GENERATION OF NONLINEAR ACOUSTIC WAVES IN STIMULATED MANDEL'SHTAM-BRILLOUIN SCATTERING IN A PLASMA

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Submitted October 19, 1970

Zh. Eksp. Teor. Fiz. 60, 1696–1701 (May, 1971)

We consider the stimulated Mandel'shtam-Brillouin scattering (SMBS) in a plasma that admits of the quasi-dynamic description. Assuming that the damping of the sound is not too large, we investigate the stationary processes in the layer. The dependence of the amplitude and of the waveform of the generated sound pulses on the intensity of the incident electromagnetic wave and their variation in the interior of the layer are determined. The question of the use of SMBS for plasma diagnostics is discussed. Corresponding estimates are presented for a laboratory plasma.

In the investigation of SMBS in nonlinear media, one usually considers the case when the damping of the sound is quite large, and therefore the acoustic wave produced upon scattering turns out to be practically sinusoidal, that is, the case when the hydrodynamic (or elastic) nonlinearity does not appear.[1,2] On the other hand, if the damping of the sound is very small (for example, in crystals at sufficiently low temperatures[3]), then as a result of the action of this nonlinearity and of the very weak dispersion for the acoustic branch, the energy of the initially harmonic wave generated during the SMBS is transferred upward through the spectrum and deformation of the wave profile takes place.[1)] The result is apparently the possibility of formation of a periodic shock wave of sound. Approximate estimates of the characteristic distances over which such a wave deformation can be attained in crystals are given in [5].

For many cases of practical interest, however, none of the foregoing approximations is valid for SMBS in a plasma. The reason is, on the one hand, that the damping is not so large that allowance for it permits one to ignore the appearance of the acoustic nonlinearity, and on the other hand one cannot neglect the damping and assume beforehand that the wave is a shock wave, for in this case only the first two harmonics take part effectively in the formation of the wave.

In the present paper we consider the interaction of electromagnetic waves with acoustic waves in a heated partly or fully ionized plasma. We note that the obtained results are qualitatively valid also for other types of interaction, particularly for a nonisothermal plasma,[6] where pulses of ion sound can be generated under the influence of the electromagnetic radiation. Similar effects should be observed also in the interaction of electromagnetic and magnetohydrodynamic waves in a plasma with a constant magnetic field.

We shall investigate the one-dimensional process of backward scattering of plane waves. Assuming that the nonlinear medium is a plasma (or liquid) filling a flat layer of thickness L and is described by the hydrodynamic equations, we represent the initial system of nonlinear equations in the form[2)]

$$\frac{\partial E_y}{\partial x}=-\frac{1}{c}\frac{\partial H_z}{\partial t},\quad \frac{\partial H_z}{\partial x}=-\frac{\varepsilon}{c}\frac{\partial E_y}{\partial t}+\frac{1}{c}\left(\frac{\partial\varepsilon}{\partial\rho_0}\right)_T\frac{\partial}{\partial t}(\rho E_y),$$
$$\frac{\partial\rho}{\partial t}+\rho_0\frac{\partial u}{\partial x}=-\frac{\partial}{\partial x}(\rho u),\qquad (1)$$
$$\frac{\partial u}{\partial t}+\frac{c_s^2}{\rho_0}\frac{\partial\rho}{\partial x}=-u\frac{\partial u}{\partial x}+\frac{c_s^2}{\rho_0^2}\frac{\partial\rho}{\partial x}\rho+\eta\frac{\partial^2 u}{\partial x^2}+\frac{1}{8\pi}\left(\frac{\partial\varepsilon}{\partial\rho_0}\right)_T\frac{\partial E_y^2}{\partial x}.$$

Here ρ and u are the deviations of the density and the velocity of the medium from the equilibrium values ρ_0 and 0, c_S is the speed of sound, and η is the viscosity coefficient; for a plasma $\eta\approx\alpha v_T^2/\nu_{eff}$, where ν_{eff} is the frequency of the collisions between the ions,[5] $\alpha\sim 1$, $v_T^2=\kappa T/M$ (M—ion mass, T—ion temperature, κ—Boltzmann's constant), and $\epsilon=1-4\pi Ne^2/m\omega^2$ (N—concentration, and e/m is the specific charge of the electron).

