THE PEIERLS DYNAMIC FORCE

V. I. AL'SHITZ, V. L. INDENBOM, and A. A. SHTOL'BERG

Crystallography Institute, USSR Academy of Sciences

Submitted December 14, 1970

Zh. Eksp. Teor. Fiz. 60, 2308–2320 (June, 1971)

The self-consistent problem of dislocation motion in a periodic lattice field with allowance for reaction of the elastic self-field is investigated. The external stress τ required for maintaining stationary dislocation motion with a constant mean velocity v is found as the eigenvalue of equations determining the self-consistent law of motion of the dislocation. The equations are solved exactly for a piecewise-parabolic and approximately for a sinusoidal Peierls relief. In the general case, the dislocation emission spectrum consists of a broad range of frequencies which are multiples of the characteristic frequency of dislocation-field variation. Stationary motion is found to be possible only for velocities exceeding a certain critical velocity v_c. Calculation of the dependence $\tau(v)$ shows that radiative friction does not ensure stability of stationary dislocation motion at nonrelativistic velocities. Instability of motion can be removed by additionally taking into account viscous friction of phonon or electron origin. With growth of viscous friction the critical velocity decreases and the minimal Peierls dynamic force tends to the static value.

TRANSLATIONAL motion of a straight-line dislocation in a crystal causes a periodic change in the atomic structure of its nucleus, as a result of which the dislocation must overcome a certain energy relief called the Peierls relief.[1] Let the dislocation move along the x axis, for which the translation period is equal to a. Then the Peierls relief is described by the periodic function $U(x) = U(x + a)$, where U is the energy per unit length of the dislocation. The static resistance of the lattice to the motion of the dislocation is determined by the Peierls force $F_\pi = -(\partial U/\partial x)_{max}$, corresponding to a starting stress $\tau_\pi = b^{-1}F_\pi$, needed to start the motion of a dislocation with Burgers vector **b**. The stress τ_π is not needed to maintain the dislocation motion, since the work of the external forces is required in this case only to compensate for the losses of mechanical energy of the dislocation on going from vertex to vertex of the Peierls relief. The Peierls dynamic force thus differs in nature from the static force F_π, and reduces in fact to radiation friction due to nonuniformity of the dislocation motion over the Peierls relief.

Depending on the degree to which the presence of the relief perturbs the uniform motion of the dislocation, a change takes place not only in the value of the radiation friction, but also in the character of the radiation itself, namely, the larger the perturbation, the broader the frequency spectrum of the radiated elastic waves. In this sense, the assumption made in[2,3] that the entire radiation occurs in a single mode corresponding to the characteristic frequency of the change of the dislocation field $\omega_1 = 2\pi v/a$ (v—average velocity of the dislocation) is equivalent to the hypothesis that the perturbation is small; this hypothesis, as we shall show, is in general incorrect. Attempts[4-6] to investigate the problem with allowance for the entire spectrum of the radiation are specially of methodological character on two counts, since the crudeness of the employed models (modifications of the Frenkel-Kontorova model) and the absence of self-consistency do not make it possible to expect a correct quantitative description.

Another important aspect of the problem, which is usually ignored, is the very possibility of stationary dislocation motion at a given average velocity. For a rigorous solution of the problem of the dynamic Peierls force it is necessary to have a self-consistent determination of the law of motion of the dislocation in a periodic potential field with allowance for the reaction of the radiation. Such an approach was first indicated in[7], of which the present article is a development and generalization.

FORMULATION OF PROBLEM

Let a linear screw dislocation with Burgers vector $\mathbf{b} = (0, 0, b)$ overcome the Peierls relief under the influence of a constant external force f and move in the XZ plane in accordance with a law $x = x(t)$. The dynamic elastic field $\sigma_{ij}(x, y, t)$ generated by the dislocation acts on the dislocation with a force $F(t) = b\sigma_{yz}[x(t), 0, t]$, which plays the role of the effective inertia force (all the forces pertain to a unit length of the dislocation). The equation of motion of the dislocation is[1)]:

$$f = -F(t) + \frac{\partial}{\partial x}U[x(t)]. \tag{1}$$

In the stationary case, Eq. (1) determines both the law of dislocation motion x(t) and the possible values

1)In principle, the entire right-hand side of (1) is the dynamic self-action of the dislocation. Its subdivision into two terms is connected with separation of the function ∂U/∂x, which is periodic and x and is due to the structure of the nucleus of the dislocation and the translational symmetry of the crystal. This part of the self-action cannot be calculated by elasticity theory, unlike the first term, which depends logarithmically on the dimension of the nucleus of the dislocation (see (5)), and for which, therefore, a continual estimate is sufficient. We shall later substitute phenomenological expressions for the static Peierls relief in (1) for U(x), neglecting the dynamic corrections, which are small so long as the dislocation velocity is $v \ll (\tau_\pi/\mu)^{1/4}c_t$ (see, for example, the estimate in [8]).

of the force f as eigenvalues that admit of periodic solutions for the function $\dot{x}(t)$ at a given average velocity $\overline{\dot{x}} = v$. In this sense, the problem is self-consistent.

