MACROSCOPIC ELECTRODYNAMICS AND THERMAL EFFECTS IN THE INTERMEDIATE STATE OF SUPERCONDUCTORS

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We obtain a complete system of equations describing the interrelated electromagnetic and thermal effects in the intermediate state of superconductors of the first kind. The kinetic coefficients (the resistivity, heat conduction, and thermoelectricity tensors) are calculated for all possible types of structures, i.e., for a layered structure and for filamentary structures with normal and superconducting filaments. Formulas are obtained for the Ettingshausen effect and are in satisfactory agreement with the available experimental data. The rate of motion of the structure under stationary conditions under the influence of current and a temperature gradient is calculated.

A superconductor of the first kind in the intermediate state constitutes a system whose volume is made up of a large number of regions occupied by normal and superconducting phases[1]. Such a system admits of a macroscopic description with the aid of the concepts of magnetic induction and magnetic intensity, which are quantities averaged over volumes with linear dimensions that are large compared with the dimensions of the normal and the superconducting regions. Peierls[2] and London[3,4] obtained electrodynamic equations that made it possible to calculate the average quantities in the state of thermodynamic equilibrium.

As shown by Gorter[5], the boundaries between the phases can move in the presence of an electric field. This motion was observed experimentally by Sharvin[6]. Since the motion of the separation boundaries is accompanied by the transition of the substance from the normal phase into the superconducting phase and vice-versa, and consequently by release or absorption of the heat of the phase transition, it is clear that the electromagnetic phenomena in a non-equilibrium system are closely related with thermal effects. An exception is the case of low temperatures, when the heat of the phase transition is small and its influence on the electrodynamics can be neglected. Sharvin and one of the authors[7,8] obtained a system of equations describing non-equilibrium and non-stationary electromagnetic phenomena at low temperatures, and making it possible to find the velocity of motion and the location of the phase separation boundaries.

In the general case the complete system of dynamic equations should contain the coupled equations of electrodynamics and heat-conduction equations. The determination of such a system of equations is the main task of the present paper. The analysis makes it possible, first, to ascertain the purely electromagnetic properties at finite temperatures and, second, to consider specific thermal and electromagnetic phenomena. The anomalously large value of the thermoelectric coefficients in the intermediate state, noted by Sharvin[6], was observed by Solomon and Otter[9,10], who investigated in detail the Ettingshausen effect. The results of the present paper agree with their data.

Unlike the case of thermodynamic equilibrium, when the macroscopic theory is not sensitive to the type of the structure of the intermediate state, in the general case the results are different for different types of structures. There exist three types of structures. First is the layered structure, in which the normal and superconducting regions are systems of alternating layers. In addition, under conditions when the concentration of one of the phases is low, two types of filamentary structures are possible, with normal or superconducting filaments. In these structures, the regions occupied by the phase with the low concentration constitute thin cylindrical filaments stretched along the direction of the magnetic field. We consider here all three types of structures.

The equations obtained by us make it possible to calculate the velocity of the phase separation boundaries. It turns out that under stationary conditions the motion is possible not only under the influence of an electric field, but also under the influence of a temperature gradient in the absence of a current. Calculations show that observation of such motion is perfectly feasible experimentally.

1. EQUATIONS OF MACROSCOPIC ELECTRODYNAMICS AND OF HEAT CONDUCTION

We assume that all quantities (electromagnetic field, temperature, location of the phase separation boundaries) change little over distances on the order of the period of the structure. This case is of greatest interest, since the period of the structure is small compared with the dimensions of the sample. In addition, this is the only case when a general description of the intermediate state with the aid of macroscopic equations is possible.

The magnetic field **h** and the electric field **e** in the normal regions satisfy Maxwell's equations (the magnetic permeability of the normal metal is assumed equal to unity)

$$\operatorname{rot}\mathbf{h} = \frac{4\pi}{c}\mathbf{i}, \quad \operatorname{div}\mathbf{h} = 0, \quad \operatorname{rot}\mathbf{e} = -\frac{1}{c}\frac{\partial\mathbf{h}}{\partial t}, \tag{1}$$

where **i** is the electric current density, and vanish in the superconducting phase. On the phase separation boundaries there should be satisfied the ordinary conditions of continuity of the tangential components of the electric field e' and of the normal component of the magnetic field h' in a coordinate system connected with the boundaries. Since the electric field in the normal phase is always much weaker than the magnetic field, and since in the superconducting phase we have $h = e = 0$, the boundary conditions reduce to vanishing of the tangential components of the vector $e + (V/c)n \times h$ and of the normal component of the vector h, where V and n are the velocity and the normal to the boundary. In addition, the tangential component of the magnetic field h should be equal on the boundary to the critical value H_c, which depends on the temperature. The boundary conditions consequently take the form

$$\mathbf{hn} = 0, \quad \left[\mathbf{n}, \mathbf{e} + \frac{V}{c}[\mathbf{n}\,\mathbf{h}]\right] = 0, \quad |\mathbf{h}| = H_c(T), \qquad (2)^*$$

whence

$$\mathbf{eh} = 0, \quad |\mathbf{h}| = H_c(T). \qquad (3)$$

Since the last equations are satisfied on all phase-separation boundaries, and the fields h and e vary little over distances on the order of the period of the structure, we can assume that conditions (3) are valid everywhere in the normal regions.

We now introduce the macroscopic quantities. Let the vector H assume in a given physically infinitesimally small volume (with dimensions large compared with the period of the structure but small compared with the distances over which the position of the phase-separation boundaries changes significantly) a value equal to the field h in the normal regions. Symbolically this can be written in the form $H = h$, and it is precisely in this sense that all the equalities between the "microscopic" and macroscopic quantities should be taken. If we denote the concentration of the normal phase by x_n, then the magnetic induction $\mathbf{B}$ and the electric field $\mathbf{E}$, defined as the averages of h and e over the volume, are obviously equal to

$$\mathbf{B} = x_n\mathbf{h}, \quad \mathbf{E} = x_n\mathbf{e}. \qquad (4)$$

It is also easy to calculate the magnetic moment per unit volume M, due to the superconducting currents flowing along the separation boundaries. Noting that the values of these currents are determined by the discontinuity of h on the boundaries, we obtain $M = -x_s H/4\pi$, where x_s is the concentration of the superconducting phase, from which we see that $H = B - 4\pi M$, i.e., H is the magnetic intensity. From (3) and (4) we see that the vectors $\mathbf{B}$, $\mathbf{H}$, and $\mathbf{E}$, satisfy not only Maxwell's equations

$$\operatorname{div}\mathbf{B} = 0, \quad \operatorname{rot}\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}, \quad \operatorname{rot}\mathbf{H} = \frac{4\pi}{c}\mathbf{j}, \qquad (5)$$

where are obtained by averaging (1), but also the conditions

$$\mathbf{B} = x_n\mathbf{H}, \quad |\mathbf{H}| = H_c(T), \quad \mathbf{EH} = 0. \qquad (6)$$

