SOVIET PHYSICS JETP VOLUME 33, NUMBER 1 JULY, 1971

RELATIVISTIC ROTATION OF SPINS S = 1/2 IN COLLISIONS OF PARTICLES ARBITRARILY MOVING WITH RESPECT TO THE OBSERVER

B. M. GOLOVIN and V. I. NIKANOROV

Joint Institute for Nuclear Research

Submitted July 30, 1970

Zh. Eksp. Teor. Fiz. 60, 28–31 (January, 1971)

The relativistic rotation of the spin $S = \frac{1}{2}$ of particles taking part in a binary reaction and moving arbitrarily with respect to the observer is considered with the aid of Lobachevskiĭ geometry. It is shown that in general relativistic rotation of spin should be taken into account not only in the final but also in the initial state of the system.

IN Stapp's paper[1] he shows that when the non-relativistic formalism is used in the relativistic region to describe the polarization of particles with spin $S = \frac{1}{2}$, it is necessary to take into consideration an extra rotation of the spin (the so-called relativistic or kinematic rotation).

A lucid description of relativistic spin rotation is given in [2–4] by Smorodinskiĭ, who employs ideas from Lobachevskiĭ geometry and proposes a convenient method for solving problems in relativistic kinematics with the aid of kinematic diagrams. These diagrams describe the kinematic states of the particles in terms of four-velocity space, several interesting properties of which were established by Chernikov.[6]

The four-velocity of each particle is represented in kinematic diagrams by a point, while the kinematic characteristics of the relative motion of the particles or the characteristics of the motion of the particles in different reference frames are numerically determined with the aid of various hyperbolic functions of the distance between the corresponding points.

The binary reaction A + B → C + D is described by the kinematic diagram shown in Fig. 1. The points A, B, C, D represent the four-velocities of the particles taking part in the reaction while the point U, lying at the intersection of the lines AB and CD, corresponds to the four-velocity of the center of mass (c.m.) of the colliding particles. The distances AU, BU, CU, DU determine (through the corresponding hyperbolic functions) the motion of the particles in the CM system or the motion of the CM system with respect to the rest frame of the corresponding particle. Thus, for example the velocity of the relative motion of the particles A and B is given by $\beta_{AB} = \tanh(AB)$; $\cosh(AB)$ determines the relativistic factor of the relative motion of these particles, and so on.

The main formulas necessary for computations with kinematic diagrams are given in the already cited papers [2,3] and in our more detailed exposition.[7]

The relativistic rotation of the spin of a particle is determined by the rotation of the vector as it is parallel-transferred along the perimeter of the triangle with vertices at the points characterizing the four-velocities of the reference frames of the observer, the center of mass of the colliding particles and, of the particle under consideration. The angle of relativistic rotation is numerically equal to the area of this triangle.

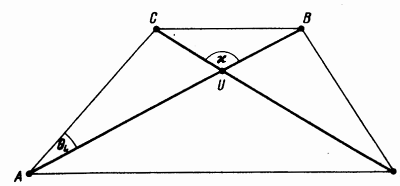

FIG. 1. Kinematic diagram of the binary reaction A + B → C + D.

Notice that in the case where the system of the observer coincides with the rest frame of one of the particles, the spin orientations in the initial state do not undergo relativistic rotation since the areas of the triangles determining these rotations are then equal to zero. There is also no relativistic rotation when colliding beams of particles of equal mass are used, for in that case the state of the observer is described by the point U and all the triangles which determine the relativistic rotations degenerate into the corresponding segments.

