HYDRODYNAMICS OF A MAGNETOACTIVE PLASMA IN THE QUASILINEAR APPROXIMATION

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Submitted October 15, 1970

Zh. Eksp. Teor. Fiz. 60, 579–587 (February, 1971)

Hydrodynamic equations for the first moments, with allowance for the influence of the waves on the translational velocities and the temperatures of different plasma components, are obtained from the kinetic equation for the "slow" distribution function of a magnetoactive spatially-homogeneous plasma. The distribution function was taken in a form that makes it possible to determine the action of the waves on the anisotropy of the pressure and the currents in the plane perpendicular to the constant magnetic field H_0. The simplest cases of waves propagating along and across H_0 are considered. In the high-frequency limit, expressions are obtained for the dragging and for the heating of the electrons; these expressions demonstrate the role of Cerenkov and cyclotron resonances in longitudinal propagation of the waves. For transverse waves propagating perpendicular to H_0, expressions are obtained for the nonlinear drift current excited by a wave perpendicular to the wave vector and to the magnetic field. In conclusion, nonlinear effects arising in the propagation of Alfven and magnetosonic waves are discussed briefly.

IN the present paper we generalize and refine the results of our earlier article[1], which was devoted to the quasilinear theory of magnetoactive plasma for high-frequency waves interacting with plasma particles in the case of Cerenkov resonance.

In the general case, in the derivation of hydrodynamic equations, when integration is carried out over all the particle velocities, it is necessary to take into account all types of interactions of particles with waves in the plasma (Cerenkov and cyclotron resonances with the harmonics) for only allowance for all the mechanisms of the interaction makes it possible to estimate in each concrete case the contribution made by the waves to the momentum and energy balance equations. In some cases, the role of one of the resonances may become decisive (for example, the role of the Cerenkov mechanism in the propagation of longitudinal waves along an external constant magnetic field H_0, or the role of the cyclotron mechanism in propagation of transverse waves along H_0), whereas in other cases (for example, in oblique propagation or propagation perpendicular to H_0) it is not correct to separate any particular interaction effect, and their summary effect is of importance.

An essential feature of the present paper (as well as of[1]) is that, besides taking into account the collisions of the particles, we take into account the dependence of the "slow" distribution function f_a^0, which describes the relaxation behavior of the plasma, on the azimuthal angle φ in the velocity space, and this makes it possible to describe effects that occur in a plane perpendicular to H_0. This is essential, for besides a constant magnetic field there is also one more preferred direction, namely the wave-propagation direction. Consequently, cylindrical symmetry is violated, as a result of which one can speak of the appearance of vector quantities, for example particle drifts due to the field of the wave in a direction perpendicular to the wave vector and the magnetic field. Such an analysis is all the more important since the influence of the magnetic field is appreciable precisely in a plane perpendicular to H_0. In addition, this reveals most clearly the role of the cyclotron resonance as the mechanism for the interaction between resonant particles and the waves (at velocities $v_z \sim (\omega \pm n\Omega_a)/k_z$, $\Omega_a = e_a H_0/m_a c$).

In the given formulation of the quasilinear theory, the collisions are taken into account in two ways: 1) with the aid of a "slow" collision integral S_a^0, which enters in the right side of the kinetic equation for f_a^0, and 2) via the effective collision frequency ν_a, which simulates their contribution to the rapidly-alternating processes[2,3]:

$$S_a^1 = -\nu_a f_a^1. \tag{1}$$

Here f_a^1 is the "fast" (pulsating) part of the distribution function and S_a^1 is the "fast" part of the collision integral.

The effective frequency $\nu_a = \nu_a(f_a^0, v, \omega, k)$ determines the final width of the curve of the resonant interaction between the particles and the plasma waves; this curve characterizes the coefficient of particle diffusion in velocity space. In earlier work on the quasilinear theory, without allowance for the collisions, the resonant-interaction curve turned out to be infinitesimally thin and was chosen in the form of a δ function (which is obtained when $\nu_a \to 0$). A consistent allowance for the collisions leads to a replacement of the δ-like curve of the resonance between the particle and the wave with a curve having a Lorentz profile, which of course is closer to the real conditions in a plasma.

Such a formulation of the theory makes it possible to take more complete account of the dissipation in the interaction of particles with oscillations, and in particular, it makes it possible to determine the heating and the dragging of the plasma components by the ions (adiabatic effects).

