathscr{E}_p(t)$$
$$-\frac{m}{4\pi\hbar^2}\int d^3r_1\, d^3p\, b(\mathbf{k}-\mathbf{p})\,\frac{e^{ip|\mathbf{r}-\mathbf{r}_1|}+e^{-ip|\mathbf{r}-\mathbf{r}_1|}}{|\mathbf{r}-\mathbf{r}_1|}V(\mathbf{r}_1)\, e^{i\mathbf{pr}_1}\mathscr{E}_p(t)$$
$$-\frac{m}{4\pi\hbar^2}\frac{1}{(2\pi)^3}\int d^3r_1\, d^3r_2\, d^3p\, d^3q\, b(\mathbf{k}-\mathbf{p})\, e^{i\mathbf{pr}_2} \qquad (5)$$
$$\times\frac{e^{iq|\mathbf{r}_2-\mathbf{r}_1|}+e^{-iq|\mathbf{r}_2-\mathbf{r}_1|}}{|\mathbf{r}_2-\mathbf{r}_1|}V(\mathbf{r}_1)\, e^{i\mathbf{q}(\mathbf{r}-\mathbf{r}_1)}\mathscr{E}_q(t).$$

The first term in (5) describes the incident wave packet moving without change; the remaining terms describe the change in the incident wave packet under the action of the interaction.

Let us consider in more detail the last term in (5). Performing the integration in it with respect to $d^3\mathbf{r}_2$, we obtain

$$I=-\frac{1}{(2\pi)^3}\frac{2m}{\hbar^2}\int d^3r_1\, d^3q\, d^3p\, b(\mathbf{k}-\mathbf{p})\,\frac{e^{i\mathbf{pr}_1}}{p^2-q^2}V(\mathbf{r}_1)\, e^{i\mathbf{q}(\mathbf{r}-\mathbf{r}_1)}\mathscr{E}_q(t). \quad (6)$$

We further perform the integration in (6) with respect to $d^3\mathbf{q}$. A proper analysis shows that the integral of interest

$$A=\int d^3q\,\frac{1}{p^2-q^2}\, e^{i\mathbf{q}(\mathbf{r}-\mathbf{r}_1)}\mathscr{E}_q(t)$$

equals

$$A=-\frac{\pi^2}{|\mathbf{r}-\mathbf{r}_1|}\{e^{-ip|\mathbf{r}-\mathbf{r}_1|}\,\Phi(\alpha_+)+e^{ip|\mathbf{r}-\mathbf{r}_1|}\,\Phi(\alpha_-)\}\mathscr{E}_p(t), \qquad (7)$$

where the error function

$$\Phi(z)=\frac{2}{\sqrt{\pi}}\int_0^z e^{-y^2}dy, \quad \alpha_\pm=\left(\sqrt{\frac{m}{2\hbar t}}\,|\mathbf{r}-\mathbf{r}_1|\pm\sqrt{\frac{\hbar p^2 t}{2m}}\right)e^{-i\pi/4}.$$

We note that the quantity $|\alpha_+|$ for values of the time $t \gg 2m/\hbar p^2$, i.e., for time intervals which one rea..y deals with in any scattering experiment, is much larger than unity. For this reason the function $\Phi(\alpha_+)$ can, to within terms of the order of $|\alpha_+|^{-1}$, be replaced by unity. As a result, the addends of the expression (5) which contain the terms $\exp[-ip|\mathbf{r}-\mathbf{r}_1|]/|\mathbf{r}-\mathbf{r}_1|$ drop out.

Let us consider in more detail the terms containing $\Phi(\alpha_-)$. If $\alpha_- e^{i\pi/4} \gg 1$ (this corresponds to performing the integration with respect to $d^3\mathbf{r}_1$ over the region of space outside the sphere of radius $r_+ = \hbar pt/m + \sqrt{2\hbar t/m}$ around the point r), then $\Phi(\alpha_-) \approx 1$. Therefore, in this region of integration the terms in the expression (5), which contain the outgoing waves, cancel each other out. In the region where the condition $\alpha_- e^{i\pi/4} \gg 1$ (this corresponds to integration over the volume of the sphere of radius $r_- = \hbar pt/m - \sqrt{2\hbar t/m}$ around the point r), the function $\Phi(\alpha_-) \approx -1$. As a result, in this region of integration with respect to $d^3\mathbf{r}_1$, the terms containing the outgoing spherical waves add up. The contribution from the remaining region of integration enclosed between the spheres of radii r_+ and r_- can be neglected.

