

WEAK COUPLING PRODUCED IN SUPERCONDUCTORS BY IRRADIATION

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The possibility of locally reducing the ordering parameter  $f$  in superconductors by means of irradiation and of creating in this way a controllable Josephson element is studied theoretically. One-dimensional equations describing the system are derived on the basis of the familiar time equations for  $f$ . The critical current  $I_c$  is calculated for various values of the wave intensity and size of the region of action of the wave. The calculation is also applicable to S-N-S and S-S'-S systems near the S and N (or S') transition temperatures. It is shown that the de Gennes method for determining  $I_c$  is not always correct.

1. INTRODUCTION

IN the last few years, time-dependent generalizations of the Ginzburg-Landau equations<sup>[1-4]</sup> could be obtained for a number of cases. These equations make it possible, for example, to investigate the question of the non-linear interaction of radiation with a superconductor, and particularly to solve the problem of the destruction of superconductivity by a microwave<sup>[2,3,5,6]</sup>. In the present paper, the possibility of a local decrease (over distances  $L$  on the order of the coherence length  $\xi$ ) of the ordering parameter  $\Delta$  in superconducting films with the aid of irradiation is considered theoretically, and the critical Josephson current  $I_c$  in such a system is calculated. The study of such a system is apparently of interest, since its experimental realization yields a Josephson junction with parameters that can be continuously varied by adjusting the wave intensity  $A$  (for example,  $I_c = I_c(A)$ ). In addition, the Josephson Effect in such a system is described by relatively simple one-dimensional equations. With the aid of these equations it is possible, in principle, to follow (if we are interested, for example, in the change of the form of the current-voltage characteristic  $I(V)$ ), the transition from strong coupling (the change of  $\Delta$  under the influence of the irradiation  $A$  is small:  $\Delta(A) \approx \Delta(0)$ ) to a weak coupling of the superconductors ( $\Delta(A) \ll \Delta(0)$ ), and also the transition from the Josephson case of weak coupling ( $L \approx \xi$ ) to the case of vanishingly weak coupling ( $L \gg \xi$ ).

2. THE SYSTEM UNDER CONSIDERATION

A possible experimental realization of the system under consideration is shown in Fig. 1. The system consists of a metallic (normal or superconducting) screen, in which a narrow slit of width  $2L$  is cut ( $L \approx \xi \approx 1 \mu$ ); at a distance  $d_1$  from the screen is placed a superconducting film, separated from the screen by an insulator layer (for example, an oxide film). The requirement concerning the thickness of the superconductor  $d_c$  will be formulated below. We note at present that since the length of the incident microwave is  $\lambda \gg L$ , the field of the wave will act on the superconductor in a band of width  $\sim L$ , if  $d_1$  and the thickness of the screen do not exceed  $2L$ . This

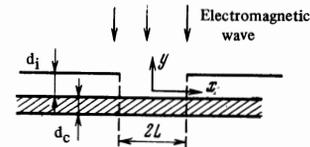


FIG. 1

requirement follows from the fact that the field of the incident wave is concentrated near the aperture at a distance no larger than  $L$  from the aperture<sup>[7]</sup>. On the other hand, if  $\lambda \ll L$  (the visible band), then the thickness of the screen and the distance  $d_1$  can be arbitrary.

3. FUNDAMENTAL EQUATIONS

We start from the Gor'kov-Éliashberg equation<sup>[2]</sup>, obtained for gapless superconductors with large concentration of paramagnetic impurities ( $\tau_S T_C \ll 1$ ,  $\tau_S$  is the free path relative to spin flip). In this case at sufficiently low frequencies ( $\omega \lesssim \tau_S |\Delta|^2$ ) the changes of  $\Delta$  with time are described by an equation of the diffusion type. In other cases (low concentration of the paramagnetic impurities, strong magnetic fields), the situation turns out to be more complicated<sup>[3,4]</sup>.

We shall, however, use the equations obtained in<sup>[2]</sup>, since it is desirable to have simpler equations for a qualitative explanation of the changes (for example, the form of  $I(V)$ ) occurring on going from weak to strong coupling. In addition, we confine ourselves to an investigation of the stationary Josephson effect, and the equations that describe this effect will have the same form in all the investigated cases<sup>[1-4,8]</sup>.

