INTERACTION BETWEEN AN ELECTRON BEAM AND NATURAL OSCILLATIONS IN A BOUNDED PLASMA

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The interaction between an electron beam and the natural oscillations of a homogeneous plasma layer is analyzed by using the collisionless kinetic equation. A correction to the distribution function is determined in the second order of perturbation theory with respect to electric field strength of the plasma oscillation. After averaging over time, the correction is found to oscillate with respect to velocity and coordinate. This phenomenon has apparently been observed recently.

A fine structure was recently observed experimentally in the distribution function of an electron beam interacting with a bounded plasma[1]. The purpose of the present communication is to attempt to explain theoretically the observed sequence of peaks in the energy distribution function of the beam.

A beam moving in a plasma is unstable under certain conditions against excitation of plasma waves with frequency

$$\omega = \omega_0[1 + {}^3/_2(kr_d)^2], \quad \omega_0 = (4\pi ne^2/m)^{1/2},$$
$$r_d = (T_e/4\pi ne^2)^{1/2},$$

where ω_0 is the plasma frequency, k the wave vector, and r_d the Debye radius (see, e.g.,[2]). In an unbounded plasma, oscillations with a continuous spectrum of wave vectors are produced. The existence of boundaries leads to the appearance of standing waves. In addition, owing to nonlinear effects, oscillations with frequencies $\omega = m\omega_0$ appear in the plasma.

Let us consider the interactions of a beam with natural oscillations in a homogeneous plasma layer of thickness L, bounded by conducting walls at x = 0 and x = L. The electric field of the standing plasma waves in such a layer is given by

$$E(x,t) = \sum_{n,m} E_{n,m} \sin kx \cos(\omega t + \varphi_{nm}), \tag{1}$$

where $\omega = m\omega_0$ if we neglect the dispersion of plasma waves at $kr_d \ll 1$, $k = n\pi/L$, n and m are integers, and φ_{nm} is the initial phase.

The kinetic equation for the beam electrons

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} - \frac{eE(x,t)}{m}\frac{\partial f}{\partial v} = 0$$

will be solved by perturbation theory, assuming the field E to be small. The first-order correction f_1 to the zeroth-approximation function f_0, corresponding to the boundary condition $f_1(x = x_0) = 0$, is equal to

$$f_1 = -\sum_{n,m}\frac{1}{2}\frac{eE_{n,m}}{m}\frac{\partial f_0}{\partial v}\left\{\frac{\cos(kx-\omega t-\varphi_{nm})}{kv-\omega} + \frac{\cos(kx+\omega t+\varphi_{nm})}{kv+\omega} - \frac{\cos[kx_0 - \omega(t-(x-x_0)/v) - \varphi_{nm}]}{kv-\omega} - \frac{\cos[kx_0 + \omega(t-(x-x_0)/v) + \varphi_{nm}]}{kv+\omega}\right\}. \tag{2}$$

In the second approximation we obtain the correction f_2 with the same boundary condition $f_2 = 0$ at $x = x_0$, and we average it over the random phases φ_{nm} 1). The averaged increment $\bar{f}_2$ is given by

$$\bar{f}_2 = -\frac{1}{v}\frac{\partial}{\partial v}\sum_{n,m}\frac{1}{4}\left(\frac{eE_{n,m}}{m}\right)^2\frac{\partial f_0}{\partial v}\int_{x_0}^{x}dx\int_0^{2\pi}\ldots\int_0^{2\pi}\prod_{n,m}\frac{d\varphi_{nm}}{2\pi}$$
$$\times[\sin(kx-\omega t-\varphi_{nm}) + \sin(kx+\omega t+\varphi_{nm})]$$
$$\times\left\{\frac{\cos(kx+\omega t+\varphi_{nm})}{kv+\omega} + \frac{\cos(kx-\omega t-\varphi_{nm})}{kv-\omega} - \frac{\cos[kx_0 - \omega(t-(x-x_0)/v) - \varphi_{nm}]}{kv-\omega} - \frac{\cos[kx_0 + \omega(t-(x-x_0)/v) + \varphi_{nm}]}{kv+\omega}\right\}. \tag{3}$$