Using the electromagnetic analogy indicated by Eshelby[10] we can treat this as a problem of self-consistent one-dimensional motion of a charged filament in its own electric field $E_x(x, y, t)$ and in a certain time-constant external periodic field $E_x(x) = E_x(x + a)$. According to[8]

$$F(t) = -\frac{\mu b^2}{4\pi c_t^2}\left\{\int_{-\infty}^{t} \frac{d^2x(t')/dt'^2}{[(t-t')^2 + r_0^2/c_t^2]^{1/2}}\,dt' + \frac{1}{2}\int_{-\infty}\left[(t-t')^2 + \frac{r_0^2}{c_t^2}\right]^{-3/2}\frac{r_0^2}{c_t^2}\frac{d}{dt'}\frac{x(t)-x(t')}{t-t'}\,dt'\right\} + O\left(\frac{V^2}{c_t^2}\right). \quad (2)$$

Here μ is the shear modulus, c_t the velocity of the shear waves, and $V(t) = \max\{\dot{x}(t')\}$ at $-\infty < t' < t$. The divergence of the elastic field on the dislocation axis is eliminated by smearing the nucleus of the dislocation over a region of radius $r_0 \sim a$.

In the case of stationary motion, the velocity of the dislocation oscillates about a mean value v with a period $T = 2\pi/\omega_1$, and these oscillations are not sinusoidal even for a sinusoidal Peierls relief. Taking into account, unlike in[2,3], all the harmonics $\omega_n = \omega_1 n$ and the law of dislocation motion, we have

$$x(t) = vt + \sum_{n=-\infty}^{\infty} \xi_n e^{i\omega_n t}. \quad (3)$$

The time is reckoned in such a way that at t = 0 the dislocation is at the vertex of the relief. Substitution of (3) in (2) gives for non-relativistic velocities $v \ll c_t$ gives

$$F(t) = \sum_{n=-\infty}^{\infty} \xi_n e^{i\omega_n t}\omega_n^2 M(\omega_n), \quad (4)$$

where the effective mass $M(\omega_n)$ is given, accurate to terms of order $\omega_{\bar{n}} r_0/c_t$, by the relation[2)]

$$M(\omega_n) = \frac{\mu b^2}{4\pi c_t^2}\left[\ln\frac{\gamma c_t}{\omega_n r_0} - i\frac{\pi}{2}\operatorname{sign}(\omega_n)\right]. \quad (5)$$

Here $\bar{n}$ is the number of harmonics that make the main contribution to expressions (3) and (4), and γ is the numerical factor of the order of unity (according to[8], $\gamma \approx 1.12$). The real and imaginary parts of $M(\omega)$ are connected by the usual Kramers-Kronig dispersion relations.

2. SOLUTION OF THE PROBLEM FOR A PIECEWISE-PARABOLIC PEIERLS RELIEF

If the Peierls relief is approximated by the system of parabolas

$$U(x) = F_\pi\frac{(x-na)^2}{a}, \quad -\frac{a}{2} \leqslant x - na \leqslant \frac{a}{2}, \quad n = 0, \pm 1, \pm 2, \ldots \quad (6)$$

then Eq. (1) is piecewise-linear and admits of an exact solution for motions such that the dislocation is situated within the limits of one relief valley during the time of one period:

$$0 \leqslant t \leqslant a/v, \quad -a/2 \leqslant x \leqslant a/2. \quad (7)$$

It will be shown below that this condition is satisfied at dislocation velocities v that exceed a certain critical velocity v_c. When $v < v_c$, Eq. (1) has no stationary solutions (see the Appendix) and consequently v_c is the minimum possible average velocity of the stationary motion of the dislocation.

Thus, let $v > v_c$ and $0 \le t \le a/v$. We expand the first term of expression (3) in a Fourier series on the segment $[0, a/v]$. Substituting the results, together with formulas (4) and (6), in Eq. (1) and equating the coefficients of like harmonics, we get

$$\xi_k = \frac{ia}{2\pi k}\Big/\left(-1 + \frac{\omega_k^2 a}{2\tau_\pi b}M(\omega_k)\right),$$
$$\xi_0 = -\frac{a}{2} - \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty}\xi_k, \quad f = -2F_\pi\sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty}\frac{\xi_k}{a}, \quad (8)$$

and consequently

$$\tau = \frac{2}{\pi}\tau_\pi\sum_{n=1}^{\infty}\frac{\nu n}{(1-\delta_n n^2)^2 + \nu^2 n^4}, \quad (9)$$

$$x(t) = vt - \frac{a}{2} + \frac{2}{\pi}a\sum_{n=1}^{\infty}\left[\frac{(1-\delta_n n^2)\cos(\omega_n t/2) + \nu n^2\sin(\omega_n t/2)}{(1-\delta_n n^2)^2 + \nu^2 n^4}\right]\frac{\sin(\omega_n t/2)}{n}. \quad (10)$$