The Gor'kov-Éliashberg equations in dimensionless variables are of the form

$$\begin{aligned} 12f + f(f^2 - 1) - \xi^2 \nabla^2 f + v^2 f &= 0, \\ 12f^2 (\chi + \Phi) + \xi \operatorname{div}(v f^2) &= 0, \\ \delta_L^2 \operatorname{rot} \operatorname{rot} v &= -v - \xi \nabla (\chi + \Phi) - f^2 v. \end{aligned} \tag{1}$$

Here  $f = |\Delta/\Delta_0|$  is the dimensionless amplitude of the ordering parameter,  $\Delta_0 = \pi \sqrt{2(T_C^2 - T^2)}$  is the equilibrium value of the ordering parameter,  $\chi$  is the phase of  $\Delta$ ,  $\Delta = \Delta_0 e^{i\chi}$ ,  $\xi = \sqrt{12Dt_0}$  is the coherence length,  $D$  is the diffusion coefficient,  $t_0 = (2\tau_S \Delta_0^2)^{-1}$ ,  $v = A/A_0 - \xi \nabla \chi$ ,  $A_0 = c\hbar/2e\xi$ , and  $\delta_L^2 = 4\pi\sigma/c^2 t_0$ . The time is

measured in units of  $t_0$ , the current in units of  $I_0 = \sigma A_0 / ct_0$ , and the potential in units of  $\Phi_0 = \hbar / 2et_0$ .

The last two equations of the system (1) describe the penetration of the field into the superconductor. At a sufficiently high frequency of the wave ( $\omega \gg 1$ ,  $\omega$ —dimensionless frequency of the field), the last equation of (1) describes the normal skin effect, and to find the  $v(\mathbf{r}, t)$  dependence it is necessary to solve this equation with the boundary conditions for the system in question. Owing to the complicated geometry of the system, the distribution of the alternating field will also be complicated. However, the exact distribution of  $v(\mathbf{r}, t)$  is of no interest to us, and we approximate the dependence  $v(\mathbf{r}, t)$  by assuming that

$$v(\mathbf{r}, t) = \begin{cases} v_{\sim}(t), & |x| < L \\ 0, & |x| > L \end{cases} \quad (2)$$

The independence of  $v_{\sim}$  of  $y$  denotes that  $d_c \ll d_{sk}$ , where  $d_{sk}$  is the skin length in the superconductor.

Let us estimate now what should be the characteristic amplitude of the field  $E$  for an appreciable decrease of  $f$  and how the field in the superconductor is connected with the field of the incident wave. If  $L \gg \delta_{sk}$ , then the change of the field along the superconductor over the skin length will be small. Therefore the problem of the penetration of the field becomes planar, and to estimate the characteristic  $E$  we can use the well-known results (see<sup>[2,6]</sup>). The characteristic field will be the smallest if<sup>[6]</sup>

$$d_c \lesssim \delta_{sk}^2 \omega / c. \quad (I)$$

In this case the wave passes through the film, and the critical field is  $E_c \approx \hbar\omega / e\xi$  (for such a field,  $v_{\sim} \approx 1$ , see (1)). If (I) is not satisfied, then the wave experiences strong reflection from the film. Therefore the field on the surface of the superconductor decreases (by a factor  $d_c c / \delta_{sk}^2 \omega$ ) and accordingly the power necessary for the destruction of the superconductivity increases. Let us present also an estimate for the connection between the field  $E$  in the superconductor and the incident field  $E_{inc}$ . This is easiest to do by assuming that (I) is satisfied. Then the film disturbs the field of the incident wave very little, and the field remains practically the same as in the absence of the superconducting film. In the case of s-polarization ( $E_{inc} \parallel \mathbf{z}$ ) we have  $E/E_{inc} \approx L\omega/c$ <sup>[7]</sup>. In the case of p-polarization ( $E \perp \mathbf{z}$ ) we have  $E/E_{inc} \approx (c/L\omega) \times [\ln(c/L\omega)]^{-1}$ <sup>[7]</sup>. It is clear that the p-polarization is preferable, for in this case it is easy to make the field  $E$  in the superconductor strong. In addition, in the case of p-polarization the magnetic field of the wave is parallel to the plane of the film and no eddies can be formed.