To derive the heat-transport equations, we used the laws of energy conservation and entropy increase, which can be written in the form

$$\dot{\mathscr{E}} + \operatorname{div}\Theta = 0, \quad \dot{S} + \operatorname{div}\frac{\mathbf{q}}{T} = \frac{R}{T}, \quad (R > 0). \qquad (7)$$

*$[\mathbf{nh}] \equiv \mathbf{n} \times \mathbf{h}$.

Here $\mathscr{E}$ and S are the energy and entropy per unit volume of the intermediate states, connected with the energies and entropies of the normal and superconducting phases by the relations

$$\mathscr{E} = x_n\mathscr{E}_n + x_s\mathscr{E}_s + x_n\frac{H_c^2}{8\pi}, \quad S = x_nS_n + x_sS_s; \qquad (8)$$

R is the dissipation function; Θ and q are the densities of the energy and heat fluxes. Differentiating the first equation of (8) with respect to the time and recognizing that

$$F_{n,s} = \mathscr{E}_{n,s} - TS_{n,s}, \quad F_n - F_s = \frac{H_c^2}{8\pi}, \quad Q = T(S_n - S_s) = -\frac{TH_c}{4\pi}\frac{\partial H_c}{\partial T} \qquad (9)$$

where F_n and F_s are the free energies, and Q is the heat of the phase transition, we obtain

$$\dot{\mathscr{E}} = \frac{H_c^2}{4\pi}\dot{x}_n - \frac{Q}{T}x_n\dot{T} + T\dot{S}. \qquad (10)$$

From (5) and (6) it follows that

$$\dot{x}_n = -\frac{c}{H_c^2}(\mathbf{H}\operatorname{rot}\mathbf{E}) + x_n\frac{4\pi Q}{TH_c^2}\dot{T}.$$

After substituting the last equation in (10) and eliminating S with the aid of the second equation of (7), we obtain

$$\dot{\mathscr{E}} + \operatorname{div}\left\{\frac{c}{4\pi}[\mathbf{EH}] + \mathbf{q}\right\} = R - \mathbf{jE} + \frac{\mathbf{q}\nabla T}{T},$$

from which we get the energy flux Θ and the dissipation function

$$\Theta = \frac{c}{4\pi}[\mathbf{EH}] + \mathbf{q}, \quad R = \mathbf{j}_\perp\mathbf{E} - \frac{\mathbf{q}\nabla T}{T}; \qquad (11)$$

here $\mathbf{j}_\perp$ is the conduction-current component perpendicular to the magnetic field $\mathbf{H}$. We have used here the condition that $\mathbf{E}$ is perpendicular to $\mathbf{H}$ (see (6)).

Introducing the specific heats C_n and C_s of the normal and the superconducting phases and substituting the obtained value of R, we get from the second equation of (7) the sought heat-conduction equation

$$C\frac{\partial T}{\partial t} + Q\frac{\partial x_n}{\partial t} = -\operatorname{div}\mathbf{q} + \mathbf{j}_\perp\mathbf{E}, \qquad (12)$$

where $C = x_nC_n + x_sC_s$ is the average specific heat of the intermediate state without allowance for the magnetic field.

For formula (11) it follows also for the dissipative function that the current $\mathbf{j}_\perp$ and the heat flux $\mathbf{q}$ are connected with the electric field and the gradient of the temperature by a relation of the form

$$j_{\perp i} = a_{ik}E_k + b_{ik}\frac{\partial T}{\partial x_k}, \quad q_i = c_{ik}E_k + d_{ik}\frac{\partial T}{\partial x_k}, \qquad (13)$$

and the coefficients a_{ik}, b_{ik}, c_{ik}, and d_{ik}, by virtue of the symmetry principle for the kinetic coefficients, satisfy the conditions

$$a_{ik}(\mathbf{H}) = a_{ki}(-\mathbf{H}), \quad d_{ik}(\mathbf{H}) = d_{ki}(-\mathbf{H}),$$
$$Tb_{ik}(\mathbf{H}) = -c_{ki}(-\mathbf{H}). \qquad (14)$$

It should be noted that the conditions (14) would follow directly from the Onsager principle only the case when all three components of $\mathbf{j}_\perp$ are independent. In our case there are two independent components, but the conditions (14) can nevertheless be satisfied by adding to a_{ik} and c_{ik} necessary terms that do not violate equations (13) because $\mathbf{E}$ is perpendicular to $\mathbf{H}$.

We note also that for current components $j_{\parallel}$ along the magnetic field there exists no connection in the form (13). Instead, there is only the condition $\mathbf{E}\cdot\mathbf{H} = 0$. It is therefore necessary to eliminate $j_{\parallel}$ from the last equation of (5). Noting that

$$[\mathbf{H}\,\mathrm{rot}\,\mathbf{H}] = -(\mathbf{H}\nabla)\mathbf{H} + \frac{1}{2}\nabla H^2 = -(\mathbf{H}\nabla)\mathbf{H} + H_c\frac{\partial H_c}{\partial T}\nabla T,$$

we easily obtain an equation containing only $j_{\perp}$:

$$(\mathbf{H}\nabla)\mathbf{H} = \frac{4\pi}{c}[\mathbf{j}_{\perp}\mathbf{H}] - \frac{4\pi Q}{T}\nabla T. \tag{15}$$

The kinetic coefficients for a layered structure, which enter in (13) depend generally speaking on the vector $\mathbf{n}$ normal to the layers. It is therefore necessary to have equations for the determination of $\mathbf{n}$. These equations are obtained from the boundary conditions of the type of (2) and from purely kinematic considerations, and they therefore coincide with the equations obtained in[8] for the case of low temperatures:

$$\mathbf{nH} = 0, \quad V = (c/H_c^2 x_n)\mathbf{n}[\mathbf{EH}],$$
$$\frac{\partial \mathbf{n}}{\partial t} + V(\mathbf{n}\nabla)\mathbf{n} = -\nabla V + \mathbf{n}(\mathbf{n}\nabla V). \tag{16}$$

The foregoing formulas enable us to calculate the rate of motion of the layered structure and the arrangement of the layers.