Bearing in mind that the attenuation of the sound in the medium is proportional to the square of the wave number, we confine ourselves to consideration of the interaction of only three harmonics of the acoustic wave,[3)] assuming their phase velocities to be equal. Then the sought solution of (1) can be written in the following form:

$$E_y=\sum_{1,2}A_{1,2}(x,t)\exp\{i[\omega_{1,2}t-k_{1,2}x+\varphi_{1,2}(x,t)]\}+\text{c.c.},$$
$$\rho=\sum_{n=1}^{3}B_n(x,t)\exp\{in(\Omega t-qx)+i\theta_n(x,t)\}+\text{c.c.},\qquad (2)$$

where $\omega=ck_1/\sqrt{\epsilon}$ is the frequency of the incident elec-

[1)]Owing to the damping of the harmonics, a quasistationary wave may become established. A similar process for drift waves in a weakly-ionized plasma was investigated by the perturbation method in [4].

[2)]We note, incidentally, that for an analysis of nonlinear acoustic effects one sometimes uses a nonlinear wave equation. However, being obtained by the method of successive approximations, it is valid only for small nonlinear distortions of the wave and is obviously not valid in the present case.

[3)]The choice of such a relatively simple model is connected with the fact that even with this model as an example it is possible to reveal all the most characteristic features of many-wave interaction, something that cannot be done in principle by considering qualitatively another case—the interaction of two harmonics.

tromagnetic wave, $-k_2 \approx k_1 = k$, $q = 2k$, and $\omega_2 = \omega_1 - \Omega$ $(\Omega = c_S q)$. The boundary conditions for the interacting waves will be represented in the form[4)]

$$A_1(0) = A_{01}, \quad A_2(L) = A_{L2}. \tag{3}$$

Reflection of the acoustic wave by the boundary of the layer x = L will henceforth be neglected.

To obtain partial differential equations with respect to the amplitudes and phases of the interacting waves, we shall use an asymptotic method[7,8] used for systems of first-order equations with small nonlinearity. As a result of averaging, we obtain the following equations for the complex amplitudes $a_j = A_j \exp(i\varphi_j)$, $b_j = B_j \times \exp(i\theta_j)$:

$$\dot{a}_1 + v_1 a_1' = i\sigma_1 a_2 b_1 - \nu_1 a_1, \qquad \sigma_1 \approx \sigma_2 = \frac{1}{2\varepsilon}\omega\left(\frac{\partial\varepsilon}{\partial\rho_0}\right)_T, \tag{4}$$

$$\dot{a}_2 - v_2 a_2' = i\sigma_2 a_1 b_1^* - \nu_2 a_2, \quad \nu_1 \approx \nu_2 = \nu, \quad v_1 \approx v_2 = c/\sqrt{\varepsilon};$$

$$\dot{b}_1 + c_s b_1' = i\gamma(b_1^* b_2 + b_3 b_2^*) + i\sigma_3 a_1 a_2^* - \eta_1 b_1,$$

$$\dot{b}_2 + c_s b_2' = i\gamma(b_1^2 - 2b_3 b_1^*) - \eta_2 b_2, \quad \dot{b}_3 + c_s b_3' = i\cdot 3\gamma b_1 b_2 - \eta_3 b_3; \tag{5}$$

$$\gamma = \frac{\Omega}{\rho_0}, \quad \sigma_3 = \frac{\rho_0 q^2}{8\pi\Omega}\left(\frac{\partial\varepsilon}{\partial\rho_0}\right)_T, \quad \eta_j = \frac{1}{2}\eta(jq)^2, \quad j = 1, 2, 3$$

(a dot denotes here differentiation with respect to t $(\partial/\partial t)$ and a prime differentiation with respect to x $(\partial/\partial x)$).

The waveform of the acoustic wave, i.e., the amplitude and phase relations between the interacting harmonics, is determined by the form of the nonlinearity and by the character of the dispersion, which in this case is expressed by a strong dependence of the absorption coefficient on the frequency. It is precisely the presence of such a unique dispersion that causes the impossibility of occurrence of periodic shockwaves in the medium in question, since many harmonics take part effectively in the formation of the shock waves. It should be noted here that strong absorption of the higher harmonics does not merely deplete the spectrum of the quasistationary nonlinear wave, but essentially violates those phase and amplitude relations between the remaining harmonics which would correspond to a periodic shock wave.

Assuming that all the times of establishment are significantly smaller than the duration of the pulse of the incident electromagnetic wave, let us consider the stationary processes in the layer. The system (4) and (5) with $\partial/\partial t = 0$ can be investigated if account is taken of the fact that at real amplitudes and interaction coefficients, σ and γ, the variables in the sound wave assume their steady-state values rapidly with increasing x, from the boundary values at x = 0 to the equilibrium values $b_j' = 0$, and subsequently follow the slow variations of the amplitudes of the electromagnetic waves. The characteristic distances over which there are noticeable changes in the amplitudes of the acoustic and electromagnetic waves are in a ratio $l_S/l \sim (1 - 10^3)c_S/c$, and therefore to describe the processes in the greater part of the layer, the parameters of the sound wave on its left-hand boundary are practically inessential. The waveform of the acoustic wave then depends only on the value of its first harmonic.