Here $\tau = f/b$ is the radiation friction (the Peierls dynamic stress),

$$\delta_n = \frac{\pi}{2}\frac{\mu}{\tau_\pi}\frac{v^2}{c_t^2}\frac{b}{a}\ln\frac{\gamma c_t}{\omega_n r_0}, \quad \nu = \frac{\pi^2}{4}\frac{\mu}{\tau_\pi}\frac{v^2}{c_t^2}\frac{b}{a}.$$

Neglecting in (9) and (10) the logarithmic dependence of δ_n on n, we can use the Poisson summation formula to transform (9) and (10) into

$$\tau = \tau_\pi\left\{\frac{\operatorname{sh}2\pi\beta''}{\operatorname{ch}2\pi\beta'' - \cos 2\pi\beta'} + \frac{\beta''}{\pi|\beta|^2} - \frac{2}{\pi}\operatorname{Im}\psi(\beta)\right\}, \quad (11)$$

$$x(t) = \frac{a}{2}\left\{\frac{e^{2\pi\beta''}(1 - e^{-2\pi\beta'' y}\cos 2\pi\beta' y) + e^{-2\pi\beta'' y}\cos[2\pi\beta'(1-y)] - \operatorname{ch}2\pi\beta''}{\operatorname{ch}2\pi\beta'' - \cos 2\pi\beta'} - \frac{4\beta''}{\pi}\sum_{k=1}^{\infty}\frac{\sin^2\pi k(1-y)}{(k+\beta')^2 + (\beta'')^2}\right\}, \quad (12)$$

where

$$\beta = \beta' + i\beta'' = \left(\frac{\sqrt{\delta_{\bar{n}}^2 + \nu^2} + \delta_{\bar{n}}}{2(\delta_{\bar{n}}^2 + \nu^2)}\right)^{1/2} + i\left(\frac{\sqrt{\delta_{\bar{n}}^2 + \nu^2} - \delta_{\bar{n}}}{2(\delta_{\bar{n}}^2 + \nu^2)}\right)^{1/2} \approx \approx \frac{1}{\delta_{\bar{n}}^{1/2}}\left(1 + i\frac{\varepsilon}{2}\right),$$

$$\varepsilon = \pi/2\ln\frac{\gamma c_t}{\omega_{\bar{n}} r_0}, \quad \psi(x) = \frac{d}{dx}\ln\Gamma(x), \quad y = \frac{vt}{a}, \quad 0 \leqslant y \leqslant 1.$$

It is seen from (10) that at sufficiently high velocities, when $\delta_{\bar{n}} \gg 1$, the deviation of the dislocation motion from uniform motion is small and condition (7) is certainly satisfied.

On the other hand, it is easy to verify that at low velocities this condition is violated, and the dislocation goes over at a certain instant $t < a/v$ into the neighboring well of the relief. For example, at $\delta_{\bar{n}} < 1$ the law of dislocation motion is described, accurate to terms of order $\epsilon\delta_{\bar{n}}^{1/2}$, not by (12) but by the function

$$x(t) \approx {}^1\!/\!_2 a(1 - 2e^{-2\pi\beta'' y}\cos 2\pi\beta' y), \quad (13)$$

which has at $0 < y < 1$ a range of variation exceeding the interval $(-a/2, +a/2)$.

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2) It follows from (4) and (5) that the radiative deceleration of the dislocation does not reduce to a differential dependence on the law of motion (owing to the factor sign ω_n), i.e., it is not local in the time, unlike the dynamic friction of a pointlike elastic-field source—an inflection on the dislocation, which, just as in the case of an electric point charge, is proportional to $\dddot{x}$ (see [10]).

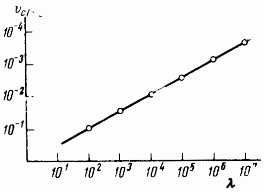

FIG. 1. Dependence of the critical velocity v_c on the height of the Peierls relief $\lambda = \mu/\tau_\pi$.

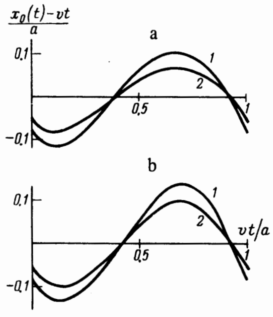

FIG. 2. Deviation of the dislocation motion from linearity for parabolic (a) and sinusoidal (b) reliefs. $x_0(t) = x(t-\xi_0/v)$; $\lambda = 10^4$; curves 1—$v = 8.34\cdot 10^{-3}\, c_t$, 2—$v = 10^{-2}\, c_t$.

