SELF-CONSISTENT REGIME OF HEATING OF MATTER BY A LASER PULSE UNDER CONDITIONS OF NONEQUILIBRIUM IONIZATION

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We investigate the hydrodynamic heating regime arising when a solid target is acted upon by a "giant" laser pulse with a flux density $q_0 \gtrsim 10^{12}$ W/cm^2 under conditions when nonequilibrium ionization takes place. Analytic expressions are obtained for the hydrodynamic parameters, electron temperature, and ionization multiplicity of the produced plasma as functions of time, radiation flux density q_0, and the properties of the target material.

AS is well known, when powerful laser-radiation fluxes interact with surfaces of condensed bodies, a rapidly expanding high-temperature plasma is produced, with an ionization multiplicity (for heavy atoms) that can reach 15–25.[1] The hydrodynamics of such a plasma, assuming thermodynamic equilibrium, was considered in a number of papers.[2-4] However, as was shown in [5,6], at high radiation flux densities ($q \gtrsim 10^{12}$–10^{13} W/cm^2) the ionization equilibrium in the plasma is strongly violated because of the rapid acquisition of energy by the electrons and the "inertia" of the ionization process. Therefore, at such radiation fluxes, the problem of heating, expansion, and ionization multiplicity of a plasma must be investigated on the basis of the hydrodynamics equations, the kinetic equation for the electron distribution function, and the equation for the ionization rate; this makes it possible to determine the nonequilibrium state of the plasma. The electron distribution function in a strong radiation field was calculated in [5], where it was shown that it is isotropic and has a quasi-maxwellian form in a coordinate system in which the medium is at rest. This circumstance greatly facilitates the calculation of the rates of the various elementary processes (ionization, recombination, etc.), and consequently makes it possible to determine the nonequilibrium plasma parameters.

In the present paper we consider the one-dimensional plane problem of formation of a multiply ionized plasma by a powerful laser radiation incident on the surface of a solid target consisting of heavy elements and filling the half-space $x \le 0$.

We also investigate the case of sharp focusing of the radiation, the equilibrium hydrodynamics of which was considered by Nemchinov.[4]

2. The system of hydrodynamic equations with allowance for absorption of laser radiation in the medium and the energy lost to ionization of the atoms and of the plasma ions by electron impact is

$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x}(\rho v)=0,\quad \frac{\partial}{\partial t}(\rho v)+\frac{\partial}{\partial x}(p+\rho v^2)=0,$$
$$\frac{\partial}{\partial t}\left(\rho\varepsilon+\frac{\rho v^2}{2}\right)+\frac{\partial}{\partial x}\left[\rho v\left(\varepsilon+\frac{v^2}{2}+\frac{p}{\rho}\right)\right]=\frac{\partial q}{\partial x}-Q_i, \tag{1}$$

where $\rho = N_0 M_i$ is the density of the medium, $p = N_0(zkT_e + kT_i)$ the pressure, ϵ the specific thermal energy, v the velocity, N_0 the number of ions per unit volume, and z the ionization multiplicity (just as in [5], z is regarded as a continuous function), M_i the ion mass, $q = q(x, t)$ the flux density of the laser radiation, Q_i the energy lost per unit time and per unit volume to ionization, and T_e and T_i the temperatures of the electrons and ions, respectively. The quantity $q(x, t)$ represents the radiation flux density at the point x:

$$q(x,t)=q_0\exp\left\{-\int_x^{x_{fr}}K\,dx\right\};$$

$q_0 = q(\infty) = \text{const}$, $x_{fr}(t)$ is the coordinate of the boundary of the plasma layer with the vacuum, $K(\rho, T_e)$ is the radiation-absorption coefficient, which for a plasma with ionization multiplicity z is given by

$$K(\rho,T_e)=\frac{C\rho^2}{\eta^{3/2}},\quad \eta=\frac{kT_e}{I(z)},\quad C=\frac{4\sqrt{2\pi}e^6\Lambda z^3}{3\sqrt{3}\,m^{3/2}c\nu^2M_i^2I^{3/2}(z)}. \tag{2}$$

Here $I(z)$ is the ionization potential of an ion of multiplicity z, for which we use below the approximate expression $I(z) = I_0 z^2$, where I_0 is a slowly varying function of z, which depends on the type of ion (for example, for a hydrogen-like ion $I_0 = I_H = 13.6$ eV), ν is the radiation frequency, and Λ is the Coulomb logarithm. In the solution of the system (1) we shall neglect the weak dependence of I_0 on z; therefore C can be regarded as constant and can easily be reduced to the form

$$C\approx10^{10}\left(\frac{10^{14}}{\nu}\right)^2\left(\frac{1}{A}\right)^2\left(\frac{I_H}{I_0}\right)^{3/2}\Lambda,$$

where A is the atomic weight of the target material.