The connection of b_2 and b_3 with b_1, and also of b_1 with $a_1 a_2^*$ is determined from (5) with $\partial/\partial t = 0$:

$$b_2 = \frac{i\cdot 3\gamma\eta_1 b_1^2}{2\gamma^2 b_1 b_1^* + 12\eta_1^2}, \qquad b_3 = -\frac{\gamma^2 b_1^3}{2\gamma^2 b_1 b_1^* + 12\eta_1^2} \tag{6}$$

$$b_1 = \sigma_3 a_1 a_2^* / \eta(|b_1|^2),$$

where

$$\eta(|b_1|^2) = \eta_1 + \frac{3\gamma^2\eta_1 b_1 b_1^*}{2\gamma^2 b_1 b_1^* + 12\eta_1^2} + \frac{3\gamma^4\eta_1 (b_1 b_1^*)^2}{(2\gamma^2 b_1 b_1^* + 12\eta_1^2)^2}$$

As seen from the last expression, the coefficient of the effective damping of the first harmonic increases with increasing $|b_1| = B_1$ because of the transfer of its energy to the second and third harmonics, and at large amplitudes of the wave it reaches the value $\eta_1^0 = 3.25\eta_1$ (see Fig. 1). The amplitude of the second harmonic at such B_1 also attains saturation and is equal to $1.5\eta_1/\gamma$, while the third harmonic increases linearly in this case, $B_3 \approx \frac{1}{2}B_1$ (Fig. 2). Recognizing that the phases of the second and third harmonics are shifted relative to the first by $\pi/2$ and π, respectively (see (6))[5)], it is easy to trace the evolution of the nonlinear acoustic wave along x. The changes that will be experienced by the form of the periodic wave with increasing amplitude are clearly seen from Fig. 3.

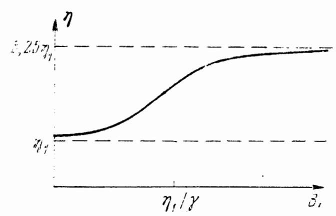

FIG. 1

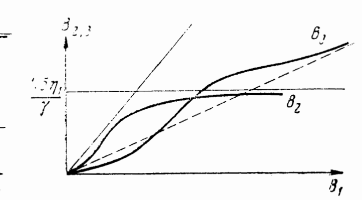

FIG. 2

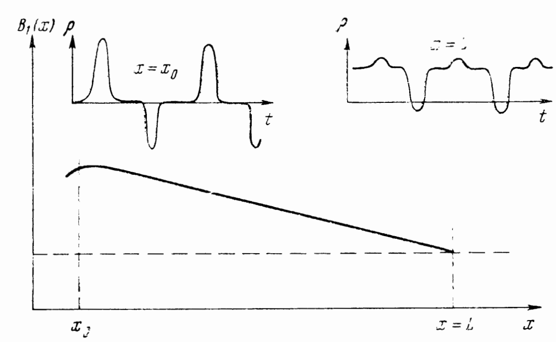

FIG. 3

To determine the law governing the decrease of the amplitude of the first harmonic along x, it is necessary to integrate (4) (with $\partial/\partial t = 0$), taking into account the connection $b_1 = \sigma_3 a_1 a_2^*/\eta(|b_1|^2)$. It is easy to see that, independently of the form of the function $\eta(|b_1|^2)$, these equations have an integral (we assume that $\nu \approx 0$)

$$A_1^2(x) - A_2^2(x) = C. \tag{7}$$

Using this, we obtain the $A_1^2(x)$ dependence at η = const

[4)]The layer boundaries are assumed to be transparent to electromagnetic waves.

[5)]It can be shown that the equilibrium state (6) is stable precisely at such phase relations between the harmonics.

$= \eta_0$ (here $\eta_0 = \eta_1$ at $B_1^2 \lesssim \eta^2/\gamma^2$ and $\eta_0 = 3.25\eta_1$ at $B_1^2 \gtrsim 20\eta_1^2/\gamma^2$):

$$A_1^2(x) = C\{A_1^2(0) + [C - A_1^2(0)]e^{-\bar{\sigma}xC}\}^{-1}A_1^2(0), \quad \bar{\sigma} = 2\sigma\sigma_3/v, \quad (8)$$

where C is the root of the transcendental equation

$$\{A_1^2(0) - [A_1^2(0) - C]e^{-\bar{\sigma}LC}\}[C + A_2^2(L)] - CA_1^2(0) = 0. \quad (9)$$

In the intermediate interval of values of B_1^2, where the dependence of η on B_1^2 is essential, it is impossible to obtain expressions for $A_{1,2}^2(x)$ in terms of elementary functions. However, by using the well-known $\eta(B_1^2)$ dependence, it is possible to obtain certain qualitative results. Inasmuch as in this interval the nonlinear damping coefficient is a monotonically growing function of B_1^2, the amplitude of the first harmonic in this region depends on B_1^2 more smoothly than when η = const. The damping of A_1 and A_2 along x is in this case also slower.