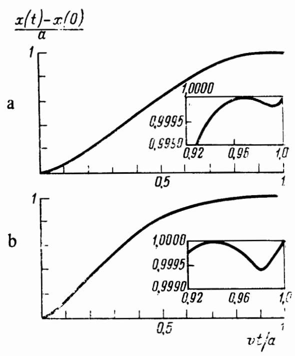

FIG. 3. Law of dislocation motion at $v = v_c$ for a parabolic relief: $\lambda = 10^4$. a—$\alpha = 0$, $v/c_t = 8.34 \times 10^{-3}$; b—$\alpha = 1$, $v/c_t = 4.79 \times 10^{-3}$.

A numerical analysis of expression (10), carried out with the aid of a computer, has shown the existence of one critical velocity v_c, above which the solutions (9)—(12) are valid, and below which, as already mentioned, Eq. (1) has no stationary solutions at all (see the Appendix). Figure 1 shows the numerically-obtained dependence of the critical velocity v_c on the height of the Peierls relief[3]. Attention is called to the high values of v_c: for a relief with $\lambda = \mu/\tau_\pi \sim 10^4$—$10^2$ we have $v_c \sim (10^{-2}$—$10^{-1})c_t$, which is at the limit of the velocities attainable by dislocations.

Figures 2a and 2b show the oscillating part of the dislocation motion over parabolic and sinusoidal reliefs at $\lambda = 10^4$ for two velocities, $v = 10^{-2}\, c_t$ and $v = v_c = 8.34 \times 10^{-3}\, c_t$. We see that with decreasing average velocity v the degree of nonuniformity of the dislocation motion increases strongly.

It is interesting to note that at velocities v close to v_c, the dislocation executes a small-scale backward motion before going over to the neighboring valley of the potential relief (Fig. 3). The existence of the return motion in the near-critical situation follows analytically from the positiveness (at any v, including $v = v_c$) of the instantaneous dislocation velocity at the instant of transition $t = a/v$, as can easily be verified by summing term by term the differentiated series (10)[4].

[3] In all the numerical calculations it was assumed that $b = a = r_0$.

[4] Calculation of $\dot{x}(a/v)$ by formula (12) requires caution, in view of the fact that term-by-term differentiation of the series (12) is incorrect at $y = 1$.

This singularity is a reflection of the nonlocal character of the effective mass of the dislocation (see (2)) and, as shown by an additional investigation, is not connected with the approximations made on going over from formula (2) to expressions (4) and (5).

The dynamic Peierls stress τ in the high-velocity region ($\delta_{\overline{n}} \gg 1$) can be calculated from formula (9), provided we neglect in it the logarithmic dependence of δ_n on n, and replace $1 - \delta_n n^2$ by $-\delta_{\overline{n}} n^2$, which yields, accurate to terms of order $\ln^{-3}(\gamma c_t/\omega_{\overline{n}} r_0)$,

$$\tau \approx \tau_\pi \Big/ \frac{\pi}{2.4}\frac{\mu}{\tau_\pi}\frac{v^2}{c_t^2}\frac{b}{a}\left[\ln^2\frac{\gamma c_t}{\omega_{\overline{n}} r_0} + \frac{\pi^2}{4}\right]. \qquad (14)$$

The fact that expression (14) is 1.2 times larger than the first term of the expansion (8) indicates that the radiation of the dislocation is essentially not of the single-mode type, even at high velocities.

3. SOLUTION OF THE LINEARIZED PROBLEM FOR A SINUSOIDAL PEIERLS RELIEF

Compared with the piecewise-linear approximation of the Peierls relief which was considered above, the sinusoidal relief

$$U(x) = \frac{F_\pi a}{\pi}\sin^2\pi\frac{x}{a} \qquad (15)$$

is much closer to reality. Substitution of the relief (15) in (1) yields the nonlinear equation

$$-\sum_{n=-\infty}^{\infty}\xi_n e^{i\omega_n t}\omega_n^2 M(\omega_n) = f - F_\pi \sin\left[\frac{2\pi}{a}\left(vt + \sum_{n=-\infty}^{\infty}\xi_n e^{i\omega_n t}\right)\right]. \qquad (16)$$

This equation can be linearized if the dislocation motion (3) differs little from linear. Expanding the sine function in the right-hand side of (16) in powers of

$$\varphi(t) = \frac{2\pi}{a}\sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty}\xi_k e^{i\omega_k t}$$

and neglecting in this expansion the terms of order $|\varphi(t)|^2$, we obtain a linear equation corresponding to a system of linear difference equations for the amplitudes ξ_n:

$$\begin{gathered}\alpha_1\xi_1 + \xi_2 e^{-i\varkappa} = iae^{i\varkappa}/2\pi,\\ \alpha_n\xi_n + \xi_{n-1}e^{i\varkappa} + \xi_{n+1}e^{-i\varkappa} = 0, \quad n \geqslant 2;\end{gathered} \qquad (17)$$

$$f = 2\pi F_\pi \operatorname{Re}\{\xi_1 e^{-i\varkappa}/a\}. \qquad (18)$$

We have introduced here the notation

$$\alpha_n = \frac{2}{\pi}n^2(-\delta_n + i\nu), \quad \varkappa = 2\pi\frac{\xi_0}{a}. \qquad (19)$$

The solution of the infinite system (17) can be represented in the form

$$\xi_k = (-1)^{k+1}e^{i\varkappa k}\frac{ia}{2\pi}\lim_{n\to\infty}\frac{\lambda_n^{k+1}}{\lambda_n^1}, \qquad (20)$$

where λ_n^k is the determinant of the matrix, in which the diagonal elements form the sequence α_k, $\alpha_{k+1}, \ldots, \alpha_n$, the near-diagonal elements are each equal to 1, and the remaining elements are equal to zero.