We write down further the equation of the ionization kinetics, with allowance for the hydrodynamic motion of the plasma:

$$\frac{\partial z}{\partial t}+v\frac{\partial z}{\partial x}=z\nu_i(z,\eta,\rho). \tag{3}$$

Here $\nu_i(z, \eta, \rho) = \nu_0\Phi(\eta)/z^3\eta^{3/2}$ is the probability of ionization of an ion of multiplicity z,

$$\nu_0=B\rho\left(\frac{I_H}{I_0}\right)^{3/2}\frac{1}{M_i}2^{3/2}\cdot10^{-8}\ \text{sec}^{-1},\qquad \Phi(\eta)=\frac{\eta^2e^{-1/\eta}}{1+\chi\eta},$$

B and χ are slowly-varying dimensionless functions of z, which depend on the type of ion, and whose values are given in [7] for a large range of z and of the properties of the corresponding ions (characteristic values are $B \approx 10$ and $\chi \approx 1$, and when z ranges from $z \approx 2-3$ to $z \approx 50-60$, the values of B and χ change by not more than the factor 1.5–2). In the right-hand side of (3) it is necessary, in principle, to include terms describing the recombination processes. It is easily seen, however (see formula (5) of [7]), that they are small under the conditions in question, since in our case the parameter $\eta \gtrsim 1$ (in the case of ionization equilibrium we have $\eta \approx 1/5 - 1/10$).

Using the formulas (2) and (3) for the absorption coefficient and the ionization frequency, respectively, we can represent Q_i in the form

$$Q_i = q_1 K\Phi, \tag{4}$$

$$q = \frac{3\sqrt{3}\, 10^{-8} m^{3/2} c I_H^{5/2} B \nu^2}{2\sqrt{\pi} e^6 \Lambda} \left(\frac{I_0}{I_H}\right)^2 = 2{,}8 \cdot 10^{12} B \left(\frac{I_0}{I_H}\right)^2 \left(\frac{\nu}{10^{14}}\right)^2 \text{W/cm}^2 \tag{5}$$

The right-hand side in Eq. (1) for the system energy can now be rewritten, with the aid of the expression for the radiation flux density q(x, t) and (4), in the form

$$\frac{\partial q}{\partial x} - Q_i = \frac{\partial q^*}{\partial x}, \tag{6}$$

$$q^* = q_0 \exp\left\{-\int_x^{x_{fr}} K\,dx\right\} - q_1 \int_0^x K\Phi\,dx. \tag{7}$$

The last term in (7) is equal to the energy lost to ionization per unit time in a plasma layer from zero to x. It is necessary to add to the system (1) and (3) the relation between the specific thermal energy and the pressure, which at sufficiently large z is given by the expression

$$\varepsilon = \frac{p}{\rho(\varkappa - 1)}, \quad p \approx NzkT_e, \quad zT_e \gg T_i, \quad \varkappa = \frac{5}{3}. \tag{8}$$

Recognizing that $z^3 = pM_i/\rho\eta I_0$, and multiplying both sides of (3) by $\rho I_0/M_i$, we obtain

$$\rho\left[\frac{\partial}{\partial t}\left(\frac{p}{\rho\eta}\right) + v\frac{\partial}{\partial x}\left(\frac{p}{\rho\eta}\right)\right] = 3K\Phi q_1. \tag{9}$$

Thus, Eqs. (1) and (9), with account taken of (6) and (7), form a system of equations for the functions $\rho(x, t)$, $v(x, t)$, $p(x, t)$, $\epsilon(x, t)$, and $\eta(x, t)$, which depend on the parameters q_0, q_1, and C.