Summarizing, we can state that it is more convenient to use a screen with large conductivity (then the skin layer in it will be thin and consequently it is possible to use a screen of small thickness) and a superconductor with low conductivity (to alternating current). The latter means that the superconductor will be more transparent for the field, and the intensity of the characteristic field  $E_c$  will be sufficiently small.

Thus, let us consider a superconductor of small thickness (the condition  $d_c \ll \{\delta_{sk}, \xi\}$  is satisfied) into which a field (2) penetrates locally. For frequen-

cies  $\omega$  larger than the reciprocal relaxation time  $t_0^{-1}$  of the ordering parameter, the penetration of the field is determined in this case by the skin effect, and the oscillating part of  $f$  will be small ( $f_{\sim} \sim v_{\sim}/\omega$ )<sup>[2]</sup>. Assuming further that  $\omega$  is much larger than the frequency of the Josephson oscillations, and averaging the first two equations of the system (1) over the period of the wave, we obtain

$$12f + f(f^2 - 1) + (v_x^2 + v^2(x))f - \xi^2 \partial^2 f / \partial x^2 = 0, \quad (3)$$

$$12f^2(\chi + \Phi) + \xi \frac{\partial}{\partial x}(v_x f^2) = 0.$$

Here  $f = \langle f + f_{\sim} \rangle$ , the angle brackets denote the averaging over the period of the wave, and  $v^2(\mathbf{x}) = \langle v_{\sim}^2(\mathbf{x}, t) \rangle$ . We have separated in explicit form the velocity  $v_T^2$ , which corresponds to the transport current. Terms of the type  $\langle f_{\sim}^2 \rangle$  have been omitted because they are small. The field corresponding to the Josephson oscillations is assumed to be quasistationary and described by a potential  $\Phi$ . Equations (3), in which

$$v^2(x) = \begin{cases} v_{\sim}^2 |x| < L \\ 0, & |x| > L \end{cases}, \quad (4)$$

describe the system under consideration.

#### 4. CALCULATION OF THE CRITICAL CURRENT

Let us analyze the stationary effect, i.e.,  $\dot{f} = \dot{\chi} = \dot{\Phi} = 0$ . We then obtain from the second equation of (3)  $v_T f^2 \equiv j = \text{const}$ . Substituting this value of  $v_T$  in the first equation of (3), we obtain<sup>1)</sup>

$$f'' + f(1 - v^2(x)) - j^2/f^3 - f^2 = 0, \quad (5)$$

where the prime denotes differentiation with respect to the dimensionless coordinate  $\zeta = x/\xi$ . The boundary conditions for Eq. (5) consist of homogeneity of  $f$  far from  $\zeta = 0$ , i.e., as follows from the equation itself,

$$f_{\pm\infty}^2 = 1 - j^2/f_{\pm\infty}^4. \quad (6)$$

In Eq. (6) it is necessary to choose the larger root; the smaller root corresponds to a metastable state of the superconducting film with the current. In addition, we shall find it necessary to have the matching conditions at  $|x| = L$  (i.e.,  $\zeta = l$ , where  $l = L/\xi$ ). They are obtained from Eq. (5) itself by integrating over an infinitesimally thin layer ( $L_-, L_+$ ) and consist in the requirement of continuity of  $f$  and  $f'$ :

$$[f]_{l=l} = 0, \quad [f']_{l=l} = 0, \quad (7)$$

where

$$[f]_{l=l} = f(l+0) - f(l-0).$$

We note that Eq. (5) also describes the structures  $S - S' - S$  at  $(1 - v^2) > 0$  or  $S - N - S$  at  $(1 - v^2) < 0$  near  $T \approx T_S \approx T_{S'}$  or  $T \approx T_S \approx T_N$  ( $T_S, T_{S'}$ , and  $T_N$  are respectively the critical temperatures of the superconductors  $S$  and  $S'$ , and also of the normal metal  $N$ ). If it is assumed that the metals  $S, S'$ , and  $N$  differ only in their critical temperatures, then the matching conditions (7) remain valid also<sup>[9]</sup>, and the coefficient of the second term in (5) is equal to  $1 - v^2 = (T - T_{S'}) / (T - T_S)$  ( $S - S' - S$  structure) or  $1 - v^2 = (T - T_N) / (T - T_S)$  ( $S - N - S$  structure); all the

<sup>1)</sup>Note added in proof (22 February 1971). An equation similar to (5) was investigated in the recently published paper<sup>[13]</sup>.

measurement units defined in (1) pertain to the superconductor  $S$ . With these stipulations, the results obtained below pertain to both the  $S - S' - S$  and  $S - N - S$  structures.