We note that when the number of harmonics participating in the formation of the sound wave increases, the region of the rapid dependence $\eta(B_1^2)$ broadens, and consequently the distance over which the amplitude and the waveform of the wave change insignificantly increases. This is due to the fact that upon an increase (decrease) in the energy fed into the wave, there takes place a sharp increase (decrease) of the amount of energy absorbed by the strongly damped higher harmonics, and as a result the amplitude of the first harmonic changes little.

Inasmuch as in the considered SMBS process the generated sound wave is essentially nonsinusoidal, in the presence (on the boundary x = L) of electromagnetic perturbations at frequencies that are multiples of the Stokes frequency (for example, $\omega_3 = 3(\omega - \Omega)$), the electromagnetic waves can become scattered by the harmonics of the sound. (The scattering of light by the second harmonic of sound in a liquid was observed in [9].) The intensity of the scattered wave can be estimated easily by assuming the field of the sound wave of the harmonic to be specified. Thus, in the simplest approximation, B_3 = const, by integrating equations analogous to (4) we obtain

$$A_3(x) = \frac{A_{3(\omega-\Omega)}(x = L)}{\operatorname{ch}\beta L}\operatorname{sh}\beta x \quad (\beta \approx \sigma B_3). \quad (10)$$

We present estimates of the considered effects for a laboratory plasma. For a hydrogen plasma we shall assume the following: molecule concentration $N_m \approx 10^{12}$ cm^{-3} (equilibrium density $\rho_0 \approx 2 \times 10^{-12}$ g/cm^3), degree of ionization $N/N_m \approx 3 \times 10^{-2}$ (N—electron concentration), frequency of collisions betwen the ions[6] $\nu_{eff} \approx 10^7$ sec^{-1}, temperature T ≈ 300°K. By specifying the frequency of the incident electromagnetic wave $\omega \approx 3 \times 10^{11}$ sec^{-1} ($\lambda_{el} \approx 6$ mm) we obtain the values of the parameters $c_S^2 \approx 2.5 \times 10^{10}$ cm^2/sec^2, $(\partial\epsilon/\partial\rho_0)_T \approx -2 \times 10^7$ cm^3/g, and $\eta \approx 10^3$/sec. Then, taking the expressions (4)–(19) into account, we obtain for a tube of length L = 1 m at electromagnetic field intensities $A_{01} \approx 3 \times 10^3$ V/cm ($p_1 \approx 15$ kW/cm^2), $A_{L2} \approx 600$ V/cm the following values for the maximum amplitudes of the sound-wave harmonics: $B_1 \approx \frac{1}{4}\rho_C$, $B_2 \approx 10^{-2}\rho_0$, $B_3 \approx B_1/2$ (these values are reached at x ≈ 1.5–2 cm). On the boundary of the tube x = L the amplitudes of the harmonics are equal respectively to $B_1 \approx \frac{1}{10}\rho_0$, $B_3 \approx B_1/2$. The intensity of the scattered third harmonic of the electromagnetic wave is in this case equal to

$$A_{3\omega}(x = 0) = 0{,}8A_{3(\omega-\Omega)}(x = L).$$

To verify the validity of the analysis let us estimate the amplitude of the fourth harmonic of the sound, which was not taken into account in the analysis. For the indicated parameters in the given field of the waves B_1, B_2, and B_3, the maximum amplitude is $B_4 \approx B_1/7$. Actually this quantity will be even smaller, since for the fourth harmonic, in view of its strong damping ($\eta_4 \approx 16\eta_1$), the deviation from synchronism is appreciable.

In conclusion we note that it is possible in principle to use SMBS for the diagnostics of a plasma with a large number of collisions. With the aid of this process, in particular, it is possible to determine the speed of sound in the plasma, and consequently to obtain an idea of its temperature. If we find the waveform of the generated sound pulses and their amplitude experimentally, then we can also easily calculate the ionization coefficient of the plasma and the effective collision frequency.

The authors are grateful to A. V. Gaponov and M. I. Petelin for a discussion of this work.

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Translated by J. G. Adashko
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