As a result, as follows from general dimensionality considerations, the problem in question is self-similar and has defining dimensional parameters q and C and a dimension-less parameter $\Theta = q_1/q_0$. The hydrodynamic regime corresponding to the self-similar solution of the obtained system of equations is similar to the self-consistent regime of evaporation and heating of matter without allowance for ionization, which was investigated in [2, 3]. A characteristic feature of the self-consistent regime is that the optical thickness of the plasma layer is independent of the time, i.e.,

$$L = \int_0^{x_{fr}} K\,dx = \text{const}.$$

This circumstance is in essence the physical reason for the existence of the self-consistent regime. In our case we have conservation of not only the optical thickness L, but also of the quantity

$$q_1 \int_0^{x_{fr}} K\Phi\,dx,$$

which is equal to the energy lost to ionization in the entire plasma layer per unit time. It must be emphasized, however, that the self-similar hydrodynamic regime is a limiting one, to which the true solution of the problem tends asymptotically.

3. We reduce the system (1), (7), and (9) to a self-similar form. It follows from dimensionality considerations that the only dimensionless combination of the coordinate x, the time t, and the defining parameters C and q_0 is in this case the variable

$$\lambda = C^{-1/5} q_0^{-2/5} x t^{-6/5}. \tag{10}$$

Since the coordinate of the boundary between the plasma and the vacuum $x_{fr}(t)$ depends only on the time and on the parameters C and q_0, it can be uniquely represented in the form

$$x_{fr}(t) = \lambda_0 t^{6/5} C^{1/5} q_0^{2/5}, \tag{11}$$

where λ_0 is the value of the self-similar variable λ corresponding to the boundary in question.

The functions v, ρ, p, and η can be represented as follows:

$$\begin{gathered} v = C^{1/5} q_0^{2/5} t^{1/5} V(\lambda), \quad \rho = C^{-3/5} q_0^{-3/5} t^{-3/5} R(\lambda), \\ p = C^{-1/5} q_0^{3/5} t^{-1/5} P(\lambda), \quad \eta = \eta(\lambda), \quad \lambda \in [0, \lambda_0], \end{gathered} \tag{12}$$

where $V(\lambda)$, $R(\lambda)$, $P(\lambda)$, and $\eta(\lambda)$ are dimensionless functions of λ and of the parameter Θ, and characterize at each instant of time (at a fixed value of Θ) the spatial distribution of the corresponding quantities. Substituting (12) in the system (1), (7), and (9), we obtain equations for the self-similar functions:

$$\begin{gathered} \frac{d}{d\lambda}[RV] - \frac{6}{5}\lambda\frac{dR}{d\lambda} - \frac{3}{5}R = 0, \\ \frac{1}{R}\frac{dP}{d\lambda} + \left(V - \frac{6}{5}\lambda\right)\frac{dV}{d\lambda} + \frac{1}{5}V = 0, \\ \frac{3}{2}\left[V - \frac{6}{5}\lambda\right]\frac{d}{d\lambda}\left(\frac{P}{R}\right) + \frac{P}{R}\frac{dV}{d\lambda} + \frac{3}{5}\frac{P}{R} \\ - \frac{R}{\eta^{3/2}}\exp\left\{-\int_\lambda^{\lambda_0}\frac{R^2}{\eta^{3/2}}d\lambda - \Theta\Phi(\eta)\right\} = 0 \\ \times\left[V - \frac{6}{5}\lambda\right]\frac{d}{d\lambda}\left(\frac{P}{R\eta}\right) + \frac{2}{5}\frac{P}{\rho\eta} = \frac{3\Phi(\eta)}{\eta^{3/2}}R\Theta. \end{gathered} \tag{13}$$

The boundary conditions for the system (13) on the vacuum side are

$$P(\lambda_0) = R(\lambda_0) = 0, \; V(\lambda_0) = 6/5\,\lambda_0. \tag{14}$$

The solution of the system (13) together with (14) will obviously contain the unknown parameter λ_0. To determine this parameter it is necessary to use the law of energy conservation on the boundary between the plasma and the surface of the condensed body, namely that the radiation flux q(0) be equal to the gasdynamic flux from the surface, with allowance for the energy lost to ionization. In self-similar form, this condition is

$$\exp\left\{-\int_0^{\lambda_0}\frac{R^2\,d\lambda}{\eta^{3/2}}\right\} = R(0)V(0)\left[\frac{1}{2}V^2(0) + \frac{5}{2}\frac{P(0)}{R(0)} + \frac{P(0)}{R(0)\eta(0)}\right] \tag{15}$$