After calculating the integrals we obtain

$$\bar{f}_2 = \frac{1}{4v}\sum_{n,m}\left(\frac{eE_{n,m}}{m}\right)^2\frac{\partial}{\partial v}\left(D_{nm}\frac{\partial f_0}{\partial v}\right), \tag{4}$$

where

$$D_{nm} = \frac{1}{v}\left\{\frac{\sin^2[{}^1/_2(k-\omega/v)(x-x_0)]}{(k-\omega/v)^2} + \frac{\sin^2[{}^1/_2(k+\omega/v)(x-x_0)]}{(k+\omega/v)^2} - \frac{2\cos k(x+x_0)}{(k-\omega/v)(k+\omega/v)}\sin\left[\frac{1}{2}\left(k-\frac{\omega}{v}\right)(x-x_0)\right] \times \sin\left[\frac{1}{2}\left(k+\frac{\omega}{v}\right)(x-x_0)\right]\right\}$$

The expression in the curly brackets in (5) is always non-negative. The oscillating structure of the distribution function (4) has a finite amplitude even when the resonance conditions $k = \omega/v$ are satisfied. If the boundary condition on the distribution function is specified at $x_0 = 0$ and the observation is carried out, for example, at x = L, then expression (5) takes the form

$$D = \frac{4}{k\omega}\frac{\xi}{1-\xi^2}\cos^2\frac{n\pi}{2}\xi,$$

where

$$\xi = \frac{\omega}{kv}, \quad \frac{\partial D}{\partial v} = -\frac{k}{\omega}\xi\left(n\pi\xi\,\mathrm{tg}\frac{n\pi}{2}\xi + 1 - 2\frac{1+\xi^2}{1-\xi^2}\right)D.$$

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1) On the other hand, if the phases of the oscillations are not random, but the measuring instrument has a large time constant $\tau \gg \omega_0^{-1}$, then all the formulas that follow remain in force, since the time averaging is equivalent in this case to averaging over the phases.

The points of the extrema of the function D(v) are determined by the roots of the equation

$$n\pi\,\mathrm{tg}\frac{n\pi}{2}\xi = \frac{1+3\xi^2}{\xi(1-\xi^2)}.$$

The amplitude of the peaks of the distribution function, which are connected with the oscillations of D(v), decreases at large ξ like ξ^{-1}, and of those connected with the oscillations of $\partial D/\partial v$ remains constant. If the zeroth distribution function $f_0(v)$ varies smoothly and slowly when the argument is changed by an amount on the order of the distance $L\omega_0$ between the peaks, then the principal term in (4) will be $(\partial D/\partial v)(\partial f_0/\partial v)$. The width of the peaks is determined by the same scale $L\omega_0$ as the distance between them. The amplitude of the oscillations of the distribution function contains an ordering parameter $(eEL/m\bar{v}^2)^2$, where v is the characteristic scale of variation of $f_0(v)$. This parameter must be small if perturbation theory is to be valid.

The phenomenon under consideration explains the main qualitative feature observed in the experiment[1], namely the fact that peaks appear only if microwave oscillations exist simultaneously. The observed shift of all the peaks towards higher energies with increasing energy and current of the beam is apparently connected with the concomitant increase of the plasma density and of the frequency ω_0. We note also that the character of the boundary conditions is not very important. A similar phenomenon should take place also in the case of dielectric walls.

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[1] S. M. Levitskiĭ and K. Z. Nuriev, ZhETF Pis. Red. 12, 172 (1970) [JETP Lett. 12, 119 (1970)].
[2] A. A. Vedenov, Teoriya turbulentnoĭ plazmy (Theory of Turbulent Plasma), VINITI, AN SSSR, 1965.

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