Thus, the expression (5) for the wave function can finally be written in the form

$$\psi(\mathbf{r},t)=\int d^3p\, b(\mathbf{k}-\mathbf{p})\, e^{i\mathbf{pr}}\mathscr{E}_p(t)$$
$$-\frac{m}{2\pi\hbar^2}\int d^3p\, b(\mathbf{k}-\mathbf{p})\int_L d^3r_1\frac{e^{ip|\mathbf{r}-\mathbf{r}_1|}}{|\mathbf{r}-\mathbf{r}_1|}V(\mathbf{r}_1)\, e^{i\mathbf{pr}_1}\mathscr{E}_p(t) \qquad (8)$$

(where L is the region of integration $|\mathbf{r}-\mathbf{r}_1| < \hbar pt/m$) or

$$\psi(\mathbf{r},t)=\int d^3p\, b(\mathbf{k}-\mathbf{p})\, e^{i\mathbf{pr}}\mathscr{E}_p(t)$$
$$+\frac{m}{(2\pi)^3\hbar^2}\int d^3p\, b(\mathbf{k}-\mathbf{p})\mathscr{E}_p(t)\int d^3q\,\hat{V}(\mathbf{q}-\mathbf{p})\frac{e^{i\mathbf{qr}}}{q} \qquad (9)$$
$$\times\left[\frac{e^{i(p+q)v_pt}-1}{p+q}-\frac{e^{i(p-q)v_pt}-1}{p-q}\right], \quad \hat{V}(\mathbf{q})=\int d^3r V(\mathbf{r})\, e^{i\mathbf{qr}},$$
$$v_p=p/m.$$

As can be seen, a consistent analysis of the collision process leads in the course of time to a cutoff of the potential.

Allowance for a possible spin dependence of the interaction and the inclusion of inelastic scattering processes offer no difficulty. It can be shown that in that case

$$\psi(\mathbf{r}s\xi_1\ldots\xi_n t)=\sum_{\lambda s'}\int d^3p\, b_{\lambda s'}(\mathbf{k}-\mathbf{p})\, e^{i\mathbf{pr}}\Phi_\lambda(\xi_1\ldots\xi_n)\mathscr{E}_{p\lambda}(t)\chi_{s'}$$
$$-\frac{m}{2\pi\hbar^2}\sum_{\lambda\lambda's'}\int d^3p\, b_{\lambda s'}(\mathbf{k}-\mathbf{p})\,\Phi_\lambda(\xi_1\ldots\xi_n)\int\ldots\int d\eta_1\ldots d\eta_n \qquad (10)$$
$$\times\Phi_{\lambda'}^*(\eta_1\ldots\eta_n)\int_{L_1} d^3r_1\frac{\exp\{ip_{\lambda\lambda'}|\mathbf{r}-\mathbf{r}_1|\}}{|\mathbf{r}-\mathbf{r}_1|}V(\mathbf{r}_1 s\eta_1\ldots\eta_n)$$
$$\times e^{i\mathbf{pr}_1}\Phi_\lambda(\eta_1\ldots\eta_n)\mathscr{E}_{p\lambda}(t)\chi_{s'},$$

where

$$(p_{\lambda\lambda'})^2=p^2+\frac{2m}{\hbar^2}(\varepsilon_\lambda-\varepsilon_{\lambda'}), \quad \mathscr{E}_{p\lambda}(t)=\exp\left\{-i\left[\frac{\hbar}{2m}p^2+\frac{\varepsilon_\lambda}{\hbar}\right]t\right\},$$
$$L_1=|\mathbf{r}-\mathbf{r}_1|<\hbar p_{\lambda\lambda'}t/m,$$

$\chi_{s'}$ is the spin function of the incident particle, $\Phi_\lambda(\xi_1 \ldots \xi_n)$ is the wave function of the stationary state of the scatterer corresponding to the energy ϵ_λ.

Let us consider now a few particular cases.