The system (13) together with(14) and (15) can be solved only numerically, and to find the numerical solution it is necessary to investigate the obtained system of equations at the singular point $\lambda = \lambda_0$. An analysis similar to that given in [2] indicates that as $\lambda \to \lambda_0$ a physically meaningful solution can be represented in the form

$$V(\lambda) = 0{,}3\lambda_0\left(1-\frac{\lambda}{\lambda_0}\right)+\frac{6}{5}\lambda,$$

$$R(\lambda) = A_0\left(1-\frac{\lambda}{\lambda_0}\right), \quad P(\lambda) = 0{,}12\lambda_0^2 A_0\left(1-\frac{\lambda}{\lambda_0}\right)^2. \tag{16}$$

The values of the coefficient A_0 and the function $\eta(\lambda)$ as $\lambda \to \lambda_0$ are determined by the equations

$$\frac{1-\Theta\Phi[\eta(\lambda_0)]}{\Theta\eta(\lambda_0)\Phi[\eta(\lambda_0)]} = 31.5, \quad A_0\Theta\frac{\Phi[\eta(\lambda_0)]}{\eta(\lambda_0)^{1/2}} = 0{,}004\lambda_0^2. \tag{17}$$

4. The asymptotic solution (16) and (17) enables us to get away from the singular point $\lambda = \lambda_0$ in numerical integration. We note that this solution satisfies the physically-natural condition $K \to 0$ and $\epsilon \to 0$ as $\lambda \to \lambda_0$, and all terms of the equation (13) for the energy of the system turn out to be of the same order of smallness when $\lambda \to \lambda_0$. This means that the plasma region bordering with the vacuum moves non-adiabatically, i.e., its motion is strongly influenced by the influx of energy due to the absorption of the radiation.

Thus, as was already indicated, the presence of the asymptotic solution (16) and (17) makes it possible, in principle, to solve our problem numerically. It is desirable, however, to obtain approximate analytic expressions for the plasma parameters at the point $\lambda = 0$,, i.e., on the boundary with the surface of the condensed body. In this case, the problem reduces to a calculation of the quantities V(0), P(0), R(0), and $\eta(0)$, which are functions of the parameters Θ and χ. To this end we use the exact integral relations that can be obtained from the system (1) and (9) under the assumption that the plasma motion is self-similar and which express the laws of conservation of mass, momentum, and energy (including also the ionization energy $\sim z^2$). Integrating (1) and (9) with respect to x from $x = 0$ to $x = x_{fr}$, and taking (12) into account, we obtain

$$0{,}6\lambda_0\int_0^1 R(y)\,dy = R(0)V(0), \tag{18}$$

$$0{,}8\lambda_0\int_0^1 R(y)V(y)\,dy = P(0)+R(0)V^2(0),$$

$$\lambda_0\left[\int_0^1\left(1{,}5P(y)+\frac{R(y)V^2(y)}{2}\right)dy\right] = 1 -$$

$$-\Theta\lambda_0\int_0^1\frac{R^2(y)\Phi(\eta)}{\eta^{3/2}}\,dy-\frac{P(0)V(0)}{\eta(0)},$$

$$\lambda\int_0^1\frac{P(y)}{\eta(y)}\,dy-\frac{P(0)V(0)}{\eta(0)} = 3\Theta\lambda_0\int_0^1\frac{R^2\Phi(\eta)}{\eta^{3/2}}\,dy, \quad y=\frac{\lambda}{\lambda_0}.$$

To determine V(0), R(0), P(0), $\eta(0)$ and q(0) we can, in first approximation, substitute in the integral relations (18) and (15) functions of the form

$$V(y) \approx V(0)(1-y)+1{,}2\lambda_0 y, \quad R(y) \approx R(0)(1-y), \quad P(y) \approx P(0)(1-y)^2, \quad \eta(y) \approx \eta_0 = \text{const}, \tag{19}$$

the dependence of which on $y = \lambda/\lambda_0$ is the same as in the exact asymptotic functions (16). As a result, (15) and (18) reduce to a system of algebraic equations, the solution of which is

$$V(0) = 0.3\lambda_0, \quad R(0) = \frac{5\cdot 10^{-3}}{\Theta}\frac{\eta_0^{1/2}}{\Phi(\eta_0)}, \quad P(0) = 0.15R(0)\lambda_0^2, \tag{20}$$

with λ_0, η_0, and q(0) determined from the transcendental equations

$$\exp\left\{-\frac{1.7\cdot 10^{-3}}{\Theta\eta_0\Phi(\eta_0)(0.185+6\cdot 10^{-2}/\eta_0)}\right\} = \frac{0.13\eta_0+4.5\cdot 10^{-2}}{0{,}185\eta_0+6\cdot 10^{-2}},$$

$$\lambda_0^5 = \frac{\Theta\Phi(\eta_0)}{\eta_0^{1/2}5\cdot 10^{-3}(0{,}185+6\cdot 10^{-2}/\eta_0)}$$

$$q(0) = q_0(0{,}13\eta_0+4{,}5\cdot 10^{-2})/(0{,}185\eta_0+6\cdot 10^{-2}). \tag{21}$$