Allowance for the collisions in the form (1) makes essential use of a model, and requires further justification in the analysis of, say, the low-frequency processes in a fully ionized plasma. However, for high-frequency processes, or, for example, a weakly-ionized plasma, the model (1) determines with sufficient accuracy the role of the collisions in the interaction of particles with waves.

In[1] we obtained an equation for f_a^0 for the propagation of high-frequency waves, and the magnetic field of the wave H was therefore neglected. In the general case, the equation of quasilinear theory for a magnetoactive spatially-homogeneous plasma has the following form:

$$\frac{\partial f_a^0}{\partial t} - \mathrm{Re}\frac{ie_a^2}{2m_a^2}\sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty}\hat{L}J_m\left(\frac{k_\perp v_\perp}{\Omega_a}\right)J_n\left(\frac{k_\perp v_\perp}{\Omega_a}\right)e^{i(m-n)(\varphi-\theta)}$$
$$\times\left\{\frac{1}{\omega-k_z v_z-n\Omega_a+i\nu_a}\left[E_z\frac{\partial}{\partial v_z}-\frac{1}{c}H_z\frac{\partial}{\partial\varphi}\right]\right.$$
$$+\frac{1}{2}\frac{e^{-i\varphi}}{\omega-k_z v_z-\Omega_a-n\Omega_a+i\nu_a}\left[\left(E_\perp e^{i\psi}+\frac{i}{c}H_\perp e^{i\eta}v_z\right)\right.$$
$$\left.\times\left(\frac{\partial}{\partial v_\perp}-\frac{i}{v_\perp}\frac{\partial}{\partial\varphi}\right)-\frac{i}{c}H_\perp e^{i\eta}v_\perp\frac{\partial}{\partial v_z}\right]+\frac{1}{2}$$
$$\times\frac{e^{i\varphi}}{\omega-k_z v_z+\Omega_a-n\Omega_a+i\nu_a}\left[\left(E_\perp e^{-i\psi}-\frac{i}{c}H_\perp e^{-i\eta}v_z\right)\right.$$
$$\left.\left.\times\left(\frac{\partial}{\partial v_\perp}+\frac{i}{v_\perp}\frac{\partial}{\partial\varphi}\right)+\frac{i}{c}H_\perp e^{-i\eta}v\frac{\partial}{\partial v_z}\right]\right\}f_a^0-\Omega_a\frac{\partial f_a^0}{\partial\varphi}=S_a^0, \quad (2)$$

$$\hat{L}=\left(\mathbf{E}+\frac{1}{c}[\mathbf{vH}]\right)\nabla, \quad \nabla=\frac{\partial}{\partial\mathbf{v}}. \quad (3)^*$$

Equation (2) is written in a cylindrical system of coordinates, the external magnetic field is directed along the z axis, the symbol $\perp$ denotes the corresponding quantities in the xy plane; φ, θ, ψ, and η denote the angles between the x axis and the corresponding vectors $\mathbf{v}_\perp$, $\mathbf{k}_\perp$, $\mathbf{E}_\perp$, and $\mathbf{H}_\perp$. For simplicity, we shall put $\theta = 0$, i.e., we choose the x axis along $\mathbf{k}_\perp$.

The amplitudes of the electric and magnetic fields of the wave are connected by Maxwell's equation

$$\frac{1}{c}\mathbf{H}(\omega,\mathbf{k})=\frac{1}{\omega}[\mathbf{kE}(\omega,\mathbf{k})], \quad (4)$$

but in the subsequent calculation it is convenient to retain the notation with H in the hydrodynamic equations. In particular, the terms with H pertain only to transverse waves (relative to k). We note also that the terms containing $E_\perp e^{i\psi}$ and $-iH_\perp e^{i\eta}$ correspond to waves with positive helicity (when viewed along the z axis), and the terms containing $E_\perp e^{-i\psi}$ and $iH_\perp e^{-i\eta}$ correspond to waves with negative helicity. In the case of wave propagation along the magnetic field ($k_\perp = 0$), they correspond to transverse waves with left-hand and right-hand circular polarizations.