1. Let V(r) be a potential that decreases with distance faster than r^{-3}. Let us evaluate the integral with respect to d^3q in the expression (9):

$$B=\int_0^\infty q\, dq\int_0^\pi\sin\theta\, d\theta\int_0^{2\pi} d\varphi\,\hat{V}(\mathbf{q}-\mathbf{p})\, e^{iqr\cos\theta}\left[\frac{e^{i(p+q)v_pt}-1}{p+q}-\frac{e^{i(p-q)v_pt}-1}{p-q}\right] \qquad (11)$$

Carrying out the integration in (11) with respect to the variable θ by parts, we obtain

$$B=\int_0^\infty q\, dq\int_0^{2\pi} d\varphi\left[\frac{e^{iqr}\hat{V}(q\mathbf{r}/r-\mathbf{p})-e^{-iqr}\hat{V}(-q\mathbf{r}/r-\mathbf{p})}{iqr}\right.$$
$$\left.-\int_{-1}^1 dx\frac{e^{iqrx}}{iqr}\frac{d\hat{V}(\mathbf{q}-\mathbf{p})}{dx}\right]\left[\frac{e^{i(p+q)v_pt}-1}{p+q}-\frac{e^{i(p-q)v_pt}-1}{p-q}\right]. \qquad (12)$$

Since the Fourier transform of a potential, which falls off faster than r^{-3}, does not have singularities, then, limiting ourselves to terms in (12) of the order of r^{-1}, we have

$$B=\frac{2\pi}{ir}\int_{-\infty}^\infty dq\,\hat{V}\left(q\frac{\mathbf{r}}{r}-\mathbf{p}\right)\left\{\frac{\mathscr{E}_p(-t)\, e^{iq(r+v_pt)}-e^{iqr}}{p+q}\right. \qquad (13)$$
$$\left.-\frac{\mathscr{E}_p(-t)\, e^{iq(r-v_pt)}-e^{iqr}}{p-q}\right\}.$$

Integrating with respect to q, we find

$$\psi_{sc}(\mathbf{r},t)=-\frac{m}{2\pi\hbar^2}\frac{1}{r}\int d^3p\, b(\mathbf{k}-\mathbf{p})\mathscr{E}_p(t)\,\hat{V}\left(p\frac{\mathbf{r}}{r}-\mathbf{p}\right)e^{ipr}\Theta(v_pt-r), \qquad (14)$$

where

$$\Theta(x)=1 \text{ for } x>0, \quad \Theta(x)=0 \text{ for } x<0.$$

Carrying out the integration with respect to d^3p, we obtain the standard result

$$\psi_{sc}(\mathbf{r}, t) = f\frac{e^{ikr}}{r}\mathscr{E}_k(t) J\left(r\frac{\mathbf{k}}{k} - \mathbf{r}_0 - \mathbf{v}_k t\right), \qquad (15)$$

where the scattering amplitude

$$f = -\frac{m}{2\pi\hbar^2}\hat{V}\left(k\frac{\mathbf{r}}{r} - \mathbf{k}\right),$$

$J(\mathbf{r})$ is a function describing the spatial distribution of the packet[4].

2. If the interaction potential decreases like or more slowly than r^{-3}, then the analysis carried out above is not correct for the reason that the Fourier transform of such a potential contains singularities and indeterminacies. As a result, integration by parts does not enable us to obtain an expansion of the wave function in inverse powers of r. Let us therefore calculate explicitly the wave function of interest.

Thus, let us consider, for example, the scattering of a particle possessing a magnetic moment by a charged particle (Schwinger scattering). In this case, as has already been indicated above, the potential of the interaction with a charge located at the point $\mathbf{R}_i$ has the form

$$V(\mathbf{r} - \mathbf{R}_i) = i\frac{Z\mu\hbar e}{mc}\sigma\left[\frac{\mathbf{r} - \mathbf{R}_i}{|\mathbf{r} - \mathbf{R}_i|^3}\nabla_r\right].$$

After the substitution of $V(\mathbf{r} - \mathbf{R}_i)$ into the expressions (8) and (9), one can obtain