The optical thickness of the plasma layer $(0, x_{fr})$ is

$$\int_0^{x_{fr}} K\,dx = \frac{1{,}7\cdot 10^{-3}}{\Theta\eta_0\Phi(\eta_0)(0{,}185+6\cdot 10^{-2}/\eta_0)}, \tag{22}$$

We now proceed to analyze the results. The solution of the system of equations (20) and (21) with $\eta_0 > 1$ can be represented in analytic form. In this case, taking (12) into account, we obtain expressions for the plasma parameters at the surface of the condensed body:

$$v(0,t) = 0.39C^{1/5}q_0^{2/5}\left(\frac{\Theta}{\chi}\right)^{3/20}t^{1/5} = 0.39C^{1/5}\chi^{-3/20}q_0^{1/4}q_1^{3/20}t^{1/5},$$

$$\rho(0,t) = 2.36C^{-3/5}q_0^{-1/5}\left(\frac{\Theta}{\chi}\right)^{-9/20}t^{-3/5} = 2.36C^{-3/5}\chi^{9/20}q_0^{1/4}q_1^{-9/20}t^{-3/5},$$

$$kT_e(0,t) = 0.23M_i^{2/3}I_0^{1/3}C^{4/15}q_0^{8/15}\left(\frac{\Theta}{\chi}\right)^{1/30}t^{4/15} = 0{,}23M_i^{2/3}I_0^{1/3}C^{4/15}q_0^{1/2}\chi^{-1/30}q_1^{1/30}t^{4/15},$$

$$p(0,t) = 0{,}59C^{-1/5}q_0^{3/5}\left(\frac{\Theta}{\chi}\right)^{-3/20}t^{-1/5} = 0.59C^{-1/5}\chi^{3/20}q_0^{3/4}q_1^{-3/20}t^{-1/5},$$

$$\eta_0 = 2\cdot 10^{-1}\left(\frac{\Theta}{\chi}\right)^{-1/2} = 2\cdot 10^{-1}\chi^{1/2}\left(\frac{q_0}{q_1}\right)^{1/2},$$

$$z = 1{,}07M_i^{1/3}I_0^{1/3}C^{2/15}q_0^{4/15}\left(\frac{\Theta}{\chi}\right)^{4/15}t^{2/15} = 1{,}07M_i^{1/3}I_0^{1/3}C^{2/15}\cdot\chi^{-4/15}\cdot q_1^{4/15}t^{2/15},$$

$$v_{fr} = 4v(0,t), \quad q(0) = 0.73q_0, \quad \int_0^{x_{fr}} K\,dx = 0{,}31. \tag{23}$$

With the aid of the expression obtained for η_0 it is easy to find the range of variation of the parameter Θ for which formulas (23) are valid. From the condition $\eta_0 > 1$ we get $\Theta < 10^{-2}\chi$, corresponding to a radiation flux density $q_0 > 25q_1/\chi \approx 10^{14}$ W/cm². A comparison of formulas (23) with the results given in [2, 3] shows that the dependence of the hydrodynamic plasma parameters on the radiation flux density q_0 is exactly the same in our case in the case of [2, 3]. However, the time dependences of the hydrodynamic quantities in [2, 3] differ somewhat from the corresponding relations (23), this being due to the slight difference in the optical thickness of the plasma layer in this case. From (23) it follows also that in the indicated region of radiation-flux densities q_0 the value of z does not depend on q_0, and then $\eta_0 \sim kT_e \sim q_0^{1/2}$, which agrees with the results of [6].

We present numerical examples for a laser pulse with parameters $q_0 \approx 10^{14}$ W/cm² and $\tau \approx 10^{-8}$ cm:

a) $I_0 = 6$ eV, $A = 100$, $\eta_0 = 1$, $kT_e = 4{,}7$ keV, $z = 30$,

b) $I_0 = 13{,}6$ eV, $A = 50$, $\eta_0 = 1$, $kT_e = 3{,}7$ keV, $z = 20$.