Equation (2) is the most general equation of the quasilinear theory for the "slow" distribution function in a magnetoactive spatially-homogeneous plasma, since it takes into account not only the collisions but also the anisotropy of f_a^0 in the xy plane. By the same token, it is possible to obtain from it expressions for the currents that arise in a plane perpendicular to $\mathbf{H}_0$ under the influence of various types of waves in the plasma, whereas no such expressions can be obtained from the equation for the function f_a^0 averaged over φ. Equation (2) describes the interaction of the plasma particles with different types of waves propagating at an arbitrary angle to $\mathbf{H}_0$, both in the case of Cerenkov resonance and in the case of cyclotron resonance. The structure of the equation offers evidence of the possibility of formation of a plateau on the distribution function for the velocity component along the magnetic field (the terms appearing in the denominators and containing v_z). The possibility of formation of a plateau for velocities perpendicular to $\mathbf{H}_0$ depends on the character of the behavior of the Bessel functions J_m and J_n, and calls for an additional investigation. For small values of the argument, the Bessel functions are expanded in power series, thus indicating that it is impossible in this case for a plateau to be formed in a plane perpendicular to the magnetic field.

The "slow" part of the collision integral S_a^0, for not very strong magnetic fields (when $\Omega_a \ll \omega_a$, where ω_a is the Langmuir frequency of the component a) and without allowance for the polarization, is obtained in a completely ionized plasma in the Landau form. On the other hand, in order for the influence of the magnetic field to be appreciable, we shall assume that H_0 is not very small, so that the condition $k_\perp v_a^\perp \ll \Omega_a$ is satisfield, where $v_a^\perp$ is the perpendicular component of the thermal velocity of the component a relative to the magnetic field. This makes it possible to carry out in (2) an expansion of the Bessel function at small values of the argument, and by the same token to simplify greatly the derivation of the hydrodynamic equations.

Confining ourselves in the sums over m and n to terms containing the ratio $k_\perp v_\perp/\Omega_a$ to a degree not higher than the first, which makes it possible to employ only values m, n = 0, ± 1, we can write Eq. (2) in the form

$$\frac{\partial f_a^0}{\partial t}+\hat{D}(f_a^0)-\Omega_a\frac{\partial f_a^0}{\partial\varphi}=S_a^0, \quad (5)$$

$$\hat{D}(f_a^0)=\frac{e_a^2}{2m_a^2}\hat{L}\sum_{j=1}^{5}\frac{A_j\hat{B}_j-\nu_a\hat{C}_j}{A_j^2+\nu_a^2}f_a^0, \quad (6)$$

where the quantities A_1 and the operators $\hat{B}_j$ and $\hat{C}_j$ (in a Cartesian coordinate system) are equal to

$$A_1=\omega-k_z v_z, \quad \hat{B}_1=\frac{k_x}{\Omega_a}\left[v_y\hat{a}_1-\frac{1}{2}(v_y\hat{a}_2+v_x\hat{a}_3)\right], \quad \hat{C}_1=\hat{a}_1,$$
$$A_{2,3}=\omega-k_z v_z\mp\Omega_a, \quad \hat{B}_{2,3}=\frac{1}{2}\left[\pm\hat{a}_3-\frac{k_x}{\Omega_a}v_y(\hat{a}_1-\hat{a}_2)\right],$$
$$\hat{C}_{2,3}=\frac{1}{2}\left[\hat{a}_2\pm\frac{k_x}{\Omega_a}(v_x\hat{a}_1-v_y\hat{a}_3)\right],$$
$$A_{4,5}=\omega-k_z v_z\mp 2\Omega_a, \quad \hat{B}_{4,5}=\frac{1}{4}\frac{k_x}{\Omega_a}(v_x\hat{a}_3-v_y\hat{a}_2),$$
$$\hat{C}_{4,5}=\pm\frac{1}{4}\frac{k_x}{\Omega_a}(v_x\hat{a}_2+v_y\hat{a}_3), \quad (7)$$
$$\hat{a}_1=E_z\nabla_z-\frac{1}{c}H_z[\mathbf{v}\nabla]_z,$$
$$\hat{a}_2=\mathbf{E}_\perp\nabla_\perp-\frac{1}{c}(v_z[\nabla\mathbf{H}]_z-\nabla_z[\mathbf{vH}]_z),$$
$$\hat{a}_3=-[\mathbf{E}\nabla]_z+\frac{1}{c}(v_z\nabla_\perp\mathbf{H}_\perp-\nabla_z\mathbf{v}_\perp\mathbf{H}_\perp).$$

The upper sign pertains here to quantities having the first index (for example A_2 and A_4), and the lower sign to quantities having the second index.