The coefficient ξ_0 is determined by the initial condition $x(0) = -a/2$, which is equivalent to the equation $x_0(\xi_0/v) = -a/2$. Here $x_0(t)$ is a function specified by formulas (3) and (20), in which we put $\xi_0 = 0$. The law of motion x(t) is obtained from $x_0(t)$ by a simple time shift:

$$x(t) = x_0(t + \xi_0 / v). \qquad (21)$$

It is easy to verify the validity of the recurrence relation:

$$\lambda_n^k = \alpha_k \lambda_n^{k+1} - \lambda_n^{k+2}, \qquad (22)$$

with the aid of which we can express ξ_1 in terms of a continued fraction

$$\xi_1 = \frac{ia}{2\pi} e^{ix} \cfrac{1}{\alpha_1 - \cfrac{1}{\alpha_2 - \cfrac{1}{\dots - \cfrac{1}{\alpha_n - \cdots}}}} \qquad (23)$$

Knowing ξ_1, we can determine the coefficients ξ_n from the recurrence equations (17).

We obtain a formula for the dynamic Peierls force by substituting (23) in (18) and changing over, as in the preceding section, from forces to stresses:

$$\tau = \tau_\pi \operatorname{Im} \left\{ \cfrac{-1}{\alpha_1 + \cfrac{-1}{\alpha_2 + \cfrac{-1}{\dots + \cfrac{-1}{\alpha_n + \cdots}}}} \right\}. \qquad (24)$$

A numerical analysis of the law of motion x(t) shows that, just as on a parabolic relief, at high velocities ($\delta_1 \gg 1$) the deviation of the dislocation velocity from uniformity is small (curve 2 on Fig. 2b). The asymptotic behavior of the dynamic Peierls force, which follows from formula (24), differs in this case from (14) only by a numerical factor on the order of unity

$$\tau \approx \tau_\pi \Big/ \frac{2}{\pi} \frac{\mu}{\tau_\pi} \frac{v^2}{c_t^2} \frac{b}{a} \left[\ln^2 \frac{\gamma c_t}{\omega_1 r_0} + \frac{\pi^2}{4} \right]. \qquad (25)$$

With decreasing average velocity v, the degree of nonuniformity of the dislocation motion increases, the amplitude of the oscillations of the function $\varphi(t)$ increases, and the reliability of the linearization becomes worse. The oscillating part of the law of dislocation motion, shown in Fig. 2b, differs from the function $\varphi(t)$ by a factor $1/2\pi$. Curve 1, plotted for the velocity that is critical for the parabolic relief, yields for $\varphi(t)$ an amplitude close to unity. Thus, within the framework of the linearized problem, the motion of the dislocation over a sinusoidal relief can be investigated only at high velocities, when $\delta_1 \gg 1$. Violation of the linearization condition ($|\varphi(t)| \ll 1$) hinders this approach to the investigation of phenomena of the critical-velocity type.

4. DYNAMIC PEIERLS FORCE UNDER VISCOUS-DISSIPATION CONDITIONS

The results obtained above were based on the assumption that the radiation of the elastic waves is the only dissipative process slowing down the dislocation motion. It is natural to extend the problem to the case when the dislocation is acted upon, in addition, by some viscous-friction force $f_{fr} = -B\dot{x}$ (B is the braking coefficient). An example may be friction of phonon or electron origin. In particular, the inequality $a\omega_{\bar{n}}/c_t \ll 1$, which we have assumed to be satisfied, is at the same time the condition for the smallness of the phonon relaxation time (at temperatures on the order of the Debye temperature and above) compared

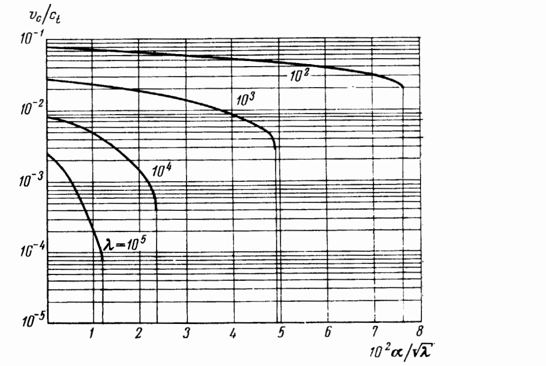

FIG. 4. Dependence of the critical velocity v_c on the viscosity α and on the height of the parabolic relief λ.

with the characteristic time of dislocation motion.