Let us integrate (5) once in each of the regions ( $|\zeta| > l$ ) and ( $|\zeta| < l$ ):

$$f'^2 + U(f) = U_0, \quad |\zeta| > l \quad (8)$$

$$f'^2 + W(f) = W_0, \quad |\zeta| < l$$

Here

$$U(f) = f^2 - \frac{1}{2}f^4 + j^2/f^2, \quad (9)$$

$$W(f) = f^2(1 - v^2) - \frac{1}{2}f^4 + j^2/f^2,$$

$U_0 = U(f_\infty)$ , and  $f_\infty$  is determined by Eq. (6), i.e., the constant  $U_0$  is chosen such as to satisfy the boundary conditions (6);  $W_0 = W(f(0))$ , since by virtue of the symmetry of the solution  $f(\zeta)$  we have at  $\zeta = 0$  the quantity  $f' = \sqrt{W_0 - W(f(0))} = 0$ .

Using Eqs. (8), we can find the critical current  $j_c$  as a function of the field of the wave  $v$  and the width of the gap in the screen  $l$ . To this end, let us consider Eqs. (8) with  $\zeta = l$  and subtract one from the other. Taking into account the explicit form of  $U$  and  $W$ , and also the matching condition (7), we obtain

$$v^2 f^2(l) = U_0 - W_0. \quad (10)$$

This equation contains the unknowns  $f(l)$  and  $f(0)$  (since  $W_0 \equiv W(f(0))$ ). A second equation for these unknowns is obtained by using the equality

$$l = \int_0^l d\zeta = \int_{f(0)}^{f(l)} \frac{df}{f'}. \quad (11)$$

The equality (11) can be rewritten by substituting  $f'$  from the second equation of the system (8) and using the explicit form of  $W(f)$  (9). Then

$$l = \int_{f_0}^{f_l} \frac{df}{\sqrt{W_0 - W(f)}} = \sqrt{\frac{2}{g_0 - g_2}} F \left\{ \arcsin \sqrt{\frac{g_1 - g_0}{g_1 - g_2}}, \sqrt{\frac{g_1 - g_2}{g_0 - g_2}} \right\} \quad (12)$$

for  $g_0 > g_1 > g_2$ . Here  $g = f^2$ ;  $g_0, g_1$ , and  $g_2$  are the roots of the equation  $g[W(\sqrt{g}) - W_0] = 0$ ,  $g_0 = f^2(0)$ , and  $F$  is an elliptic integral of the first kind<sup>[10]</sup>.

Equations (10) and (12) are algebraic equations with respect to  $f_0$  and  $f_l \equiv f(l)$ . Finding the solutions of these equations we can, in principle, determine the critical current  $j_c$ , which is the maximum value of  $j$  at which the solutions (10) and (12) exist. Such a problem can be solved for arbitrary values of  $l$  and  $v$  only numerically. We shall therefore consider analytically only the most interesting particular cases and analyze the solutions qualitatively. We note that for a qualitative analysis of the solutions (8) it is convenient to consider the phase plane  $(f, f')$  (see Fig. 2a). The solution of equations (8) is mapped (for  $v^2 < 1$ ) by the phase trajectory  $DABC$ , with the trajectory  $ADC$ , defined by the first equation of (8) and passing through the saddle point  $D$  representing the solution at  $|\zeta| > l$ , while the trajectory  $ABC$ , defined by the second equation of (8), represents the solution at  $|\zeta| < l$ . The condition (10) denotes that at  $f = f_1$  the trajectories intersect. On the other hand, the condition (11) con-

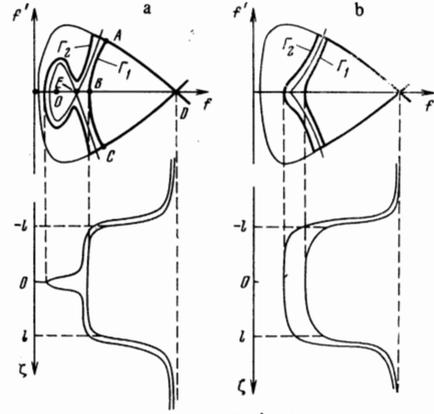


FIG. 2. Phase trajectories on the  $(f, f')$  plane and the corresponding dependence of the solutions  $f$  of the system (8) on the coordinate  $\zeta$  for two values of the current:  $j_1$  (a) and  $j_2 > j_1$  (b).

nects the "length" of the trajectory  $AB$  with the dimension  $l$ .