The arrangement of filaments in filament-like structures can be characterized by specifying at each point a unit vector $\mathbf{v}$ directed along the filaments. It is easily determined if one knows the magnetic field: $\mathbf{v} = \mathbf{H}/H_c$. To determine the rate of motion of the filaments, we note that in a filamentary structure the vector $\mathbf{n}$ normal to the boundaries runs through all possible values in a plane perpendicular to $\mathbf{H}$. Therefore the boundary conditions (2) will be satisfied only if the velocity $\mathbf{V}$ of the filament as a whole satisfies the condition $\mathbf{e} + \mathbf{V}\times\mathbf{h}/c = 0$, from which we get

$$\mathbf{V} = (c/H_c^2 x_n)[\mathbf{EH}]. \tag{17}$$

If the kinetic coefficients a_{ik}, b_{ik}, c_{ik}, and d_{ik} are known, then the complete system of equations describing the dynamic properties of the layered structure consists of the first two equations (5), and equations (6), (15), (12), (13), and (16). For filamentary structures, the kinetic coefficients do not depend on $\mathbf{n}$, and therefore there is no need for equations of the type (16). After solving the system, the velocity of motion of the filamentary structure can be determined with the aid of (17).

2. LAYERED STRUCTURE

We shall carry out the calculation of the kinetic coefficients separately for each of the three types of structures. We shall assume throughout that the free path of the electrons is much smaller than the period of the structure. The properties of the normal and superconducting phases coincide in this case with the properties of a bulky normal or a superconducting metal. In particular, the electric field in the normal layers, and also the heat fluxes in the normal and in the superconducting layers are expressed in terms of the "microscopic" values of the current and the temperature gradients $\nabla\tau$ in the following manner:

$$\mathbf{e} = (1/\sigma)\mathbf{i} + r[\mathbf{hi}],$$
$$\mathbf{q}_n = -\varkappa_n\nabla\tau_n + \lambda[\mathbf{h}\nabla\tau_n], \quad \mathbf{q}_s = -\varkappa_s\nabla\tau_s, \tag{18}$$

where σ and r are the electric conductivity and the Hall coefficient of the normal metal, κ_n and κ_s are the thermal conductivity coefficients of the metal in the normal and in the superconducting states, and λ is the Leduc-Righi coefficient.

We consider first a layered structure. We introduce a coordinate system (ξ, η, ζ) with origin at a certain point in the middle of the normal layer, such that the ζ axis is directed along the normal to the boundaries of the layer and the ξ axis along the direction of the magnetic field $\mathbf{h}$. Since by assumption all the quantities vary little over distances on the order of the period of the structure, we have for the temperatures in the normal (τ_n) and in the adjacent superconducting layers $(\tau_{s\pm})$ the following expansion in ζ:

$$\tau_n = \tau_n^{(0)}(\xi,\eta) + \zeta\tau_n^{(1)}(\xi,\eta),$$
$$\tau_{s\pm} = \tau_{s\pm}^{(0)}(\xi,\eta) + \left(\zeta \mp \frac{a_n + a_s}{2}\right)\tau_{s\pm}^{(1)}(\xi,\eta), \tag{19}$$

where a_n and a_s are the thicknesses of the normal and of the superconducting layers.

From the conditions for the continuity of the temperature and of the normal component of the heat flux at $\zeta = \pm a_n/2$ we obtain

$$\tau_n^{(0)} \pm \tau_n^{(1)} a_n/2 = \tau_{s\pm}^{(0)} \mp \tau_{s\pm}^{(1)} a_s/2,$$
$$-\varkappa_s\tau_{s\pm}^{(1)} + \varkappa_n\tau_n^{(1)} - \lambda H_c\frac{\partial\tau_n^{(0)}}{\partial\eta} = -QV. \tag{20}$$

We have taken here into account the effect of heat release during motion of the separation boundaries.

The gradient of the macroscopic temperature T is equal to, by definition,

$$\frac{\partial T}{\partial\xi,\eta} = \frac{\partial\tau_n^{(0)}}{\partial\xi,\eta}, \quad \frac{\partial T}{\partial\zeta} = \frac{\tau_{s+}^{(0)} - \tau_{s-}^{(0)}}{a_n + a_s}.$$

We note that these formulas lead to the equalities $\tau_{s+}^{(1)} = \tau_{s-}^{(1)} \equiv \tau_s^{(1)}$ and $\partial T/\partial\zeta = x_n\tau_n^{(1)} + x_s\tau_s^{(1)}$, i.e., the gradient of the macroscopic temperature is, as it should be, the gradient of the "microscopic" temperature averaged over the volume. From conditions (20) we obtain, in addition, a connection between the "microscopic" gradients and the macroscopic quantities:

$$\tau_n^{(1)} = \frac{\varkappa_\perp}{\varkappa_n\varkappa_s}\left\{x_s\left(\lambda H_c\frac{\partial T}{\partial\eta} - QV\right) + \varkappa_s\frac{\partial T}{\partial\zeta}\right\},$$
$$\tau_s^{(1)} = \frac{\varkappa_\perp}{\varkappa_n\varkappa_s}\left\{-x_n\left(\lambda H_c\frac{\partial T}{\partial\eta} - QV\right) + \varkappa_n\frac{\partial T}{\partial\zeta}\right\}, \tag{21}$$

where

$$1/\varkappa_\perp = x_n/\varkappa_n + x_s/\varkappa_s.$$

The macroscopic heat flux, which is defined as the average of $q_{n,s}$ over the volume, is obviously $q = x_n q_n + x_s q_s$. Substituting here (18) and taking (21) into account, we obtain

$$q_\xi = -\varkappa_\parallel\frac{\partial T}{\partial\xi}, \quad \varkappa_\parallel = x_n\varkappa_n + x_s\varkappa_s,$$
$$q_\eta = -\varkappa_\parallel\frac{\partial T}{\partial\eta} - x_n\lambda H_c\frac{\varkappa_\perp}{\varkappa_n}\frac{\partial T}{\partial\zeta} - cQ\lambda\frac{x_s\varkappa_\perp}{\varkappa_n\varkappa_s}E_\eta,$$
$$q_\zeta = -\varkappa_\perp\frac{\partial T}{\partial\zeta} + x_n\lambda H_c\frac{\partial T}{\partial\eta}\frac{\varkappa_\perp}{\varkappa_n} - \frac{cQ}{H_c}\left(1 - \frac{\varkappa_\perp}{\varkappa_n}\right)E_\eta.$$

Here and below we neglect the squares of the small quantities r and λ.