The necessary generalization of the equation of motion of the dislocation is attained by adding to the left side of (1) the additional force f_{fr}. The obtained equation can be investigated in analogy with the foregoing analysis of Eq. (1). The law of motion is obtained from (10) for a parabolic relief and from (3) and (20) for a sinusoidal relief, by replacing ν in them by $\nu + 2\alpha\sqrt{\nu}/n$, where $\alpha = (Bc_l/b\mu)\sqrt{\lambda a/b}$ (for typical values $B \sim 10^{-4}$ P, $c_t \sim 10^5$ cm/sec, $b \sim a \sim 10^{-8}$ cm, $\mu \sim 10^{11}$ dyne/cm^2, and $\lambda \sim 10^{+4}$ we have $\alpha \sim 1$). The eigenvalue τ, i.e., the dynamic Peierls stress, is made up of the radiative and viscous stresses

$$\tau(v) = \tau_{rad}(v) + b^{-1}Bv, \qquad (26)$$

with the radiative stress $\tau_{rad}(v)$ determined by formulas (9) and (24) with the already indicated renormalization of the parameter ν.

In this case the spectrum of the allowed velocities for the parabolic relief is also bounded from below, and the critical velocity decreases with increasing viscosity (Fig. 4). It is easy to verify, however, that there exists a certain limiting $\alpha = \alpha_c$, above which there is no critical velocity. In fact, summing the series (9) and (10) with $\alpha\sqrt{\epsilon} > 1$, $\delta_n = \delta_{\bar{n}}$, and with renormalized ν, we obtain a law of motion satisfying the condition (7):

$$x(t) = -\frac{a}{2} + a\frac{1 - \exp\{-2F_\pi y/Bv\}}{1 - \exp\{-2F_\pi/Bv\}} + \frac{a}{8\alpha^2\varepsilon} \frac{\operatorname{ch}\{2\pi^2 Bv(y - 1/2)/\delta_{\bar{n}}F_\pi\} - \operatorname{ch}\{\pi^2 Bv/\delta_{\bar{n}}F_\pi\}}{\operatorname{sh}\{\pi^2 Bv/\delta_{\bar{n}}F_\pi\}} + O\left\{\frac{1}{(2\alpha\sqrt{\varepsilon})^4}\right\} \qquad (27)$$

and the dynamic Peierls stress

$$\tau = \tau_\pi \left\{ \operatorname{cth}\frac{F_\pi}{Bv} - \frac{1}{4\alpha^2\varepsilon} \operatorname{cth}\frac{\pi^2 Bv}{\delta_{\bar{n}}F_\pi} \right\} + O\left\{\frac{1}{(2\alpha\sqrt{\varepsilon})^4}\right\}. \qquad (28)$$

Formulas (27) and (28), obtained in the approximation $\sqrt{\nu} \ll \alpha$, are not violated[5)] at sufficiently large α up to the velocity region corresponding to $\delta_{\bar{n}} \gg 1$. But at such high velocities, as we have already seen, the degree of nonuniformity of motion of the dislocation is small, and satisfaction of condition (7) is ensured. Thus, from the existence of a critical velocity at $\alpha = 0$ and from the absence of such a velocity at

[5)] It should be noted that the inequality $\alpha\sqrt{\epsilon} > 1$ actually does not limit the velocity from below, since at low velocities $n \sim \delta_{\bar{n}}^{-1/2}$ and $\omega_{\bar{n}}$ and consequently ϵ, are independent of the velocity.

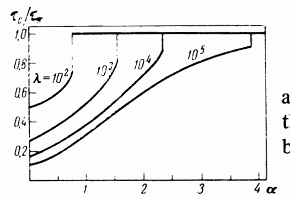

FIG. 5. Maximum values of the radiative stress $\tau_c = \tau_{rad}(v_c)$ as a function of the viscosity α and the height of the parabolic relief λ.

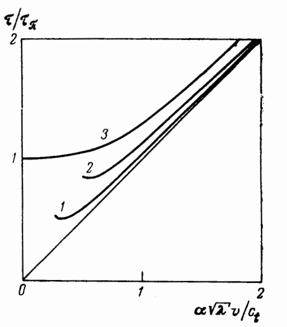

FIG. 6. Dynamic Peierls stress for a parabolic relief as a function of the velocity v and of the viscosity α. $\lambda = 10^4$. Curves 1—α = 0.4; 2—α = 1.0; 3—α = 3.0 $> \alpha_c$.

$\alpha \gg \epsilon^{-1/2}$ follows the existence of a certain $\alpha_c \sim \epsilon^{-1/2}$, which separates these regions. A numerical analysis has confirmed this conclusion. Figure 4 shows the numerically obtained dependence of the critical velocity v_2 on the viscosity α and on the height λ of the Peierls relief.