In order to understand the conditions that must be imposed on  $v^2$  and  $l$  in order to obtain an analytic solution, let us consider first the case when there is no current. Then we can obtain from (10) and (12) the dependence of  $f_0$  on  $v^2$  at different  $l$ . In the absence of current ( $j = 0$ ) and  $\zeta = \pm \infty$  we get  $f_\infty = 1$  (see (6)), and therefore  $U_0 = \frac{1}{2}$  (see (9) and the definition of  $U_0$ ). Using this value and the explicit form of  $W(f_0)$  (9), we rewrite (10) in the form

$$v^2(g_1/g_0 - 1) = 1/2g_0 - 1 + \frac{1}{2}g_0. \quad (13)$$

We are interested in the case when the irradiation decreases  $f_0$  strongly and there is actually a weak coupling, i.e.,  $g_0 = f_0^2 \ll 1$ . Then in the right-hand side of (13) we can neglect the last two terms:

$$v^2(g_1/g_0 - 1) = 1/2g_0. \quad (13')$$

We turn now to Eq. (12). In the absence of current we obtain  $g_1 = 0$ ,  $g_2 = -[g_0 + 2(v^2 - 1)]$ .

Let us consider the asymptotic form of  $F$  in (12) in different cases. Let

$$|g_2| \gg g_0, \quad g_1/(g_0 - g_2) \ll \min\{1, \sqrt{g_0 - g_2}\}. \quad (14)$$

Under these conditions the elliptic integral in (12) reduces to the hyperbolic arc cosine of  $g_l/g_0$ <sup>[10]</sup>. We then obtain from (12)

$$g_l/g_0 = \text{ch}^2(l\sqrt{1 - v^2}). \quad (15)$$

Substituting this ratio in (13'), we obtain the dependence of  $f_0$  on the intensity of irradiation

$$f(0) = 1/v\sqrt{2} \text{sh}(l\sqrt{1 - v^2}). \quad (16)$$

Using the obtained expressions (15) and (16), we can easily verify that conditions (14) are satisfied if

$$l \ll 1/v^2 \ll 1. \quad (17)$$

Then (15) and (16) can be approximately rewritten in the form

$$f(0) = 1/2v \text{sh}(vl), \quad f_l/f_0 = \text{ch}(vl). \quad (18)$$

It follows therefore that in the case of a broad gap ( $l \gg 1$ )  $f_0$  is exponentially small and decreases with increasing  $l$  like  $e^{-vl}$ , if the field of the wave greatly

exceeds the characteristic value  $E_c$  ( $v \gg 1$ ). For a narrow gap, however, ( $l \ll 1/v$ ) we have  $f_0 \sim 1/lv^2$ .

In the other limiting case

$$|g_2 + g_0| \ll g_0, \quad g_1 \gg g_0, \tag{19}$$

we obtain from (12)

$$f_0 = l^{-1}K(1/\sqrt{2}),$$

where  $K$  is the complete elliptic integral of the first kind. The value  $f_l \equiv \sqrt{gl}$  is obtained from (13'):

$$f_l = 1/v\sqrt{2}. \tag{20}$$

The conditions (19), and also the condition for the smallness of  $f_0$ , are satisfied if

$$1 \ll l \ll 1/\sqrt{|1-v^2|}.$$

It is then seen from (20) that on the edge of the slit of the screen  $f_l = 1/\sqrt{2}$ , and at the center at  $\zeta = 0$  it decreases with increasing  $l$  in accordance with a power law.

It can also be shown that when  $1 \ll 1/\sqrt{1-v^2} \ll l$  we have

$$f_0 = \sqrt{1-v^2} + e^{-l\sqrt{2(1-v^2)}},$$

i.e., the dependence of  $f_0(v)$  is the same, with exponential accuracy, as in the case of an infinitely broad slit.