Thus, when account is taken of only the first degree of k_x (the case of small spatial dispersion for the wave-vector component perpendicular to $\mathbf{H}_0$) in the case of particle-wave interaction, only the Cerenkov

*$[\mathbf{vH}] \equiv \mathbf{v} \times \mathbf{H}$.

and the single and double cyclotron resonances contribute in the normal and anomalous Doppler effect, whereas the higher cyclotron harmonics are discarded. However, even in this approximation, the quasilinear term, which determines the diffusion of the particle from the waves, has, as can be seen from (7), a rather complicated form, which gives no less complicated expressions in the hydrodynamic equations. It is therefore advantageous to confine oneself in their approximation to the simplest cases of propagation of waves along and across $\mathbf{H}_0$, so as to reveal completely the hydrodynamic effects occurring when particles interact with waves.

The hydrodynamic equations for the first moments of the distribution function f_a^0 (the equations for the momentum and energy balance of the component a) will be calculated, just as in[1], in Grad's "zeroth" approximation with allowance for the anisotropy of the temperatures along and across $\mathbf{H}_0$, i.e., we use the function f_a^0 in the form

$$f_a^0 = f_a^\perp(\mathbf{v}_\perp, t) f_a^z(v_z, t) = (\sqrt{\pi})^{-3}(v_a^\perp)^{-2}(v_a^z)^{-1}\exp\left\{-\frac{(\mathbf{v}_\perp - \mathbf{u}_a^\perp(t))^2}{(v_a^\perp)^2} - \frac{(v_z - u_a^z(t))^2}{(v_a^z)^2}\right\}, \quad v_a^\perp = \left(\frac{2T_a^\perp(t)}{m_a}\right)^{1/2}, \quad v_a^z = \left(\frac{2T_a^z(t)}{m_a}\right)^{1/2}. \tag{8}$$

Such a choice of the distribution function makes it possible to take into account the influence of waves on the currents in a plane perpendicular to $\mathbf{H}_0$, and on the anisotropy of the temperatures.

We shall also assume that the translational velocities u_a are much lower than the thermal ones, and we shall neglect the square of the ratio of these velocities compared with unity. We shall consider several cases below.

A. High-frequency Waves

Such waves satisfy the conditions

$$|\omega \pm n\Omega_a| \gg k_z v_a^z, \qquad n = 0, 1, 2. \tag{9}$$

corresponding to the case of a "cold" plasma. We shall assume in addition that

$$|\omega \pm n\Omega_a| \gg \nu_a, \qquad n = 0, 1, 2, \tag{10}$$

i.e., we shall consider waves outside the resonance region, where they are strongly absorbed. Under these conditions we can separate four characteristic cases of wave propagation:

Case A1. Longitudinal waves along $\mathbf{H}_0$ ($k_x = 0$, $\mathbf{E}_\perp = 0$, $\mathbf{H} = 0$). In this case, as follows from (7), the only nonzero operator from among $\hat{B}_j$ and $\hat{C}_j$ will be $\hat{C}_1 = E_z\nabla_z$. This indicates that the only mechanism for the interaction of the longitudinal waves along $\mathbf{H}_0$ with the plasma particles is the Cerenkov resonance. This case is considered in detail in[1].

Case A2. Transverse waves along H_0 ($k_x = 0$, $E_z = 0$, $H_z = 0$). Here

$$\hat{B}_2 = -\hat{B}_3 = {}^1/_2\hat{a}_3, \quad \hat{C}_2 = \hat{C}_3 = {}^1/_2\hat{a}_2, \tag{11}$$

and the remaining operators $\hat{B}_j$ and $\hat{C}_j$ are equal to zero. The responsible interaction mechanism is cyclotron resonance with normal and anomalous Doppler effects.