An analysis of the $\tau_{rad}(v)$ dependence shows that at high velocities ($\sqrt{\nu} \gg \alpha$) the viscosity does not come into play in the radiative deceleration and $\tau_{rad}(v)$ is determined by the asymptotic formulas (14) and (25). With increasing velocity v, the viscosity exerts an ever-increasing influence on the character of the radiative stress. Thus, at $\alpha > \alpha_c$ in the velocity region $F_\pi/B < v < c_t\sqrt{\alpha/\lambda}$, according to (28), the radiative stress decreases like $1/v$:

$$\tau_{rad} \approx \tau_\pi F_\pi / 3Bv. \tag{29}$$

Figure 5 shows the maximum radiative stress $\tau_c = \tau_{rad}(v_c)$ for a parabolic Peierls relief as a function of the viscosity and of the height of the relief. When $\alpha > \alpha_c$, the maximum radiative stress is attained at v = 0 and, as can be seen from formulas (26) and (28), $\tau_c = \tau_{rad}(0) \approx \tau_\pi$. On the other hand, allowance for the radiative losses greatly distorts the linear dependence of the stress on the velocity which is customary for viscosity. Figure 6 shows the stress $\tau(v)$ obtained numerically for $\lambda = 10^4$ and α = 0.4, 1, and 3. It should be noted that, in accord with formula (28), when $\alpha = 3 > \alpha_c$ the dynamic Peierls force is practically constant in a wide velocity interval ($0-10^{-3}\, c_t$), and is close to the static value, i.e., an effect of the dry-friction type is observed.

Finally, it is seen from Fig. 3 that when $\alpha < \alpha_c$ and $v \approx v_c$, there remains a small return motion of the dislocation prior to the transition to the neighboring valley of the relief.

5. DISCUSSION

Stationary motion of a dislocation in a periodic potential Peierls relief, investigated by a self-consistent method for a piecewise-parabolic and in part for a sinusoidal relief, reveals a number of common properties that apparently do not depend on the concrete form of the relief.

At high velocities, when the kinetic energy of the dislocation greatly exceeds the Peierls energy ($\delta_1 \gg 1$), the potential relief exerts only a weak perturbing influence on the dislocation motion, and the fundamental harmonic ω_1 predominates in the radiation (the contribution of the higher harmonics to the dissipation, however, amounts, according to (14), to not less than 20% of the contribution of the fundamental harmonic). In this velocity region, the radiative friction is small and decreases with increasing velocity like v^{-2} (see (14), (25)). With decreasing velocity v, the degree of nonuniformity of the dislocation motion increases (see Fig. 2), and accordingly the radiative losses increase, the radiation of the higher harmonics becoming more and more effective. Unfortunately, at the same time the problem for the sinusoidal relief becomes less amenable to linearization, and therefore an investigation of low-velocity effects has been carried out only for a parabolic relief.

In the case of a parabolic relief, a decrease of the velocity is possible down to a certain critical velocity v_c, below which no stationary motion is realized. This is connected with the absence of large-scale backward displacements of the dislocations during the course of motion, although small-scale oscillations do take place at velocities close to v_c (Fig. 3). We note that the existence of the critical velocity is in itself not peculiar to the dislocation. The presence of such a critical velocity is characteristic, for example, of stationary motion of a small sphere on a wavy surface. The minimum average velocity corresponds in this case to motion with zero kinetic energy on the top of the relief. The fact that this peculiarity of the dislocation motion escaped notice in [2-6] is connected with the non-self-consistent formulation of the problems considered in these papers. In [7], the phenomenon of critical velocity was not observed, in spite of the self-consistent approach to the solution of the problem, since it was not noted that conditions (7) are violated at low dislocation velocities.

The phenomenon of the existence of the critical velocity remains in force also under conditions of viscous dissipation but, as shown by an exact solution of the problem for a parabolic relief, at not too high viscosity values. When the viscosity exceeds a certain limiting value α_c, the stationary motion is realized at all velocities v. As expected, the critical velocity v_c decreases with increasing viscosity (Fig. 4) and increases noticeably with height of the relief (Figs. 1 and 4). The radiation stress assumes a maximum value at $v = v_c$, and it is seen from Fig. 5 that for the cases in question $\tau_{rad}(v_c)$ lies in the range from 0.1 τ_π to τ_π.

Without allowance for viscosity, the Peierls dynamic force reduces to radiative deceleration τ_{rad} and is characterized by a decreasing function of the velocity, corresponding to instability of the stationary motion of the dislocation. Viscous dissipation adds to this force a term linear in the velocity, so that the total Peierls dynamic stress (26) for not too high a viscosity goes through a minimum at a certain velocity v_0, and increases linearly at higher velocities (see, for example, curves 1 and 2 on Fig. 6). The stationary motion of the dislocation is stable only if $v > v_0$. v_0 decreases with increasing viscosity, and at $\alpha > \alpha_c' > \alpha_c$ the minimum on the $\tau(v)$ curve vanishes, and the stationary motion of the dislocation becomes stable at all the velocities v. In a wide interval of velocities $0 < v < F_\pi/B$, the Peierls dynamic force is close to static (see formula (28) and curve 3 of Fig. 6) and does not vanish as $v \to 0$—this is an example of dynamic "dry friction."