It follows thus from the presented expressions that if the dimension of the gap in the screen is smaller than  $\xi$  (i.e.,  $l < 1$ ), then to obtain an appreciable decrease of  $f$  it is necessary to have fields that greatly exceed the characteristic field  $E_c = \hbar\omega/e\xi$ . For a broad slit ( $l \gg 1$ ),  $f_0$  decreases significantly at  $v \approx 1$ , i.e., at  $E \approx E_c$ .

Let us consider now the situation when a current  $j$  flows through the film. We analyze first the case of large  $v$ , when in the absence of current the expressions (18) are valid and  $f_0 \ll 1$ . So long as  $f_0$  is small, the critical current is also small (this is confirmed by the final result). In this case we can obtain for the roots  $g_i$  in (12) approximately  $g_1 = -j^2/v^2 g_0$ ,  $g_2 = -2v^2$ . In a limit analogous to (14), i.e., when the second argument in  $F$  is close to unity, we obtain in analogy with the current-free case

$$g_i/g_0 = \text{ch}^2(lv) + (j^2/v^2 g_0^2) \text{sh}^2(lv). \tag{21}$$

Equation (10) at small  $f_0$  takes the form (compare with (13'))

$$v^2(g_i/g_0 - 1) = 1/2g_0 - j^2/g_0^2. \tag{22}$$

Substituting the expression (21) in (22), we obtain

$$f_0^2 = f_c^2 [1 \pm \sqrt{1 - j^2/j_c^2}]. \tag{23}$$

Here

$$f_c = [2v \text{sh}(lv)]^{-1}, \quad j_c = f_c/2 \text{ch}(lv) = [2v \text{sh}(2lv)]^{-1}.$$

Thus, one value of the current corresponds to two values of  $f_0$ , and consequently to two solutions  $f(x)$ . One value decreases with increasing current from  $\sqrt{2}f_c$  to  $f_c$ , and the second increases from zero to  $f_c$ . The first solution, as will be shown below, corresponds to a phase difference of zero for the ordering parameter (as  $j \rightarrow 0$ ), and the second to a phase dif-

ference  $\pi$  (as  $j \rightarrow 0$ ). A similar situation takes place in a point contact, when in the resistive state  $f(x, t)$  oscillates in time, varying between the minimal and maximal solutions which can be obtained by letting  $j \rightarrow 0^{[11]}$ .

In dimensional form, the critical current is given by

$$I_c = I_0 A_0 / 2A \text{sh}(2LA/\xi A_0), \tag{24}$$

where  $I_0 A_0 \sim A_0^2 t_0^{-1} \sim \xi^{-4} \sim (T_S - T)^2$  (in accordance with the notation of (1)), i.e., the critical current decreases when  $T \rightarrow T_S$  like  $(\Delta T)^2$ . A similar dependence on  $T$  is given by (24) also for the system  $S - N - S$ , for in this case, as already indicated,  $v^2 = (T_N - T)/(T - T_S)$  (when  $v \gg 1$ ) and (24) can be rewritten in the form

$$I_c = I_0 \frac{(T_S - T)^{1/2}}{2(T - T_N)^{1/2} \text{sh}(2L/\xi_N)} \quad \xi_N = \xi \left( \frac{T_N - T}{T - T_S} \right)^{1/2}.$$

In the case of a broad slit in the screen ( $l \gg 1$ ) we shall carry out an analysis of the solutions by considering the phase plane. When  $v^2 < 1$ , the phase trajectories have the forms shown in Figs. 2a and 2b, respectively, for the two values of the current  $j_1 < j_2$  and  $j_2 \approx j_c$ . Indeed, when  $l \gg 1$ , Eq. (11) can be satisfied only in the case when the trajectory passes near the singular saddle point  $E$ , since near  $E$  the trajectories are determined by the expression  $f' = \pm \sqrt{(f - f_E)^2 \pm \beta^2}$ . We see that there are two such trajectories:  $\Gamma_1$  (the minus sign under the root in the expression for  $f'$ ) and  $\Gamma_2$  (the plus sign). The solutions  $f(x)$  corresponding to  $\Gamma_1$  and  $\Gamma_2$  when  $j < j_c$  will be close to each other everywhere except in the segment  $|x| \lesssim 1$  (Fig. 2a). With increasing  $j$ , the loop on the phase trajectory becomes compressed and vanishes at  $j = j_0$ , and the singular points  $O$  and  $E$  merge (Fig. 2b). This value of the current will be critical for  $l \rightarrow \infty$ , since when  $j > j_0$  the second equation of (8) has no singular points. The current  $j_0$  is determined from the equalities  $\partial W/\partial f = \partial^2 W/\partial f^2 = 0$  and is equal to the critical current for a homogeneous irradiated film