Let the particle B with mass m_B be incident on the particle A of mass m_A and, as a result of the reaction, let the particles C, D having masses m_C, m_D respectively be produced (see Fig. 1). Then the relativistic rotation of the spin of the particle C in the rest frame of the particle A is given by the area of triangle AUC and can be found by means of the formula

$$\sin^2\left(\frac{\Omega_c}{2}\right) = \frac{1}{2}\sin^2\theta_L[\mathrm{ch}(AU)-1]\left[\sqrt{1-\rho^2+\rho^2\,\mathrm{ch}^2(AU)}-1\right] \times\left\{1+\frac{1}{\rho}\cos\varkappa + \frac{\sqrt{1-\rho^2+\rho^2\,\mathrm{ch}^2(AU)}\left[\mathrm{ch}(AU)-\sqrt{1-\rho^2+\rho^2\,\mathrm{ch}^2(AU)}\right]}{\rho^2[\mathrm{ch}^2(AU)-1]}\right\}. \quad (1)$$

Here Ω_C is the value of the relativistic rotation of the spin,

$$\rho = m_A / m_C,$$

$$\mathrm{ch}(AU) = \frac{m_A + m_B\,\mathrm{ch}(AB)}{\sqrt{m_A^2 + m_B^2 + 2m_A m_B\,\mathrm{ch}(AB)}}$$

θ_L, κ are the scattering angles of the particle C in the rest frame of the particle A and in the c.m.s. of the colliding particles respectively. In order to compute

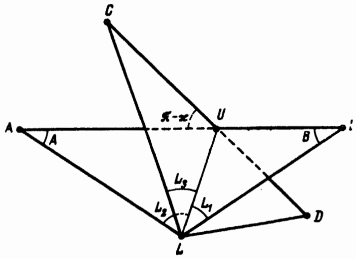

FIG. 2. Kinematic diagram of the binary reaction when the observer represented by the point L is moving.

the relativistic rotation of the spin of particle D, we must put $\rho = m_A/m_D$ in this expression and assume that the angle θ_L corresponds to the scattering angle (in the rest frame of particle A) of the particle D, and change the sign of $\cos \kappa/\rho$. It is easy to see that for elastic scattering of particles of equal mass ($m_A = m_B = m_C = m_D$) formula (1) takes the form

$$\sin(\Omega_c/2) = \sin(\kappa/2 - \theta_L) \quad (2)$$

and coincides with the well-known Stapp formula.[1]

Let us consider the case (Fig. 2) when the system of the observer (point L) does not coincide with any of the points A, B, C, D, U of the kinematic diagram of the reaction and, generally speaking, does not lie in the plane ABCD. Note that under these conditions, in contrast to the previous case when the system of the observer coincided with the rest frame of one of the particles, it is also necessary to take into account the relativistic rotations of the spins of the particles in the initial state. These rotations are given by the areas of the triangles LUA and LUB.

The relativistic rotations of the spins of the reaction products (C, D) are given by the areas of the triangles LUC and LUD. Spin rotations in all cases take place around the normals to the planes of the triangles that determine these rotations. By solving the corresponding triangles, we determine the kinematic diagram parameters necessary for further computations.[7]

The relativistic rotations of the initial state spins are given by the area of triangle ALU for particle A and by the area of triangle BLU for particle B:[7]

$$\sin\frac{\Omega_A}{2} = \left\{\frac{1 + 2\,\mathrm{ch}(LU)\,\mathrm{ch}(AU)\,\mathrm{ch}(AL) - \mathrm{ch}^2(LU) - \mathrm{ch}^2(AU) - \mathrm{ch}^2(AL)}{2[\mathrm{ch}(LU)+1]\,[\mathrm{ch}(AU)+1]\,[\mathrm{ch}(AL)+1]}\right\}^{1/2} \quad (3)$$

The corresponding formulas for $\sin(\Omega_B/2)$ and $\sin(\Omega_C/2)$ are obtained from (3) by making the substitutions A → B, A → C respectively. Putting AL = 0 (the frame of the observer coincides with the rest frame of the particle A), we obtain $\sin(\Omega_A/2) = 0$, which confirms the assertion made above that no relativistic rotations of the initial spin states take place under these conditions.

In order to analyze the final states we consider the relativistic rotation of the spin of the particle C—which requires the solution of triangle LUC. The hyperbolic function of the length of the side LC of this triangle can be determined uniquely only in the case when the scattering angles of particle C are known in both the system of the observer and the system of the center of mass.

