# HEAT TRACK AND SELF-FOCUSING OF A POWERFUL BEAM IN A MEDIUM

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A non-stationary equation of propagation of heat following absorption of a powerful beam of a specified profile, with an intensity dip near the axis, is derived and investigated. The dynamics of formation and the profile of the nonlinear variations of the refractive index of the medium are investigated. Self-focusing conditions for the near-axis part of the beam are obtained for the case of heat spreading in time and space. It is shown that the non-stationarity of the process is essential for self-focusing. Nonstationarity ensures satisfaction of the self-focusing condition at any instant of time and in continuously decreasing near-axis regions.

## INTRODUCTION

SELF-focusing and self-defocusing of powerful beams in a medium, due to the different nonlinear effects, have been the subject of a number of recent papers (see, e.g.,[1-15], the review[4], and the literature cited therein).

For not too short radiation pulses, especially for beams of cw lasers, the main cause of the change of the refractive index is the heating of the medium upon absorption of the light. The change of the refractive index is equal to

$$\delta n(\mathbf{r}) = \frac{\partial n}{\partial T}\delta T + \frac{\partial n}{\partial \rho}\delta\rho \approx \frac{dn}{dT}\delta T,$$

since usually $\delta\rho \approx -\alpha\delta T$ if the expansion has time to settle. For example, in a solid medium we have upon local heating

$$\frac{\delta\rho}{\rho} = -\frac{\alpha}{3}\frac{1-\sigma}{1+\sigma}\delta T$$

with allowance for the counterpressure of the surrounding unheated medium, where $\alpha$ is the coefficient of thermal expansion and $\sigma$ is the Poisson coefficient.

For self-focusing, i.e., for a decrease of the divergence angle $\theta$, it is necessary that the derivative $n'_T$ be negative, for in this case it follows from the refraction condition $\theta'_z \approx n'_r$ that $\theta'_z < 0$.

For the ordinary most widespread media (liquids, gases) $n'_T < 0$ and therefore one observes thermal defocusing for rays with usual profiles, with the intensity decreasing away from the axis $(I'_r < 0)$[5-8].

In some media (plasma, almost all glasses) $n'_T > 0$ and thermal self-focusing can be observed[1,9-12]. It was noted in a number of papers[13-15] that even in media with $n'_T < 0$ it is possible to obtain self-focusing of the main part of the beam, using a special profile of the intensity distribution with a decrease near the axis, for in the region where $I'_r > 0$ we have $n'_r = n'_I I'_r \approx n'_T T'_r < 0$ if $T \sim I$. However, the dependence of the temperature at a given point on the intensity of the light at the given point is so simple only in the case when the heat does not have time to diffuse from the heat-release volume ($t < a^2/\chi$ and $\delta T = \alpha It/C\rho$, where a is the dimension of the heat-release region, i.e., the radius of the beam, $\alpha$ is the coefficient of light absorption per centimeter of path, $\chi$ is the temperature conductivity, C is the specific heat, and $\rho$ is the density of the medium). For large durations and especially for cw lasers, it is necessary to take into account the spreading of the heat.

In the present paper we investigate the distribution of the temperature in time and in space upon absorption of a beam with a special profile in which the intensity decreases near the axis. The distribution of the intensity is assumed to be specified and is used to calculate the temperature field and to analyze the tendency of variation of the beam divergence, assuming geometrical optics. Such a simplified analysis is usually employed for self-focusing problems and provides, without solving the self-consistent problem (which is usually done by computer calculations), a sufficient description of the process, especially in the case of a layered medium and external focusing (when the change of the distribution of the intensity in the layer of the medium can be disregarded), either on sections of the beam or in short time intervals.

## TEMPERATURE DISTRIBUTION AND NONLINEAR REFRACTION IN THE CASE OF A LIGHT BEAM OF SPECIAL PROFILE

Given a light beam with a reduced intensity near the axis, for example with a radial distribution of the intensity $I(r) = I_n r^{2n}\exp(-r^2/a^2)$, then in the case of not too large an absorption ($\alpha a \ll 1$) the power of the heat-release sources $q = \alpha I(r)$ will depend mainly on the radius, i.e., the problem is nearly one-dimensional and cylindrical. In this case the equation of thermal conductivity takes the form

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} - \frac{1}{\chi}\frac{\partial T}{\partial t} = -f_n r^{2n}\exp\left(-\frac{r^2}{a^2}\right), \tag{1}$$

with $T|_{t=0} = 0$ ($f_n = \alpha I_n/k$ and $k = \chi C\rho$ is the thermal conductivity of the medium).