$$j_0 = \frac{2}{3\sqrt{3}}(1-v^2)^{1/2}.$$

It is easy to obtain the correction for  $j_0$ , connected with the finite dimension of the aperture in the screen  $l$ . To this end it is necessary to take into account the fact that when  $j > j_0$  the two roots  $g_1$  and  $g_2$  in (12) are imaginary. We present the result:

$$j_c = j_0 + 1/l^4(1-v^2)^{1/2}.$$

Let us calculate, finally, the connection between the current through a junction with a phase difference  $\varphi$ , and confine ourselves to the case of a narrow slit ( $l \ll 1$ ). We have

$$\varphi = \int_1^2 \frac{dx}{d\zeta} d\zeta = \int_1^2 \frac{j}{f^2} d\zeta. \tag{25}$$

Here the points 1 and 2 are located near  $\zeta \rightarrow -\infty$  and  $\zeta \rightarrow +\infty$ , where  $f$  is practically independent of the coordinates ( $j \ll 1$ ). At these points we have from the second equation of the system (3)

$$\chi_{1,2} + \Phi_{1,2} = 0.$$

Calculating the difference of these equations

$$\varphi = V, \quad (26)$$

we obtain a relation between the time variation of the phase difference  $\varphi$  and the potential difference  $V$ . In the considered case of small currents, Eq. (25) can be rewritten in the form

$$\begin{aligned} \varphi &\approx \int_1^2 d\xi \left( \frac{j}{f^2} - \frac{j}{f_\infty^2} \right) \\ &= 2 \int_{f_0}^{f_1} \left( \frac{j}{f^2} - \frac{j}{f_\infty^2} \right) \frac{df}{f'} + 2 \int_{f_1}^{f_\infty} \left( \frac{j}{f^2} - \frac{j}{f_\infty^2} \right) \frac{df}{f'} \end{aligned} \quad (27)$$

Using expressions (21) and (23), we can easily verify that at sufficiently small  $l$  ( $lv \ll 1$ ) the contribution to  $\varphi \sim \pi/2$  from the first integral can be neglected, i.e., the phase difference on the slit of the screen itself (or on the N layer, if we speak of the S - N - S system) is negligibly small. Substituting in (27)  $f'$  from the first equation of (8), we obtain

$$j = j_c \sin \varphi, \quad (28)$$

where  $j_c$  is defined in (23). Thus, in the case of a sufficiently narrow slit we have a situation typical for a Josephson junction, when Eqs. (26) and (28) are valid. We note the following circumstance. Usually the S - N - S system is regarded on the basis of the de Gennes method<sup>[12]</sup>, according to which the current through the junction is connected with the phase difference on the N layer, and  $j_c$  is expressed in terms of  $f(0)$ . To find  $f(0)$ , one neglects the dependence of this quantity on  $j$ , this being attributed to the smallness of  $j$  (in the dimensionless form  $j \ll 1$ ). However, our analysis shows that this method is not always correct. We have found that  $\varphi$  in (28) is the change of the phase not inside the region of action of  $v$ , but outside it (i.e., outside the N layer), at a length on the order of  $\xi$ . This is connected with the fact that the smallness of  $j$  compared with the critical current of the non-irradiated film still does not mean that  $f(0)$  is independent of  $j$ , since  $f(0)$  itself is small.

In conclusion, we note that to observe the effect, the

condition  $d_c \ll \delta_{sk}$  is not significant. In fact, when  $\delta_{sk} \ll \xi$ , the change of  $f$  under the influence of the light occurs over a length  $\xi$ , in spite of the fact that the wave penetrates only in the near-surface region. Therefore  $f$  will decrease upon irradiation, remaining constant over the thickness of the film when  $d_c \ll \xi$ <sup>[3]</sup>.

To ascertain the form of the current-voltage characteristic it is necessary to solve the time-dependent equations (3).

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