$$\psi_{sc}(\mathbf{r}, t) = \frac{2m\alpha}{i\hbar^2}\sigma\left[\mathbf{k}\frac{\mathbf{r} - \mathbf{R}_i}{|\mathbf{r} - \mathbf{R}_i|}\right]\left\{\frac{e^{ik|\mathbf{r}-\mathbf{R}_i|}e^{i\mathbf{k}\mathbf{R}_i}}{k|\mathbf{r} - \mathbf{R}_i| - \mathbf{k}(\mathbf{r} - \mathbf{R}_i)}\right.$$
$$\times J\left(|\mathbf{r} - \mathbf{R}_i|\frac{\mathbf{k}}{k} - \mathbf{r}_0 + \mathbf{R}_i - \mathbf{v}t\right) \qquad (16)$$
$$-\frac{\mathbf{k}(\mathbf{r} - \mathbf{R}_i)[k^2\rho_0^2 + k^2(\mathbf{r} - \mathbf{R}_i)^2 - (\mathbf{k}, \mathbf{r} - \mathbf{R}_i)^2]^{1/2} + k^2|\mathbf{r} - \mathbf{R}_i|\rho_0}{[k^2(\mathbf{r} - \mathbf{R}_i)^2 - (\mathbf{k}, \mathbf{r} - \mathbf{R}_i)^2][k^2\rho_0^2 + k^2(\mathbf{r} - \mathbf{R}_i)^2 - (\mathbf{k}, \mathbf{r} - \mathbf{R}_i)^2]^{1/2}}$$
$$\left.\times e^{i\mathbf{k}\mathbf{r}} J(\mathbf{r} - \mathbf{r}_0 - \mathbf{v}t)\right\},$$

where $J(\boldsymbol{\rho})$—a sharp function of $\boldsymbol{\rho}$ in the vicinity of zero—characterizes the motion of the packet[4], $\alpha = Z\mu\hbar e/mc$, and $\rho_0 = |\mathbf{r} - \mathbf{v}t - \mathbf{R}_i|$.

Using the connection arising from J between $\mathbf{r}$, $\mathbf{r}_0$ and $\mathbf{v}t$, we rewrite ρ_0 in the form $\rho_0 = |\mathbf{r}_0 - \mathbf{R}_i|$. It follows from this that ρ_0 is the distance between the source and the scatterer.

Thus, in contrast to the expressions obtained in the theory of scattering by a short-range potential, the wave function (16) contains a nontrivial dependence on the distance ρ_0. We note here, however, that the standard integration over the packet of the expression (1) does not lead to a similar dependence. The expression (16) agrees with (1) only when $\rho_0 \gg |\mathbf{r} - \mathbf{R}_i|$, i.e., only in the case when the distance between the source and scatterer is much larger than the distance between the scatterer and the observation point. We draw attention also to the fact that the wave function (16) consists of two different—in their properties—addends: one addend contains a spherical wave whose phase depends on the coordinate of the scatterer, and the second addend contains a plane wave whose phase does not depend on the coordinate of the scatterer (such a dependence exists only in the amplitude of this wave). As a result, after averaging over the coordinates of the scatterer (we consider, for instance, scattering on atomic electrons), the first addend will contain the form factor of the system, and the second will not. The absence of dependence of the phase of the wave in the second addend on the coordinate of the scatterer leads to the vanishing of its contribution in elastic scattering on neutral bodies.

Indeed, the coefficient α in the expression (16) depends on the sign of the charge which scatters the particle with a magnetic moment. In the case of elastic scattering on a neutral atom, the wave function ψ_{sc} is a superposition of Z electron-scattered waves ψ_e and a nucleus-scattered wave ψ_n. Since for the nucleus the coefficient $\alpha_n = -Z\alpha_e$, the terms containing the plane wave $e^{i\mathbf{kr}}$ cancel out[1)] and we obtain a result similar to the result obtained in the theory of scattering by short-range potentials. This is understandable since the effective potential, which describes elastic scattering on a neutral body, is short-range.

It is interesting to note that for charged particles there exists an interaction which leads to the appearance of the above-considered anomalous terms in elastic scattering on neutral bodies. We have in mind an interaction of the form

$$V' = \beta r^{-3}[\mathbf{r}\mathbf{p}]\sigma,$$

where σ is the spin operator of the scatterer, $\beta = 2e\mu\hbar/mc$, μ is the magnetic moment of the scatterer (see, for example, the Breit Hamiltonian (3)). This interaction differs from the Schwinger interaction only in that α has been replaced by β and σ_n—by σ (this interaction virtually has the same nature as the Schwinger interaction, only in this case we are dealing with the interaction of the magnetic moment of the scatterer with the electric field of the impinging charged particle). Thus, by making the indicated substitution in (16), we obtain a wave function that describes scattering on the potential V′. Although the atom is not polarized, the wave functions from the various scatterers do not now cancel out since the σ_i's are spin operators of different electrons. We shall not also take the nucleus into consideration since its magnetic moment is much smaller than that of an electron.