We introduce the dimensionless variables

$$\Phi = T/f_n a^{2n+2}, \quad \xi = r/a, \quad \tau = \chi t/a^2.$$

Then the equation (1) is transformed into

$$\frac{\partial^2\Phi}{\partial\xi^2} + \frac{1}{\xi}\frac{\partial\Phi}{\partial\xi} - \frac{\partial\Phi}{\partial\tau} = -\xi^{2n}e^{-\xi^2}. \tag{2}$$

Using operator notation, and also the properties of the $\delta$ function (see, e.g.,[16]), we rewrite (2) in the form

$$L\Phi = -\int_0^\tau\int_0^\infty \xi'^{2n}\exp(-\xi'^2)\delta(\xi-\xi')\delta(\tau')d\xi'\,d\tau',$$

where

$$L = \frac{\partial^2}{\partial\xi^2} + \frac{1}{\xi}\frac{\partial}{\partial\xi} - \frac{\partial}{\partial\tau}, \quad \delta(\tau') = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{iq\tau}\,dq,$$

$$\delta(\xi-\xi') = \xi'\int_0^\infty kJ_0(k\xi')J_0(k\xi)\,dk.$$

Substituting these expressions in (2) and applying the inverse operator, we obtain

$$\Phi = \frac{1}{2\pi}\int_0^\tau\int_0^\infty\left\{\xi'^{2n}\exp(-\xi'^2)\xi'\int_{-\infty}^{\infty}\int_0^\infty \frac{kJ_0(k\xi')J_0(k\xi)\exp(iq\tau')}{k^2+iq}\,dk\,dq\right\}d\xi'\,d\tau'.$$

Calculating the integrals with respect to q, k, and $\xi'$, we have ultimately

$$\Phi = \frac{\Gamma(n+1)}{4}\int_0^\tau \frac{\exp(-\xi^2/4\tau')}{\tau'(1+1/4\tau')^{n+1}}\,{}_1F_1(n+1,1,\xi^2/4\tau'(1+4\tau'))\,d\tau',$$

where ${}_1F_1$ is the confluent hypergeometric function.

In the simplest case when n = 0 we have a Gaussian profile of the intensity, and since ${}_1F_1(1, 1, z) = e^z$, it follows that

$$\Phi = \frac{1}{4}\left\{\mathrm{Ei}(-\xi^2) - \mathrm{Ei}\left(-\frac{\xi^2}{1+4\tau}\right)\right\},$$

where

$$\mathrm{Ei}(z) = \int_{-\infty}^{z}\frac{e^t}{t}\,dt$$

is the integral exponential function.

This result agrees with the result of the article[5], which is devoted to thermal defocusing of a Gaussian beam. A characteristic property of this solution is the persistent logarithmic growth of the temperature in time, as can be readily verified, for example, by assuming the distance $\xi$ to the axis to be small and bearing in mind the fact that $\mathrm{Ei}(x) = C + \ln|x| + x$ when $x \ll 1$. In this case we have for $\xi \ll 1$

$$\Phi \approx {}^1\!/_4\ln(1+4\tau) - 4\tau\xi^2/(1+4\tau) + O(\xi^2).$$

We see that in this problem there is no steady state of the temperature (we note that the temperature gradient $\Phi'_\xi$ becomes established nevertheless, corresponding to a slow rise of the entire temperature curve at large values of the time).

We consider the case of a beam with decreasing intensity near the axis. In the simplest case when n = 1 (quadratic-parabolic near-axis profile) we have

$$\Phi = \int_0^\tau \frac{\exp(-\xi^2/4\tau')}{4\tau'(1+1/4\tau')^2}\,{}_1F_1\left(2,1,\frac{\xi^2}{(4\tau')^2(1+1/4\tau')}\right)d\tau'.$$

But ${}_1F_1(2, 1, z) = (z + 1)e^z$, and therefore

$$\Phi = \frac{1}{4}\left\{\frac{\exp[-\xi^2/(1+4\tau)]}{\xi^2}\left(2+\frac{\xi^2}{1+4\tau}\right) - \frac{e^{-\xi^2}}{\xi^2}(\xi^2+2) + \mathrm{Ei}(-\xi^2) - \mathrm{Ei}\left(-\frac{\xi^2}{1+4\tau}\right)\right\}.$$