In the "cold" plasma approximation, the balance equations have the form

$$\begin{gathered}
\frac{du_a^{x,y}}{dt} \mp \Omega_a u_a^{y,x} = \int v_{x,y} S_a^0\, d\mathbf{v},\\
\frac{du_a^z}{dt} - \frac{k_z}{\omega}\frac{e_a^2 E_\perp^2 \nu_a}{2m_a^2}\frac{\omega^2 + \Omega_a^2}{(\omega^2 - \Omega_a^2)^2} = \int v_z S_a^0\, d\mathbf{v},\\
\frac{dT_a^\perp}{dt} - \frac{e_a^2 E_\perp^2 \nu_a}{2m_a}\frac{\omega^2 + \Omega_a^2}{(\omega^2 - \Omega_a^2)^2} = m_a \int \frac{v_\perp^2}{2} S_a^0\, d\mathbf{v},\\
\frac{1}{2}\frac{dT_a^z}{dt} = m_a \int \frac{v_z^2}{2} S_a^0\, d\mathbf{v}.
\end{gathered} \tag{12}$$

The upper sign in the first equation pertains to the equation for u_a^x, and the lower to the equation for u_a^y; expressions for the integrals in the right-hand sides are given in[1].

In the stationary state, in analogy with[1], it is possible to obtain from (12) the currents and the temperature differences produced by the transverse waves along $\mathbf{H}_0$:

$$\begin{gathered}
u_e^\perp = 0, \qquad u_e^z = \frac{k_z}{\omega}\frac{e^2 E_\perp^2 \nu_e \tau_{ei}^u (\omega^2 + \Omega_e^2)}{2m_e^2(\omega^2 - \Omega_e^2)^2},\\
T_e^\perp = T_e^z + \frac{5\sqrt{2}}{12(1+\sqrt{2})}\frac{e^2 E_\perp^2 \nu_e \tau_{ei}^u (\omega^2 + \Omega_e^2)}{m_e(\omega^2 - \Omega_e^2)^2},\\
T_e^z = T_i + \frac{e^2 E_\perp^2 \nu_e \tau_{ei}^T (\omega^2 + \Omega_e^2)}{3m_e(\omega^2 - \Omega_e^2)^2}.
\end{gathered} \tag{13}$$

Comparing this with the corresponding expressions for the longitudinal waves (see[1]), we see that the longitudinal and transverse waves complement each other, so to speak, and in particular the longitudinal waves produce an electron temperature anisotropy with $T_e^z > T_e^\perp$, and the transverse waves do the opposite. In both cases, however, this anisotropy is small since it is determined by the small parameter of the quasilinear theory.

Case A3. Transverse linearly-polarized waves propagating perpendicular to $\mathbf{H}_0$ with $\mathbf{E} \parallel \mathbf{H}_0$ ($k_z = 0$, $E_\perp = 0$, $H_x = H_z = 0$). In this case, just as in case A4, it is meaningless to separate the Cerenkov or the cyclotron resonances, for when $k_z = 0$ the particles, in order to fall into resonance with the wave, would have to have an infinitely large velocity along the z axis. The character of the interaction of the waves with the particles is not resonant here, but adiabatic.

The form of the operators $\hat{B}_j$ and $\hat{C}_j$, neglecting the terms $\sim k_x^2/\omega\Omega_a$, is given by the equations

$$\begin{gathered}
\hat{B}_1 = E_z \frac{k_x v_y}{\Omega_a}\nabla_z, \qquad \hat{C}_1 = E_z\nabla_z,\\
\hat{B}_{2,3} = \mp\frac{1}{2}E_z\frac{k_x}{\omega}\left(v_z\nabla_y \pm \frac{\omega \mp \Omega_a}{\Omega_a} v_y \nabla_z\right),\\
\hat{C}_{2,3} = \frac{1}{2}E_z\frac{k_x}{\omega}\left(v_z\nabla_x \pm \frac{\omega \mp \Omega_a}{\Omega_a} v_x \nabla_z\right),\\
\hat{B}_4 = \hat{C}_4 = \hat{B}_5 = \hat{C}_5 = 0.
\end{gathered} \tag{14}$$

The energy and momentum balance equations are of the form

$$\begin{gathered}
\frac{du_a^x}{dt} - \frac{k_x}{\omega}\frac{e_a^2 E_z^2 \nu_a}{2m_a^2(\omega^2 + \nu_a^2)} - \Omega_a u_a^y = \int v_x S_a^0\, d\mathbf{v},\\
\frac{du_a^y}{dt} + \Omega_a u_a^x = \int v_y S_a^0\, d\mathbf{v}, \qquad \frac{du_a^z}{dt} = \int v_z S_a^0\, d\mathbf{v},\\
\frac{dT_a^\perp}{dt} = m_a \int \frac{v_\perp^2}{2} S_a^0\, d\mathbf{v}, \qquad \frac{1}{2}\frac{dT_a^z}{dt} - \frac{e_a^2 E_z^2 \nu_a}{2m_a(\omega^2 + \nu_a^2)} = m_a \int \frac{v_z^2}{2} S_a^0\, d\mathbf{v}.
\end{gathered} \tag{15}$$