Let us consider further and in more detail scattering with spin flip, since the wave scattered with spin flip does not interfere with the incident wave and, therefore, even in the region of small scattering angles it is possible, in principle, to separate the incident and scattered beams. The total cross section is given in this case by the expression

$$\sigma_{\uparrow\downarrow} = \frac{\int |\mathbf{j}_{sc}\mathbf{r}|\, r\, d\Omega}{|\mathbf{j}_{in}|},$$

where $\mathbf{j}_{sc}$ is the flux of the scattered particles whose scatterers have had their spins flipped, and $\mathbf{j}_{in}$ is the flux of the incident particles. Let the wavelength of a particle be much larger than the radius a of the atom (which, for simplicity, is assumed to possess a single electron), so that $ka \ll 1$. In this case the form factor of the atom is equal to one and the total scattering cross section can be written in the form (under the condition that $\varphi \gg 1/\sqrt{kr}$)

1)There is no such canceling out for inelastic Schwinger (Coulomb) scattering processes (see the general formula (10)).

$$\sigma_{\uparrow\downarrow} = \frac{4Z^2e^2\mu^2}{\hbar^2c^2}\Big[\ln\frac{8\gamma^3\sin^4\varphi\,(kr)^4}{\sqrt{\gamma^2+\sin^2\varphi}}\Big\{2\gamma^2+(1-\gamma^2)\sin^2\varphi + 2\gamma[\gamma^2+(1-\gamma^2)\sin^2\varphi-\sin^4\varphi]^{1/2}\Big\}^{-1} - \frac{1}{2}\sin^2\varphi - \frac{2\gamma}{\sqrt{\gamma^2+1}}\arcsin\frac{\gamma\cos\varphi-\sqrt{\gamma^2-\sin^2\varphi}}{\gamma^2+1} + 4C - 2\Big], \quad (17)$$

where $\gamma = r_0/r$, $\sin\varphi = \rho_\perp/r$, $C = 0.58$ is Euler's constant, and Z is the magnitude of the charge of the impinging particle. As can be seen, the total cross section $\sigma_{\uparrow\downarrow}$ depends on the relationship between the distances from the scattering center to the source r_0, to the detector r and the transverse dimensions $\rho_\perp$ of the packet. If $\gamma \gg \sin\varphi$ and $r \gg \rho_\perp$, then the expression for $\sigma_{\uparrow\downarrow}$ can be simplified:

$$\sigma_{\uparrow\downarrow} = \frac{16Z^2e^2\mu^2}{\hbar^2c^2}\ln k\rho_\perp.$$

Now let the opposite limiting condition be fulfilled, i.e., let $ka \gg 1$. In that case

$$\sigma_{\uparrow\downarrow} = \frac{4Z^2e^2\mu^2}{\hbar^2c^2}\Big[\ln\frac{\gamma^3\sin^4\varphi\, r^4}{2a^4\sqrt{\gamma^2+\sin^2\varphi}}\{2\gamma^2+(1-\gamma^2)\sin^2\varphi + [\gamma^2+(1-\gamma^2)\sin^2\varphi-\sin^4\varphi]^{1/2}\}^{-1} + 4C - \frac{1}{2}\sin^2\varphi - \frac{2\gamma}{\sqrt{\gamma^2+1}}\arcsin\frac{\gamma\cos\varphi-\sqrt{\gamma^2-\sin^2\varphi}}{\gamma^2+1}\Big] \quad (18)$$

and, for $\gamma \gg \sin\varphi$, $r \gg \rho_\perp$, the cross section is equal to

$$\sigma_{\uparrow\downarrow} = \frac{16Z^2e^2\mu^2}{\hbar^2c^2}\ln\frac{\rho_\perp}{a}.$$

The above-considered dependence of the cross section on the distances r and r_0 and the transverse dimension $\rho_\perp$ of the packet is due to the long-range character of the potential, with the result that at large distances from the scatterer, besides the outgoing spherical wave, there exists a scattered plane wave. It is the interference of these waves that leads to the above-discussed dependence of the cross section on r, r_0 and $\rho_\perp$. The effective cutoff of the potential which appears in the nonstationary theory also leads, in this theory, to finite total cross sections for long-range potentials. This is, generally speaking, understandable since, because of the finite dimension of the packet, the regions of the potential where the particle had not been do not make any contribution to the scattering.

It will, apparently, be most convenient to observe the above-indicated peculiarities of scattering by the potentials V and V′ by investigating the processes of polarization and depolarization of colliding particle beams.

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