T_B, MeV	T_A, MeV	Ω_A, deg	Ω_B, deg	Ω_C, deg
600	0	0	0	7.9
600	1	0.7	0.6	7.9
600	10	1.1	1.9	7.9
600	100	6.7	5.9	6.7

If, however, the scattering direction of this particle is given in the system of the observer only, then, generally speaking, cosh (LC) may have two non-coinciding values corresponding to two different scattering angles in the c.m.s. which lead to one and the same direction of the motion of the particle with respect to the observer. Evidently, to each of these values of cosh (LC) corresponds a definite value of relativistic rotation of the spin.

The formula for calculating the relativistic rotation of the particle C may be written in the form of (3) with the substitution A → C. Similarly, the relativistic rotation of the spin of the particle D may be found. It is easy to show that when the point L coincides with the point A (with the rest system of A), the formula for $\sin(\Omega_C/2)$ takes the form of (1).

For illustration we give in the table numerical values of the angles of relativistic rotation of spins in elastic pp-scattering for $\kappa = 90°$ in the case when L lies in the plane ABCD and the angle between the initial velocities of the protons is 90°.

The first row of the table corresponds to the scattering of a proton with energy T = 600 MeV on a proton initially at rest, and, consequently, the angle of rotation of the spin of the particle in this row coincides with the Stapp rotation.

As is well known, the matrix element of the elastic scattering of particles with spin S = ½ has, in the nonrelativistic approximation, the form

$$M = \langle S_c S_D A \rho_{in} A^+ \rangle. \quad (4)$$

Here A is the scattering amplitude,[8] ρ_{in}—the density matrix of the initial spin states and $S_{C,D}$—the spin operators acting on the wave function of the final state.

The generalization of this formula to the scattering of relativistic particles on stationary targets which has in it the operators $R_{C,D}$ of the relativistic rotation of the spins of the particles C, D, is given in [9]:

$$M = \langle R_c R_D S_c S_D A \rho_{in} A^+ \rangle. \quad (5)$$

In the general case of scattering of a relativistic particle on a moving target, it is, as has been noted already, necessary to take into consideration relativistic rotation of spins in the initial state. The structure of the matrix element then takes the form

$$M = \langle R_c R_D S_c S_D A R_A R_B \rho_{in} A^+ \rangle, \quad (6)$$

which differs from (5) by the appearance of R_A and R_B on the density matrix of the initial state.

A more detailed exposition of the subject considered in this article may be found in the preprint [7].

The authors express their gratitude to L. I. Lapidus for a discussion of the subject touched upon in this paper and to Ya. A. Smorodinskiĭ for valuable advice and for looking through the manuscript before its publication.

[1] H. P. Stapp, Phys. Rev. **103**, 425 (1956).
[2] Ya. A. Smorodinskiĭ, Sb. Voprosy élementarnykh chastits (Shkola teoreticheskoĭ i éksperimental'noĭ fiziki v Nor-Amberde) (Collected Problems in Elementary Particle Physics (School of Theoretical and Experimental Physics in Nor-Amberd)), AN Arm. SSR, III, 1963, p. 242.
[3] Ya. A. Smorodinskiĭ, Atomnaya énergiya **14**, 110 (1963).
[4] Ya. A. Smorodinskiĭ, Zh. Eksp. Teor. Fiz. **43**, 2217 (1962) [Sov. Phys.-JETP **16**, 1566 (1963)].
[5] N. I. Lobachevskiĭ, Tri sochineniya po geometrii (Three Papers on Geometry), Gostekhizdat, 1956.
[6] N. A. Chernikov, Nauchnye doklady vyssheĭ shkoly (Scientific Reports of Higher Institutions of Learning) **2**, 158 (1958).
[7] B. M. Golovin and V. I. Nikanorov, JINR Preprint R2-5272, Dubna (1970).
[8] L. Wolfenstein and J. Ashkin, Phys. Rev. **85**, 947 (1952).
[9] S. M. Bilen'kiĭ, L. I. Lapidus, and R. M. Ryndin, Zh. Eksp. Teor. Fiz. **51**, 891 (1966) [Sov. Phys.-JETP **24**, 593 (1967)].

Translated by A. K. Agyei
4