In the stationary state in an electron-ion plasma we obtain for the temperatures exactly the same expressions as in the case A1 (see[1]), whereas for the electron current components we have

$$u_x=\frac{k_x}{\omega}\frac{e^2E_z{}^2\nu_e\tau_{ei}{}^u}{2m_e{}^2(\omega^2+\nu_a{}^2)}\frac{1}{1+(\Omega_e\tau_{ei}{}^u)^2},\qquad(16)$$
$$u_y=-u_x\Omega_e\tau_{ei}{}^u,\qquad u_z=0.$$

If $\Omega_e\tau_{ei}^u \gg 1$ (the momentum relaxation time due to the electron-ion collisions is larger than the Larmor period of the electrons), then $|u_y| \gg |u_x|$. The minus sign in front of u_y denotes that the current is excited in a direction opposite to the positive direction of the y axis, which in this case coincides with the direction of the vector $\mathbf{k}\times\mathbf{H}_0$. We shall call this the nonlinear drift current, since it is perpendicular to $\mathbf{H}_0$ and to the Poynting vector of the transverse wave. We note that this current is larger by a factor $\Omega_e\tau_{ei}^u$ than the nonlinear current excited as a result of "direct" momentum transfer by the wave along the x axis, but is smaller by a factor $\Omega_e\tau_{ei}^u$ than the nonlinear current excited by the longitudinal waves along the z axis in case A1 (see[1]). In[1] it was indicated that for case A3 we have $u_x = u_y = 0$, this being due to the discarding of the term containing the magnetic field of the wave in the high-frequency limit in the initial kinetic equation. It is precisely this term which makes a contribution to the balance equation leading to the appearance of the nonlinear drift current. Excitation of this current is apparently connected with the fact that relative motion of the electrons and ions sets in in the field of the transverse wave, and the particles are in crossed electric and magnetic fields.

Case A4. Transverse linearly polarized waves propagating perpendicular to $\mathbf{H}_0$, with $\mathbf{E}\perp\mathbf{H}_0$. This case does not differ in principle from the preceding one, and complements it in much the same way as A2 complements the case A1.

In the stationary state, the expressions for the temperatures are found to be the same as in the case A2, and for the electron current components we have

$$u_x=\frac{k_x}{\omega}\frac{e^2E_y{}^2\nu_e\tau_{ei}{}^u(\omega^2+\Omega_e{}^2)}{2m_e{}^2(\omega^2-\Omega_e{}^2)^2}\frac{1}{1+(\Omega_e\tau_{ei}{}^u)^2},\qquad(17)$$
$$u_y=-u_x\Omega_e\tau_{ei}{}^u,\qquad u_z=0,$$

which is analogous to (16).

B. Low-frequency Waves

Greatest interest attaches to an analysis of the magnetohydrodynamic waves, in which the frequencies are smaller than or comparable with the ion cyclotron frequency. In this region of frequencies the waves exert a strong influence on the ions. We confine ourselves for simplicity to two cases.

Case B1. Alfven wave along $\mathbf{H}_0$.

As is well known, when low-frequency magnetohydrodynamic waves are considered, it is possible to employ the "cold" plasma approximation (see, for example,[4]), and therefore the formulas obtained above for the high-frequency waves can be used also for the low-frequency ones.

In the Alfven wave, the electric vector is perpendicular to $\mathbf{k}$, and consequently the mechanism of interaction of these waves with the plasma particles is cyclotron resonance (in analogy with the case A2). The balance equations, with allowance for the condition $\omega < \Omega_a$, are

$$\frac{du_a{}^{x,y}}{dt}\mp\Omega_a u_a{}^{y,x}=\int v_{x,y}S_a{}^0\,d\mathbf{v},$$
$$\frac{du_a{}^z}{dt}-\frac{k_z}{\omega}\frac{e_a{}^2E_\perp{}^2\nu_a}{2m_a{}^2(\Omega_a{}^2+\nu_a{}^2)}=\int v_zS_a{}^0\,d\mathbf{v},$$
$$\frac{dT_a{}^\perp}{dt}-\frac{e_a{}^2E_\perp{}^2\nu_a}{2m_a(\Omega_a{}^2+\nu_a{}^2)}=m_a\int\frac{v_\perp{}^2}{2}S_a{}^0\,d\mathbf{v},\qquad(18)$$
$$\frac{1}{2}\frac{dT_a{}^z}{dt}=m_a\int\frac{v_z{}^2}{2}S_a{}^0\,d\mathbf{v}.$$