Thus, the presented numerical estimates show that in the case of a laser pulse with given parameters, the heavy-element atoms up to $z \approx 30$ are ionized almost completely.

If $\eta \leq 1$, i.e., $q_0 < 10^{14}$ W/cm^2, then the solution of (20) and (21) cannot be represented in explicit form. In this case the algebraic equations (20) and (21) can easily be solved numerically for a given value of the parameter Θ.

We note also that the ionization multiplicity and the temperature of a fixed plasma layer that experiences strong expansion, i.e., in the region $x \to x_{fr}$, both tend to constant values, just as in the model where a flat layer of given mass is used.[6] Indeed, changing over to the mass coordinate

$$m = \int_{x_{fr}}^{x} \rho \, dx,$$

we readily obtain with the aid of (16)

$$z \sim m^{1/6} t^{1/30}, \quad kT_e \sim m^{2/3} t^{1/15}, \tag{24}$$

i.e., the indicated quantities for a selected particle (mass) of the plasma are saturated in practice even within the framework of the considered hydrodynamics. We present expressions for the parameter $\eta(\lambda)$ as $\lambda \to \lambda_0$ as a function of the flux density q_0:

$$\Theta \ll 1 \ (q_0 \gg q_1), \ \eta = 1.7 \cdot 10^{-1} \chi^{1/2} \ (q_0 / q_1)^{1/2}; \tag{25}$$

$$\Theta \gg 1 \ (q_0 \ll q_1), \ \Theta = {}^{2}/_{63} e^{1/\eta} \eta^{-3}. \tag{26}$$

In conclusion, we generalize the results to the case of sharp focusing of the radiation. As shown by Memchinov in [4], when the radiation is focused on the surface of a condensed body, there exists a stationary regime of evaporation and heating of the substance, wherein the incident radiation is absorbed only in a region whose dimensions are of the order of the focusing spot r_f. This circumstance is a consequence of the rapid decrease of the density, and consequently also of the absorption coefficient, with increasing distance, owing to the spherical character of the hydrodynamic motion. As a result, the system of hydrodynamic equations (1) admits of a solution wherein

$$\int_0^{r_f} k \, dx = \text{const},$$

where r_f = const, and the spatial distribution of the hydrodynamic parameters of the plasma does not depend on the time. Formally, stationary values of the plasma parameters can be obtained in the case of focused radiation, accurate apart from coefficients of the order of unity in the region $r \approx r_f$, if the time is eliminated from (12) with the aid of the relation

$$\int_0^t v(0, t) \, dt = r_f$$

In addition, in place of the radiation flux density q_0 it is convenient to introduce the total flux $Q = \pi r_f^2 q_0$. As a result of the indicated substitution we obtain

$$v \approx \pi^{-1/3} C^{1/6} Q^{1/3} r_f^{-1/2} V(\Theta), \quad \rho \approx C^{-1/2} r_f^{-1/2} R(\Theta),$$
$$p \approx \pi^{-2/3} C^{-1/6} Q^{2/3} r_f^{-3/2} P(\Theta),$$
$$zkT_e \approx \pi^{-2/3} M_i C^{1/3} Q^{2/3} r_f^{-1} \frac{P(\Theta)}{R(\Theta)}. \tag{27}$$

Physically, formulas (27) correspond to the limiting values of the plasma parameters that can be attained in the case of sharp focusing of the radiation.

At flux densities $q_0 \gtrsim 10^{14}$ W/cm^2, the coefficients $V(\Theta)$, $R(\Theta)$, and $P(\Theta)$ in (27) can be represented with the aid of (23) in explicit form:

$$v \approx 0{,}47 \left(\frac{\Theta}{\chi}\right)^{1/8} \pi^{-1/3} C^{1/6} Q^{1/3} r_f^{-1/2}$$
$$\rho \approx 1{,}3 \left(\frac{\Theta}{\chi}\right)^{-3/8} C^{-1/2} r_f^{-1/2},$$
$$p \approx 0{,}5 \left(\frac{\Theta}{\chi}\right)^{-1/20} \pi^{-2/3} C^{-1/6} Q^{2/3} r_f^{-3/2},$$
$$kT_e \approx 0{,}3 \pi^{-4/9} M_i^{2/3} I_0^{1/3} C^{2/9} Q^{4/9} r_f^{-2/3},$$
$$z = 1{,}2 \left(\frac{\Theta}{\chi}\right)^{1/4} \pi^{-2/9} M_i^{1/3} I_0^{-1/3} C^{1/9} Q^{2/9} r_f^{-1/3}. \tag{27'}$$