The last three equations are equivalent to a single vector equation

$$\mathbf{q} = -\varkappa_{\parallel}\nabla T - (\varkappa_{\perp} - \varkappa_{\parallel})\mathbf{n}(\mathbf{n}\nabla T) + x_n\lambda\frac{\varkappa_{\perp}}{\varkappa_n}[\mathbf{H}\nabla T]$$
$$-\frac{cQ}{H_c^2}\left(1 - \frac{\varkappa_{\perp}}{\varkappa_n}\right)\mathbf{n}([\mathbf{nH}]\mathbf{E}) - cQ\lambda\frac{x_s\varkappa_{\perp}}{\varkappa_n\varkappa_s}\left\{\mathbf{E} - \mathbf{n}(\mathbf{nE}) - \frac{\mathbf{H}}{H_c^2}(\mathbf{HE})\right\}. \quad (22)$$

Comparing (22) with (13), we obtain the kinetic coefficients c_{ik} and d_{ik}:

$$c_{ik} = -\frac{cQ}{H_c^2}\left(1 - \frac{\varkappa_{\perp}}{\varkappa_n}\right) n_i[\mathbf{nH}]_k - cQ\lambda\frac{x_s\varkappa_{\perp}}{\varkappa_n\varkappa_s}\left\{\delta_{ik} - n_i n_k - \frac{H_i H_k}{H_c^2}\right\},$$
$$d_{ik} = -\varkappa_{\parallel}\delta_{ik} - (\varkappa_{\perp} - \varkappa_{\parallel}) n_i n_k - x_n\lambda\frac{\varkappa_{\perp}}{\varkappa_n}\varepsilon_{ikl}H_l. \quad (23)$$

To calculate the macroscopic current $\mathbf{j}_{\perp}$, we note that since the normal regions are always elongated in the direction of the magnetic field, the derivatives of the macroscopic quantities along the field direction coincide with the derivatives of the corresponding "microscopic" quantities. In particular, we have $(\mathbf{H}\nabla)\mathbf{H} = (\mathbf{h}\nabla)\mathbf{h}$, whence

$$\nabla H^2/2 - [\mathbf{H}\,\mathrm{rot}\,\mathbf{H}] = \nabla h^2/2 - [\mathbf{h}\,\mathrm{rot}\,\mathbf{h}]$$

or

$$H_c\frac{\partial H_c}{\partial T}\nabla T - \frac{4\pi}{c}[\mathbf{Hj}] = H_c\frac{\partial H_c}{\partial T}\nabla\tau_n - \frac{4\pi}{c}[\mathbf{hi}].$$

By taking the vector product of the last equation with $\mathbf{H} = \mathbf{h}$, we obtain the connection between the macroscopic and "microscopic" currents:

$$\mathbf{j}_{\perp} = \mathbf{i}_{\perp} + \frac{cQ}{TH_c^2}[\mathbf{H}, \nabla T - \nabla\tau_n]. \quad (24)$$

Substituting here (18) and (21) we get

$$j_{\eta} = \frac{cQ}{TH_c}\left\{\frac{x_s\varkappa_{\perp}}{\varkappa_n\varkappa_s}\lambda H_c\frac{\partial T}{\partial\eta} + \frac{x_s\varkappa_{\perp}}{\varkappa_n\varkappa_s}\frac{cQ}{H_c}\frac{E_{\eta}}{x_n} - \left(1 - \frac{\varkappa_{\perp}}{\varkappa_n}\right)\frac{\partial T}{\partial\zeta}\right\}$$
$$+\frac{\sigma}{x_n}E_{\eta} + r\sigma^2 H_c\frac{E_{\zeta}}{x_n},$$
$$j_{\zeta} = \frac{\sigma}{x_n}E_{\zeta} - \frac{r\sigma^2 H_c}{x_n}E_{\eta},$$

which can be written in the form

$$\mathbf{j}_{\perp} = \frac{\sigma}{x_n}\mathbf{E} - \frac{r\sigma^2}{x_n}[\mathbf{HE}] + \frac{x_s}{x_n}\left(\frac{cQ}{H_c}\right)^2\frac{\varkappa_{\perp}}{T\varkappa_n\varkappa_s}\{\mathbf{E} - \mathbf{n}(\mathbf{nE})\}$$
$$+ x_s\lambda\frac{cQ}{T}\frac{\varkappa_{\perp}}{\varkappa_n\varkappa_s}\left\{\nabla T - \mathbf{n}(\mathbf{n}\nabla T) - \frac{\mathbf{H}}{H_c^2}(\mathbf{H}\nabla T)\right\}$$
$$-\frac{cQ}{TH_c^2}\left(1 - \frac{\varkappa_{\perp}}{\varkappa_n}\right)[\mathbf{nH}](\mathbf{n}\nabla T). \quad (25)$$

The kinetic coefficients a_{ik} and b_{ik} which enter in (13) are therefore

$$a_{ik} = \frac{\sigma}{x_n}(\delta_{ik} + r\sigma\varepsilon_{ikl}H_l) + \frac{x_s}{x_n}\left(\frac{cQ}{H_c}\right)^2\frac{\varkappa_{\perp}}{T\varkappa_n\varkappa_s}(\delta_{ik} - n_i n_k),$$
$$b_{ik} = x_s\lambda\frac{cQ}{T}\frac{\varkappa_{\perp}}{\varkappa_n\varkappa_s}\left\{\delta_{ik} - n_i n_k - \frac{H_i H_k}{H_c^2}\right\} - \frac{cQ}{TH_c^2}\left(1 - \frac{\varkappa_{\perp}}{\varkappa_n}\right)[\mathbf{nH}]_i n_k. \quad (26)$$

By comparing (26) with (23) we can readily verify the validity of the Onsager relations (14).