We see therefore that the contributions of the electrons and of the ions to the current along the z axis are approximately equal and are determined by the corresponding collision frequencies. As to the heating, it is the ions that are predominantly heated, since the contribution of the waves to the growth of $T_a^\perp$ in the third equation of (18) is proportional to m_a (under the condition $\Omega_a > \nu_a$).

Case B2. Magnetosonic wave propagating perpendicular to $\mathbf{H}_0$.

Although such a wave is sometimes called "longitudinal" in the literature in the sense that the displacement vector of the velocity of matter is parallel to $\mathbf{k}$, we are dealing with a transverse linearly-polarized wave in which the magnetic vector is directed along the z axis and the electric vector along the y axis, i.e., it is perpendicular to the propagation direction.

For such a wave one can employ the results obtained above for the case A4 with allowance for the condition $\omega < \Omega_i$. This case, like the case B1, is characterized by predominant heating of the ions with the transverse pressure component exceeding the longitudinal one.

A number of experimental studies[5-8] have been made of strong heating of plasma by ion-cyclotron and magnetosonic waves with the conditions $\omega \lesssim \Omega_i$ and $\Omega_i < \omega \ll \Omega_e$ satisfied. The results of these investigations could not be explained on the basis of the linear theory of cyclotron and Cerenkov damping of waves, nor could they be attributed to the influence of collisions. An attempt was therefore made in[9,10] to connect them with the excitation of high-frequency small-scale instabilities due to the relative motion of the ions and electrons in the electric field of the low-frequency wave. Without excluding a similar heating mechanism, which is a secondary effect in this analysis (the wave excites two-stream instability, which leads to a rapid turbulent heating), we note that in the present paper the heating of the ions (under the condition $\omega < \Omega_i$) is explained within the framework of the quasilinear approximation as being due to direct transfer of energy from the wave to the particles in cyclotron resonance, as a result of collisions. There is also qualitative agreement with the experimental data concerning the growth of the transverse component of the pressure relative to the magnetic field ($T^\perp > T^z$) when the plasma is heated by a fast magnetosonic wave[8]. A quantitative comparison of the results is unfortunately impossible since the conclusion of the present investigations were obtained under the assumption that the wave energy and the temperature anisotropy are low, whereas in[8] they used waves with large amplitudes.

We note that if the mechanism for heating the plasma by low-frequency transverse waves is cyclotron resonance, then in propagation of longitudinal waves, as can be readily shown in accordance with the same scheme[11], the predominant role in the plasma heating is assumed by the Cerenkov resonance, and the electrons are heated. The predominant heating of any particular species of particles is determined by the ratio of the number of these particles in the region of the resonant velocities.

In conclusion, it must be emphasized once more that allowance for the anisotropy of the velocities in a plane perpendicular to H_0 makes it possible to refine significantly the picture of the interaction of waves with resonant particles. In addition, the appearance of adiabatic (hydrodynamic) effects in the quasilinear approximation is possible only when account is taken of the finite width of the curve of resonant interaction of the particles with the waves, this being connected with the introduction of the effective frequency ν_a, which simulates the contribution of the collisions to the rapidly-alternating processes. Such an analysis is equivalent to taking into account the finite correlation time of the electro microfields in the plasma, as was done in[12].

Under conditions of a "cold" plasma, the current direction is determined by the corresponding component of the wave vector, and the pressure anisotropy is determined by the corresponding component of the electric vector of the wave. In the propagation of waves perpendicular to the magnetic field, the wave causes a drift current perpendicular to H_0 and $\mathbf{k}$, the magnitude of which is larger by a factor of $\Omega_a \tau_{ab}^u$ than the nonlinear current excited by the wave along $\mathbf{k}$.

In the present paper we considered the simplest cases of wave propagation, but the described scheme can be used in the arbitrary case of oblique propagation. In the analysis of low-frequency waves, it is necessary to refine also the role of ionic collisions.

The authors are grateful to Yu. L. Klimontovich for advice and interest in the work.

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Translated by J. G. Adashko
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