Using (20), (21), and (27) for $q_0 \gtrsim 10^{12}$ W/cm^2, $\nu = 3 \times 10^{14}$ sec^{-1}, $A = 50$, $I_0 = 4.2$ eV, and $r_f \approx 10^{-2}$ cm we obtain $zkT_e \approx 8$ keV and $v \approx 10^7$ cm/sec. At $z = 15$ we have $kT_e \approx 0.53$ keV and $I(z) \approx 1$ keV, which is somewhat higher than the experimental values.[1] This discrepancy can be attributed to the fact that the ionization is close to equilibrium at $q_0 \approx 10^{12}$ W/cm^2.

5. Let us ascertain further the limits of applicability of the model in question. The point is that the self-similar regime is obviously asymptotic, and can therefore be observed in reality if the time of its establishment, τ_a, is smaller than the duration of the laser pulse. The value of τ_a can be estimated from the following considerations. At the initial instant of time an electronic thermal conductivity wave propagates inside the target. With increasing ion temperature, a rarefaction wave is also produced, and overtakes the thermal wave after a certain time. This is the time of establishment of the self-consistent regime. Formally, the time τ_a can be determined from the condition $\alpha = c_0 \tau_a / l_0 \approx 1$, where c_0 is the speed of sound and l_0 is the length heated by the electronic thermal conductivity. It is easy to obtain an expression for the parameter α by considering the stage of electronic thermal conductivity and using the dimensionality considerations (2):

$$\alpha = 10^{-5} \Lambda^{1/3} A^{-1/2} q_0^{-1/3} \tau_a^{1/3} n_0^{2/3},$$

where q_0 is the radiation flux density (erg/cm^2 sec) and n_0 is the target density, i.e., $\tau_a \approx 10^{15} q_0 A^{3/2} / n_0^2 \Lambda$. For $n_0 \approx 2 \times 10^{23}$, $A = 50$, $\Lambda = 10$, $q \approx 10^{14}$ W/cm^2 $= 10^{21}$ erg/cm^2 sec we have $\tau_a \approx 10^{-9}$ sec, i.e., say for a giant pulse ($\tau_p \approx 10^{-8}$) the self-consistent regime has time to become established under these conditions. In addition, for atoms with nuclear charge $z_0 \gtrsim 10$ we have $z(\tau_a) < z_0$ (see (27′)). Let us estimate further the loss to radiation of the plasma layer, which we have not taken into account. Using the known formulas[8] for the bremsstrahlung and for the photorecombination plasma radiation integrated over the spectrum, we find, if $kT_e \approx 2$ keV, $z \approx 30$, $N_0 \approx 10^{20}$ cm^{-3}, and the plasma has a linear dimension $l \approx 10^{-2}$, that the radiation flux from the plasma flare, $q \approx 5 \times 10^{10}$ W/cm^2, is much smaller than the flux of interest to us, that of the absorbed laser radiation ($q_0 \gtrsim 10^{12}$ W/cm^2), and can be disregarded in the general balance (1).

Thus, the approach developed in this article makes it possible to refine the earlier results[2, 3] at large radiation-flux densities ($q_0 > 10^{12}$ W/cm^2). The most significant result lies in the possibility of directly determining the nonequilibrium state of the plasma and the nonequilibrium degree of its ionization. As already noted above, at large radiation flux densities the degree of ionization of the plasma decreases compared with the equilibrium value, and the contribution of the ionization energy to the total internal energy of the plasma decreases. In thermodynamic equilibrium, the ionization energy constitutes the main part of the internal energy in an incompletely ionized plasma. Thus, at high radiation flux densities, the absence of equilibrium causes the plasma behavior to become more and more similar to that of an ideal gas with $H = 5/3$, the pressure and internal energy of which are determined by the electrons, and whose inertial properties are determined by the ions.

We note also that the presence of the indicated lack of equilibrium causes the plasma produced as a result of the heating to have a relatively higher temperature. This circumstance can be used, apparently, in those cases when it is desirable to increase the plasma temperature, but it should be noted that this conclusion is valid only until the energy lost to radiation or thermal conductivity begins to play an important role.

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