As already noted, for the current component along the magnetic field there exists no universal connection with the electric field and the temperature gradient. Therefore for the intermediate states there is no electric conductivity tensor in the usual sense of the word. However, there exists a resistivity tensor ρ_{ik}, which enters in the formula for $\mathbf{E}$ and $\mathbf{q}$ in terms of the total current $\mathbf{j}$ and the temperature gradient:

$$E_i = \rho_{ik}j_k + \alpha_{ik}\partial T/\partial x_k,$$
$$q_i = T\alpha_{ki}(-\mathbf{H})j_k - \varkappa_{ik}\partial T/\partial x_k. \quad (27)$$

Here κ_{ik} is the heat-conduction tensor and α_{ik} are the thermoelectric coefficients.

The ratio of the third term in (25) to the first is of the order of

$$\frac{c^2Q^2}{TH_c^2\varkappa\sigma} \sim \frac{c^2H_c^2}{T_c\varkappa\sigma} \sim \varkappa_{\mathrm{GL}}^2\left(\frac{\xi_0}{l}\right)^2, \quad (28)$$

where κ_{GL} is the parameter of the Ginzburg-Landau theory (for a pure superconductor), ξ_0 is the coherence length, and l is the mean free path of the electrons. In pure metals with $l \gg \xi_0$ this ratio is small, and the terms proportional to Q^2 can be neglected. In this case we obtain from (25) and (22)

$$\rho_{ik} = \frac{x_n}{\sigma}\left\{\delta_{ik} - \frac{H_i H_k}{H_c^2}\right\} - x_n r\varepsilon_{ikl}H_l,$$
$$\varkappa_{ik} = \varkappa_{\parallel}\delta_{ik} + (\varkappa_{\perp} - \varkappa_{\parallel})n_i n_k + x_n\lambda\frac{\varkappa_{\perp}}{\varkappa_n}\varepsilon_{ikl}H_l, \quad (29)$$
$$\alpha_{ik} = \frac{x_n}{\sigma}\frac{cQ}{TH_c^2}\left(1 - \frac{\varkappa_{\perp}}{\varkappa_n}\right)[\mathbf{nH}]_i n_k.$$

We have neglected here also small terms proportional to the products rQ and λQ.

In superconducting alloys of the first kind with a mean free path on the order of ξ_0, the ratio (28) is of the order of unity. On the other hand, the coefficients r and λ are exceedingly small and can be neglected. We then obtain

$$\rho_{ik} = x_n\left\{\sigma + x_s\left(\frac{cQ}{H_c}\right)^2\frac{\varkappa_{\perp}}{T\varkappa_n\varkappa_s}\right\}^{-1}\left\{\delta_{ik} - \frac{H_i H_k}{H_c^2} + x_s\left(\frac{cQ}{H_c}\right)^2\frac{\varkappa_{\perp}}{T\sigma\varkappa_n\varkappa_s}n_i n_k\right\},$$
$$\varkappa_{ik} = \varkappa_{\parallel}\delta_{ik} + \left\{\varkappa_{\perp} - \varkappa_{\parallel} + x_n\left(1 - \frac{\varkappa_{\perp}}{\varkappa_n}\right)^2 c^2Q^2\left(\sigma TH_c^2 + x_s c^2Q^2\frac{\varkappa_{\perp}}{\varkappa_n\varkappa_s}\right)^{-1}\right\}n_i n_k,$$
$$\alpha_{ik} = x_n cQ\left(1 - \frac{\varkappa_{\perp}}{\varkappa_n}\right)\left(\sigma TH_c^2 + x_s c^2Q^2\frac{\varkappa_{\perp}}{\varkappa_n\varkappa_s}\right)^{-1}[\mathbf{nH}]_i n_k. \quad (30)$$

Let us compare the order of magnitude of the tensor α_{ik} with the coefficient of the thermal emf α_N of a normal metal, the order of magnitude of which, as is known, is $(1/e)(T/\epsilon_F)$ (e is the electron charge). From (29) and (30) we see that

$$\frac{\alpha}{\alpha_N} \sim \frac{ecH_c}{T\sigma}\frac{\varepsilon_F}{T} \sim \varkappa_{\mathrm{GL}}\frac{\xi_0}{l}\frac{\varepsilon_F}{T},$$

i.e., at not too large values of the mean free path, the thermoelectric coefficient of the intermediate state greatly exceeds the value characteristic of normal metals.

3. STRUCTURE WITH NORMAL FILAMENTS

We consider one linear normal filament moving with velocity $\mathbf{V}$ and surrounded by a superconducting phase. Neglecting the small r and λ, let us consider the distribution of the "microscopic" temperature τ in a plane perpendicular to the filament axis. Since the velocity $\mathbf{V}$ is small, we can assume that the temperature τ satisfies the Laplace equation $\Delta\tau = 0$. We choose the origin at the center of the filament and seek the temperature distribution in the form

$$\tau_s = -\mathbf{g}_s\mathbf{r} + \frac{2\mathbf{dr}}{r^2} \quad (r > a),$$
$$\tau_n = -\mathbf{g}_n\mathbf{r} \quad (r < a),$$

where $\mathbf{g}_n$, $\mathbf{g}_s$, and d are constant vectors, and a is the radius of the filament. On the phase separation boundary, i.e., at r = a, it is necessary to satisfy the usual boundary conditions

$$\tau_n = \tau_s,$$
$$-\varkappa_s(\nabla\tau_s\mathbf{n}) + \varkappa_n(\nabla\tau_n\mathbf{n}) = -Q(\mathbf{Vn}),$$

where $\mathbf{n} = \mathbf{r}/r$. These conditions make it possible to express d and $\mathbf{g}_n$ in terms of $\mathbf{g}_s$ and $\mathbf{V}$:

$$\mathbf{d} = \frac{a^2}{2}\left\{\frac{\varkappa_n - \varkappa_s}{\varkappa_n + \varkappa_s}\mathbf{g}_s - \frac{Q}{\varkappa_n + \varkappa_s}\mathbf{V}\right\},$$
$$\mathbf{g}_n = \frac{2\varkappa_s}{\varkappa_n + \varkappa_s}\mathbf{g}_s + \frac{Q}{\varkappa_n + \varkappa_s}\mathbf{V}. \tag{31}$$

We note that if τ is taken to mean an electric potential, then g plays the role of the electric field, and the normal filament is equivalent to a two-dimensional dipole with moment d, placed in the field $\mathbf{g}_s$. From (31) we see that the filament has a "spontaneous" moment proportional to the velocity and to the heat of the phase transition, as well as the moment "induced" by the field $\mathbf{g}_s$, proportional to the difference between the thermal conductivities of the normal and the superconducting phases.