The derivative of the temperature with respect to the coordinate takes the form

$$\Phi_\xi' = \frac{e^{-\xi^2}}{2\xi}\left\{(\xi^2+1) - \exp\left(\frac{4\tau\xi^2}{1+4\tau}\right)\left[1+\frac{\xi^2}{(1+4\tau)^2}\right]\right\}. \qquad (3)$$

We consider now the near-axis region of the beam $\xi \ll 1$ for arbitrary $\tau$. In this case

$$\Phi_\xi' = \frac{2\tau\xi}{(1+4\tau)^2}\left\{1 - \frac{\xi^2}{1+4\tau}(1+2\tau+8\tau^2)\right\}.$$

So long as the expression in the curly brackets (3) is positive, $\Phi'_\xi > 0$ and under these conditions the temperature on the axis is always smaller than the temperature near the axis, even as $\tau \to \infty$, i.e., the self-focusing condition is satisfied in media with $n'_T < 0$. We see that the condition under which self-focusing will take place is as follows:

$$\xi^2 < \frac{1+4\tau}{1+2\tau+8\tau^2} \quad \text{for } \xi \ll 1.$$

For example, by specifying the dimension of the near-axis region of interest to us, $r = \xi a$, we find that self-focusing will occur for

$$\tau < a^2/2r^2.$$

This result is quite unexpected—self-focusing in the near-axis region of a beam with decreasing intensity is the consequence of the non-stationarity of the process even at very large values of the time, i.e., in this formulation of the problem there is no steady state of the temperature.

We point out that were we to assume that such a regime is established, we would obtain only defocusing. Indeed, from the equation for steady-state heat outflow we have

$$\frac{1}{\xi}\frac{d}{d\xi}\left(\xi\frac{d\Phi}{d\xi}\right) = -f(\xi),$$

where $f(\xi)$ is a dimensionless function of the heat-release density. Integrating, we obtain for any form $f(\xi) > 0$

$$\Phi_\xi'(\xi) = -\frac{1}{\xi}\int_0^\xi \xi f(\xi)\,d\xi \leq 0,$$

i.e., there is always defocusing or the absence of refraction.

We note that the formation of a region of small temperature gradients near the axis leads to a decrease of the nonlinear refraction near the axis (the focal distance for the near-axis rays becomes very large and we obtain an almost parallel beam near the axis).

Let us investigate the temperature gradient in our case for arbitrary $\xi$ and different instants of time.

For any $\xi$ at small $\tau \ll 1$ (initial stage of heating) we obtain, expanding the exponential:

$$\left\{(\xi^2+1) - \exp\left(\frac{4\tau\xi^2}{1+4\tau}\right)\left[1+\frac{\xi^2}{(1+4\tau)^2}\right]\right\} = 4\tau\xi^2(1-\xi^2),$$

i.e., we get $\Phi'_\xi > 0$ when $\xi < 1$, and thus we get focusing, and $\Phi'_\xi < 0$ when $\xi > 1$, i.e., we plainly have defocusing of the edge part of the beam—a situation typical of "banana" self-focusing.

Solving the transcendental equation $\Phi'_\xi = 0$ we can determine for which values of $\tau$ for the given $\xi$ are the self-focusing conditions still satisfied. It is easily seen that for $\tau \sim 1$ self-focusing appears still in a sufficiently large region. For $4\tau \gg 1$ we have

$$\{\ \} = {}^1\!/_2\xi^2(1/2\tau - \xi^2) \text{ for } \xi \ll 1,$$

i.e., we obtain the previous conditions for near-axis focusing

$$\tau < 1/2\xi^2.$$

Let us estimate the extent to which the focal lengths increase. Since the refraction angle is

$$\theta \approx \frac{\partial n}{\partial r}l \approx n_T'\frac{\partial T}{\partial r}l,$$

and the focal distance is $L_f \approx r/\theta \sim r/T_r'$, it follows that $L_f \sim \xi/\Phi_\xi'(\xi, \tau)$, all other conditions being equal. But for $\xi \ll 1$ in the case of developed "banana" self-focusing (at $4\tau \approx 1$) we have $\Phi_\xi'(\xi, \tau) \approx \xi/2$ and for $4\tau \gg 1$ we have $\Phi_\xi'(\xi, \tau) \approx \xi/8\tau$, i.e.,

$$L_f/L_{f_1} \approx 4\tau.$$

However, when the beam profile deviates from quadratic-parabolic, a strong caustic is observed, and the lengthening of the focal distance is smeared out by changes inside the elongated caustic, as was observed in a number of experiment,[15] in which an increased intensity on the beam axis was observed for a very long time.


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141