The investigated problem is to a considerable degree model-dependent, since at sufficiently high dislocation velocities ($V/c_t \gtrsim (\tau_\pi/\mu)^{1/4}$) the quasistatic approach used by us in writing down Eq. (1) is no longer applicable and it is necessary to take into account the dynamic properties of the atomic configuration of the dislocation nucleus. In addition, we ignored in the solution the flexibility of the dislocation and the effect of ejection of double inflections. It should be noted, however, that an analogous character should be possessed also by motion of an inflection on the secondary Peierls relief, which frequently determines the mobility of the dislocation under practical conditions.

In conclusion, the authors are deeply grateful to V. M. Chernov for great help during the preparation of the article.

APPENDIX

We assume that a stationary solution of (1) exists when $v < v_c$. Then, as shown in Sec. 2, the dislocation goes out from the region $[-a/2, a/2]$ during the period $(0 < t < a/v)$. In this case Eq. (1) takes the form

$$-\sum_{k=-\infty}^{\infty}\xi_k e^{i\omega_k t}\omega_k^2 M(\omega_k) = f - 2F_\pi\frac{x}{a} + 2F_\pi\Big[\sum_{k=1}^{\infty}\eta\Big(x+\frac{a}{2}-ka\Big) - \sum_{k=0}^{\infty}\eta\Big(-x-\frac{a}{2}-ka\Big)\Big], \qquad (A.1)$$

where

$$\eta(z) = \begin{cases} 1, & z > 0, \\ 0, & z \leqslant 0. \end{cases}$$

Expanding the right-hand side of (A.1) in a Fourier series on the segment $[0, a/v]$ and equating coefficients of like harmonics, we express the law of motion x(t) in terms of the known function $x^{(0)}$, which is defined by formula (10):

$$x(t) = x^{(0)}(t) + \sum_{m,k}[x^{(0)}(t-t_{m,k}) - x^{(0)}(-t_{m,k})]\theta\{\dot{x}(t_{m,k})\}. \qquad (A.2)$$

Here $t_{m,k}$ is the instant of the m-th transition through the k-th vertex of the Peierls relief (see Fig. 7). The values of $t_{m,k}$ (m = 1, 2, ...) are determined by the equations

$$x(t_{m,k}) - x(0) = ka, \quad k = 0, \pm 1, \pm 2, \ldots \qquad (A.3)$$

The ends of the temporal interval (0, a/v) enter in the series $t_{m,k}$ only if the function x(t) approaches the corresponding limiting value x(0) or x(a/v) from the outside of the interval $[-a/2, a/2]$. The function $\theta(x)$ is defined as

$$\theta(x) = \begin{cases} 1, & x > 0 \\ -1, & x < 0, \end{cases}$$

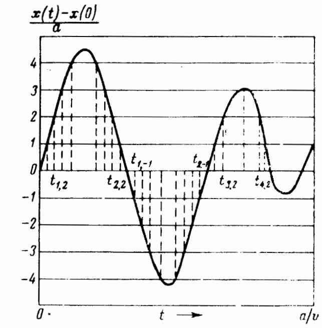

FIG. 7. Illustrating the determination of the instants t_m, k of passage of a dislocation through the vertex of the Peierls relief.

The value of $\theta(0)$ is not uniquely defined. If $t_{m,k} \in (0, a/v)$ is a point of maximum or minimum of the function x(t), then $\theta\{\dot{x}(t_{m,k})\} = 0$. If $t_{m,k} \in (0, a/v)$ is an inflection point, then $\theta\{\dot{x}(t_{m,k})\} = \theta\{x(t_{m,k} + \Delta t) - x(t_{m,k} - \Delta t)\}$. If the extremum point coincides with one of the ends of the interval (0, a/v), then $\theta\{\dot{x}(t_{m,k})\} = -1$.

Thus, the problem of the existence of a stationary solution of Eq. (A.1) reduces to the problem of the existence of a solution of Eq. (A.3). Equation (A.3) was investigated numerically by successive approximations. The transition from the n-th approximation to the (n + 1)-st was by means of the formulas

$$x^{(n+1)}(t) = x^{(0)}(t) + \sum_{m,k}[x^{(0)}(t-t_{m,k}^{(n)}) - x^{(0)}(-t_{m,k}^{(n)})]\theta\{\dot{x}(t_{m,k}^{(n)})\}, \qquad (A.4)$$

where $t_{m,k}^{(n)}$ are the solutions of Eqs. (A.3) in the n-th approximation.

A numerical analysis has shown that when $v < v_c$ the iteration process (A.4) diverges, and consequently Eq. (A.1) has no stationary solutions. The value of v_c itself was found as the largest root of the system of equations

$$x^{(0)}(t, v) - x^{(0)}(0, v) = a, \quad \dot{x}^{(0)}(t, v) = 0. \qquad (A.5)$$

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Translated by J. G. Adashko
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