A system of normal filaments behaves in analogy with a system of flat dipoles. We shall need in what follows to know the gradient of the macroscopic temperature T in a plane perpendicular to the filament axis, $\nabla_\perp T$. The quantity $\mathbf{G} = -\nabla_\perp T$ plays the role of the average electric field in a system of dipoles. Generally speaking, the connection between the macroscopic field G and the field $\mathbf{g}_s$ acting on the dipole depends on the arrangement of the filaments[11]. In the case when the filaments are randomly arranged (or when they form a square lattice), there is the well known Lorentz formula, which in the planar case can be written in the form

$$\mathbf{G} = \mathbf{g}_s - 2\pi\mathbf{P}, \tag{32}$$

where P is the moment per unit area, given by $\mathbf{P} = N\mathbf{d} = (x_n/\pi a^2)\mathbf{d}$ (N is the number of filaments per unit area). Formulas (32) and (31) make it possible to express $\mathbf{g}_n$ and $\mathbf{g}_s$ in terms of G and V:

$$\mathbf{g}_s = \mathbf{G}\left(1 + x_n\frac{\varkappa_n - \varkappa_s}{\varkappa_n + \varkappa_s}\right) - x_n\frac{Q}{\varkappa_n + \varkappa_s}\mathbf{V},$$
$$\mathbf{g}_n = \mathbf{G}\frac{2\varkappa_s}{\varkappa_n + \varkappa_s}\left(1 + x_n\frac{\varkappa_n - \varkappa_s}{\varkappa_n + \varkappa_s}\right) + \frac{Q}{\varkappa_n + \varkappa_s}\left(1 - x_n\frac{2\varkappa_s}{\varkappa_n + \varkappa_s}\right)\mathbf{V}. \tag{33}$$

We have taken into account here the condition $x_n \ll 1$, and only satisfaction of this condition realizes the structure in question. We note that formulas (33) can be obtained also from the condition $\mathbf{G} = x_n\mathbf{g}_n + x_s\mathbf{g}_s$.

Substituting $\nabla_\perp\tau_n = -\mathbf{g}_n$ and $\nabla_\perp T = -\mathbf{G}$ in (24), we obtain the sought connection of the current with the electric field and with the temperature gradient:

$$\mathbf{j}_\perp = \frac{\sigma}{x_n}\mathbf{E}\left\{1 + \frac{c^2Q^2}{\sigma T H_c^2(\varkappa_n + \varkappa_s)}\left(1 - x_n\frac{2\varkappa_s}{\varkappa_n + \varkappa_s}\right)\right\} + \frac{cQ}{TH_c^2}\left\{\frac{\varkappa_n - \varkappa_s}{\varkappa_n + \varkappa_s} - x_n\frac{2\varkappa_s(\varkappa_n - \varkappa_s)}{(\varkappa_n + \varkappa_s)^2}\right\}[\mathbf{H}\nabla T]. \tag{34}$$

The macroscopic heat flow in a plane perpendicular to H is

$$\mathbf{q}_\perp = x_s\mathbf{q}_s + x_n\mathbf{q}_n = x_s\varkappa_s\mathbf{g}_s + x_n\varkappa_n\mathbf{g}_n. \tag{35}$$

Substituting (33), we obtain

$$\mathbf{q}_\perp = -\varkappa_s\left(1 + 2x_n\frac{\varkappa_n - \varkappa_s}{\varkappa_n + \varkappa_s}\right)\nabla_\perp T + \frac{cQ}{H_c^2}\frac{\varkappa_n - \varkappa_s}{\varkappa_n + \varkappa_s}[\mathbf{EH}]\left(1 - x_n\frac{2\varkappa_s}{\varkappa_n + \varkappa_s}\right) \tag{36}$$

On the other hand, the flux along the field H, as in the preceding case, is equal to $\mathbf{q}_\parallel = -\kappa_\parallel\nabla_\parallel T$.

The obtained formulas suffice to express E and q in terms of j and T. We have

$$\mathbf{E} = \frac{x_n T H_c^2(\varkappa_n + \varkappa_s)}{\sigma T H_c^2(\varkappa_n + \varkappa_s) + c^2Q^2}\left\{\mathbf{j} - \frac{\mathbf{H}}{H_c^2}(\mathbf{jH})\right\} - \frac{x_n cQ(\varkappa_n - \varkappa_s)}{\sigma T H_c^2(\varkappa_n + \varkappa_s) + c^2Q^2}[\mathbf{H}\nabla T],$$
$$\mathbf{q} = -\varkappa_\parallel\frac{\mathbf{H}}{H_c^2}(\mathbf{H}\nabla T) - \left\{\varkappa_s + x_n\frac{\varkappa_n - \varkappa_s}{\varkappa_n + \varkappa_s}\left[2\varkappa_s + \frac{c^2Q^2(\varkappa_n - \varkappa_s)}{(\varkappa_n + \varkappa_s)\sigma T H_c^2 + c^2Q^2}\right]\right\}\left\{\nabla T - \frac{\mathbf{H}}{H_c^2}(\mathbf{H}\nabla T)\right\} - \frac{x_n cQ(\varkappa_n - \varkappa_s)T}{\sigma T H_c^2(\varkappa_n + \varkappa_s) + c^2Q^2}[\mathbf{Hj}].$$

The last formulas determine the tensors ρ_{ik}, κ_{ik}, and α_{ik} in the case of an arbitrary ratio of the mean free path l to the parameter ξ_0. In pure metals, the formulas simplify greatly:

$$\rho_{ik} = \frac{x_n}{\sigma}\left(\delta_{ik} - \frac{H_iH_k}{H_c^2}\right),$$
$$\varkappa_{ik} = \varkappa_\parallel\frac{H_iH_k}{H_c^2} + \varkappa_s\left(1 + 2x_n\frac{\varkappa_n - \varkappa_s}{\varkappa_n + \varkappa_s}\right)\left(\delta_{ik} - \frac{H_iH_k}{H_c^2}\right), \tag{37}$$
$$\alpha_{ik} = \frac{x_n cQ(\varkappa_n - \varkappa_s)}{\sigma T H_c^2(\varkappa_n + \varkappa_s)}\varepsilon_{ikl}H_l.$$

4. STRUCTURE WITH SUPERCONDUCTING FILAMENTS

The problem of the temperature distribution is perfectly analogous in this case to that considered in the preceding section. For $\mathbf{g}_n = -\nabla_\perp\tau_n$ and $\mathbf{g}_s = -\nabla_\perp\tau_s$, we obtain the following expressions:

$$\mathbf{g}_n = \mathbf{G}\left(1 - x_s\frac{\varkappa_n - \varkappa_s}{\varkappa_n + \varkappa_s}\right) + x_s\frac{Q}{\varkappa_n + \varkappa_s}\mathbf{V},$$
$$\mathbf{g}_s = \mathbf{G}\frac{2\varkappa_n}{\varkappa_n + \varkappa_s}\left(1 - x_s\frac{\varkappa_n - \varkappa_s}{\varkappa_n + \varkappa_s}\right) - \frac{Q}{\varkappa_n + \varkappa_s}\left(1 - x_s\frac{2\varkappa_n}{\varkappa_n + \varkappa_s}\right)\mathbf{V}. \tag{38}$$

Substituting (38) in (24) and (35), we obtain the current and heat-flow components perpendicular to the field H:

$$\mathbf{j}_\perp = \sigma\mathbf{E}\left\{1 + x_s\left[1 + \frac{c^2Q^2}{\sigma T H_c^2(\varkappa_n + \varkappa_s)}\right]\right\} + \frac{cQ}{TH_c^2}\frac{\varkappa_n - \varkappa_s}{\varkappa_n + \varkappa_s}x_s[\mathbf{H}\nabla T],$$
$$\mathbf{q}_\perp = -\varkappa_n\left(1 - 2x_s\frac{\varkappa_n - \varkappa_s}{\varkappa_n + \varkappa_s}\right)\nabla_\perp T + x_s\frac{cQ}{H_c^2}\frac{\varkappa_n - \varkappa_s}{\varkappa_n + \varkappa_s}[\mathbf{EH}]. \tag{39}$$

The heat flow along the magnetic field is, as before, equal to $-\kappa_\parallel\nabla_\parallel T$. Solving the foregoing equations with respect to E and q, we obtain the following formulas for the resistance, thermal conductivity, and thermoelectric coefficients in the case of a structure of superconducting filaments

$$\rho_{ik} = \frac{1}{\sigma}\left\{1 - x_s\left[1 + \frac{c^2Q^2}{\sigma T H_c^2(\varkappa_n + \varkappa_s)}\right]\right\}\left(\delta_{ik} - \frac{H_iH_k}{H_c^2}\right),$$
$$\varkappa_{ik} = \varkappa_\parallel\frac{H_iH_k}{H_c^2} + \varkappa_n\left(1 - 2x_s\frac{\varkappa_n - \varkappa_s}{\varkappa_n + \varkappa_s}\right)\left(\delta_{ik} - \frac{H_iH_k}{H_c^2}\right),$$
$$\alpha_{ik} = x_s\frac{cQ}{\sigma T H_c^2}\frac{\varkappa_n - \varkappa_s}{\varkappa_n + \varkappa_s}\varepsilon_{ikl}H_l. \tag{40}$$

We recall that the condition for the realization of the structure of superconducting filaments is the smallness of the concentration of the superconducting phase.

We note that in all three types of structures the resistivity tensor ρ_{ik} is such that the electric resistance of the intermediate state to current flowing along the magnetic field is zero. It can be shown that superconductivity of the intermediate state takes place in this case. On the other hand, if current flows in the layered structure along the layers but transversely to the magnetic field then the resultant motion of the layers gives rise to electric resistance.

5. THE ETTINGSHAUSEN EFFECT

Solomon and Otter[9,10] carried out an experimental investigation of the Ettingshausen effect, wherein an electric field causes heat to flow perpendicular to the direction of the electric and magnetic fields. Let us therefore consider this effect in greater detail.

If we neglect the Leduc-Righi effect, then, as seen from (22), in a layered structure the heat flux, which is proportional to the electric field, is equal to

$$\mathbf{q}_E = -\frac{cQ}{H_c^2}\left(1-\frac{\varkappa_\perp}{\varkappa_n}\right)\mathbf{n}([\mathbf{n}\mathbf{H}]\mathbf{E}). \tag{41}$$

Introducing the velocity of the structure $\mathbf{V} = V\mathbf{n}$, we can rewrite the last formula in the form

$$\mathbf{q}_E = x_n Q\mathbf{V}\left(1-\frac{\varkappa_\perp}{\varkappa_n}\right) = x_n Q\mathbf{V}\frac{x_s(\varkappa_n-\varkappa_s)}{x_s\varkappa_n + x_n\varkappa_s}. \tag{42}$$

At low temperatures we can neglect the thermal conductivity of the superconducting phase compared with the thermal conductivity of the normal phase. The heat flow q_E is then equal to $x_n QV$, corresponding to the transport of excess entropy of the normal regions with a velocity $\mathbf{V}$. Such a simple model was used by Solomon and Otter, but they have assumed that it is valid at all temperatures. In contrast to the results of such a model, the complete formula (41) predicts, in agreement with experiment, the vanishing of the effect as $T \to T_c$. It should be noted that formula (42) was obtained for a "good" layered structure, in which the vector $\mathbf{n}$ is constant everywhere. On the other hand, in the geometry employed in[9,10] (a plate in a perpendicular magnetic field), a "random" layered structure was usually realized, in which the vector $\mathbf{n}$ assumed all possible values. The Ettingshausen effect in a "random" structure can be described by noting that in the presence of an electric field such a structure drifts as a whole with a velocity $\mathbf{V} = (c/H_c^2 x_n)\mathbf{E}\times\mathbf{H}$ (see[8]). Substituting this expression in (42), we obtain

$$\frac{q_E}{E} = \frac{cQ}{H_c}\frac{\varkappa_n-\varkappa_s}{\varkappa_n+(x_n/x_s)\varkappa_s}. \tag{43}$$

At low concentrations of one of the phases, a filamentary structure is realized. For a structure with normal filaments we obtain for the Ettingshausen effect from (36) the expression

$$\frac{q_E}{E} = \frac{cQ}{H_c}\frac{\varkappa_n-\varkappa_s}{\varkappa_n+\varkappa_s}\left(1-2x_n\frac{\varkappa_s}{\varkappa_n+\varkappa_s}\right). \tag{44}$$

The analogous expression for the structure with the superconducting filaments is

$$\frac{q_E}{E} = x_s\frac{cQ}{H_c}\frac{\varkappa_n-\varkappa_s}{\varkappa_n+\varkappa_s}. \tag{45}$$

Figure 1 shows the results of a comparison of the dependence of the effect on the concentration of the normal phase $x_n = H/H_c$ with the experimental data of Solomon and Otter[9,10] for the highest ($T/T_c = 0.82$) and for the lowest ($T/T_c = 0.46$) temperatures used in[9,10]. Curves 1, 2, and 3 are constructed respectively in accordance with formulas (43), (44), and (45). The agreement is perfectly satisfactory. The sharp decrease of the effect at $x_n \approx 0.5$, which becomes particularly pronounced at $T/T_c = 0.46$, is apparently connected with the alignment of the layers transversely to the magnetic field, which was observed in[9,10] at these concentrations. It is seen from (41) that such an alignment should indeed lead to a decrease of the effect.

Figure 2 shows the dependence of the effect of the temperature as $x_n \to 0$. The solid curve corresponds to formula (43) for the layered structure, and the dashed one to formula (44) for the structure with the normal filaments. The discrepancy between theory and experiment can be attributed entirely to the uncertainty of the intermediate-state structure in the experiments of Solomon and Otter[9,10]. It would be quite desirable to carry out experiments for a plate in an inclined field, where a good layered structure is observed[6].

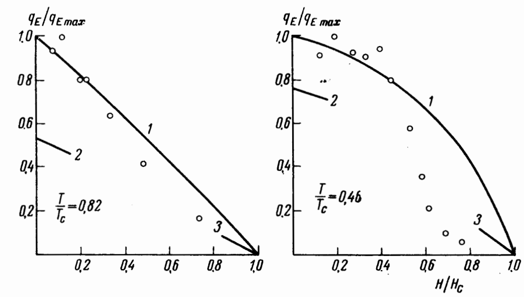

FIG. 1

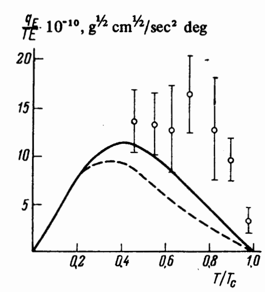

FIG. 2

6. VELOCITY OF MOTION UNDER THE INFLUENCE OF A CURRENT AND A TEMPERATURE GRADIENT

Let us find the velocity of a layered structure of the intermediate state, arising in a plane-parallel plate placed in an inclined magnetic field $\vec{\mathscr{H}}$, under the influence of a current and a temperature gradient. If we choose the z axis normal to the plate and the y axis along the projection of $\vec{\mathscr{H}}$ on the plane of the plate, then in the absence of a current and a temperature gradient we have

$$H_x=0,\ H_y=\mathscr{H}_y,\ H_z=H_c\sqrt{1-\mathscr{H}_y^2/H_c^2},$$
$$\mathbf{E}=0,\ x_n=(\mathscr{H}_z/H_c)\,(1-\mathscr{H}_y^2/H_c^2)^{-1/2},$$
$$n_x=1,\quad n_y=n_z=0. \tag{46}$$

If a weak current flows in the plane of the plate, the magnetic field of which can be neglected, and if $\nabla T = 0$, then the electric field is equal to $\rho_{ik}j_k$. Substituting here ρ_{ik} from (29) and the electric field obtained in this manner into the expression for the velocity (16), we get

$$V=\left(1+\frac{c^2Q^2}{\sigma TH_c^2}\frac{\varkappa_\perp x_s}{\varkappa_n\varkappa_s}\right)^{-1}\left\{\frac{cj_y}{\sigma H_c}\sqrt{1-\frac{\mathscr{H}_y^2}{H_c^2}}+cr j_x\right\}.$$

At $Q = 0$ the last formula goes over, as it should, to the corresponding formulas of[8].

If there is a temperature gradient in the plane of the plate, but there is no current, then the electric field is determined by the relation $E_i = \alpha_{ik}\partial T/\partial x_k$. Substituting here the expression obtained above for the tensor α_{ik}, with the aid of the second formula of (16), we obtain

$$V=-\frac{c^2Q\varkappa_\perp x_s}{\sigma TH_c^2\varkappa_n\varkappa_s+c^2Q^2\varkappa_\perp x_s}\left\{(\varkappa_n-\varkappa_s)\frac{\partial T}{\partial x}+\lambda H_c\sqrt{1-\frac{\mathscr{H}_y^2}{H_c^2}}\frac{\partial T}{\partial y}\right\}. \tag{47}$$

The motion of the structure under the influence of a temperature gradient directed along n is thus due to the difference between the heat-conduction coefficients of the normal and superconducting phases. On the other hand, if the temperature gradient is perpendicular to n, then the motion is due to the Leduc-Righi effect in the normal phase. Let us estimate the order of magnitude of the velocity of motion under the influence of the temperature gradient. From (47) it follows that $V \sim (c^2/4\pi\sigma)(\nabla T/T)$. When $\sigma \sim 10^{20}\ \text{sec}^{-1}$ and $\nabla T/T \sim \text{cm}^{-1}$, this yields $V \sim 1$ cm/sec.

In conclusion, we are grateful to P. L. Kapitza, who called our attention to the possibility of the motion of the structure under the influence of the temperature gradient, and to Yu. V. Sharvin for a useful discussion.

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