ELECTRON HEATING AND INCOHERENT HARD RADIATION PRODUCED BY INTERACTION OF ULTRASHORT INTENSE LASER PULSES WITH MATTER

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We consider the main mechanisms of heating of the electronic component of a plasma by interaction with picosecond laser pulses in a wide range of laser intensities. It is shown that in some intensity regions the hard bremsstrahlung of the plasma is determined not by the electron temperature, but by the energy corresponding to the electron oscillations in the field of the laser radiation.

1. INTRODUCTION

WHEN an arbitrary medium is exposed to light with intensity $I > I_{cr}$, where

$$I_{cr} = \frac{mc\omega^2\Delta}{4\pi e^2} = 2\left(\frac{\omega}{\omega_a}\right)^2 I_a \tag{1}$$

(e, m—charge and mass of electron, ω—circular frequency of light wave, Δ—ionization energy of the atoms, $\omega_a = \Delta/\hbar$, $I_a = cm\Delta^3/8\pi e^2\hbar^2$—intensity corresponding to a wave amplitude E equal to the intraatomic intensity E_a), the medium becomes completely ionized "instantaneously" (within one or several cycles of the wave). Indeed, when $I > I_{cr}$, the amplitude of the electron vibrational velocity in the field $eE/m\omega$ exceeds the velocity of the electrons in the atom

$$eE/m\omega > \sqrt{\Delta/m}. \tag{2}$$

This in turn means that the ionization of the atoms in the wave field is due to the tunnel effect,[1,2] and the probability of ionization per unit time is determined by the Oppenheimer formula[3] averaged over the period of the wave:

$$w \sim \omega_a(E_a/E)^{1/2}\exp(-E_a/E). \tag{3}$$

Under conditions typical of the optical band, $\omega_a/\omega \sim 10$, and the ionization probability per period is $2\pi w/\omega \sim 1$ already at $I = 3I_{cr}$.

Consequently, radiation with intensity $I > I_{cr}$ can interact only with a plasma (in particular, an electron-nuclear plasma if Δ is the energy of the complete ionization of the atom). The produced plasma, in turn, by subsequently interacting with optical high-intensity radiation, can serve as a source of incoherent bremsstrahlung of hard quanta. The obvious possibility of such an effect is connected with the heating of the electronic component of the plasma, by absorption of optical radiation, to high temperatures T_e corresponding to emission of hard quanta. This is the basis of a procedure for measuring the temperature T_e, particularly that of a laser plasma.[4] But hard incoherent radiation of a plasma situated in an intense light-wave field is possible also at relatively low temperatures T_e, or more accurately, in those cases when the plasma electrons do not have time to become strongly heated during the time of the laser pulse. The point is that the bremsstrahlung spectrum of a plasma interacting with the optical radiation corresponds to the temperature T_e only in the case when $\epsilon_e \equiv \frac{3}{2}T_e \gtrsim \epsilon_{vib}$, where ϵ_{vib} is the average vibrational energy of the electron in the field of the wave, equal to (see the figure, curve 1)

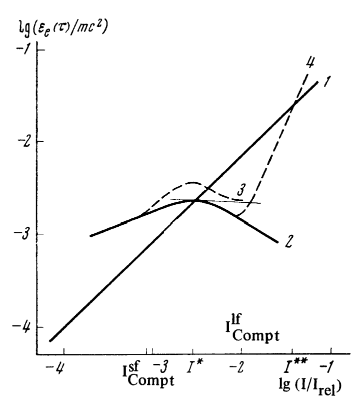
$$\varepsilon_{vib} = e^2E^2/4m\omega^2 = 2\pi e^2 I/mc\omega^2. \tag{4}$$

Under the opposite condition, i.e., when

$$\varepsilon_{vib} \gg \varepsilon_e, \tag{5}$$

the spectrum of the hard bremsstrahlung of the plasma is determined not by the temperature T_e, but by the vibrational energy ϵ_{vib}.

Let us agree to assume that the electrons are heated during the laser-radiation pulse if $\epsilon_e(\tau) \gtrsim \epsilon_{vib}$ at the end of the pulse, and are not heated if $\epsilon_e(\tau) < \epsilon_{vib}$. Our problem is to determine the radiation conditions under which the electrons have time to become heated, and consequently when the hard bremsstrahlung of the plasma corresponds to their temperature, and under what conditions there is no heating and the spectrum of the hard radiation is determined by the fast electron vibrations, which vanish together with the laser pulse. In accordance with the experimental possibilities (see



Qualitative plots of $\epsilon_e(\tau, I)$ against I in a logarithmic scale (τ fixed) for the case of typical values of the parameters. 1—Energy $\epsilon_{vib}(I)$, 2—$\epsilon_e(\tau, I)$ with only the bremsstrahlung mechanism taken into account, 3—$\epsilon_e(\tau, I)$ with allowance for the bremsstrahlung and Compton mechanism, short-focus lens), 4—the same for a long-focus lens.

below) for obtaining intense optical radiation in the wide range of values of interest to us (10^{14}–10^{20} W/cm^2), all the quantitative estimates will be carried out for picosecond laser pulses, i.e., for irradiation times $\tau \sim 10^{-11}$–10^{-12} sec. We shall first consider (Sec. 2) the case of nonrelativistic temperature, $T_e \ll mc^2$, and nonrelativistic intensities $I \ll I_{rel}$, where

$$I_{rel} = \frac{m^2\omega^2c^3}{4\pi e^2} = \frac{mc^2}{\Delta} I_{cr}. \tag{6}$$

When $I \gtrsim I_{rel}$, the wave amplitude E satisfies the condition

$$eE/m\omega \gtrsim c \tag{7}$$

and an important role in the interaction between the optical radiation and the electrons (whose translational velocity may be also nonrelativistic) is assumed by relativistic effects, particularly the many-quantum Compton effect.[5] Section 3 is devoted to the relativistic case.

Let us dwell briefly on the experimental possibilities of obtaining spectral radiation intensities $I > I_{cr}$. For neodymium-glass laser radiation (λ = 1.06 μ) and an ionization energy Δ = 15 eV, the intensity is $I_{cr} = 7 \times 10^{13}$ W/cm^2. At the present state of quantum electronics, intensities of this order and even of the order of $I_a \sim 10^2 I_{cr}$ are experimentally feasible. Such intensities can be realized most easily in the picosecond range of radiation-pulse durations, with neodymium and ruby lasers operating in the mode-locking regime. Furthermore, the use of this laser regime, together with specially developed focusing systems, also yields high optical-radiation intensities, and one can hope to realize experimentally in the near future $I \sim 10^{18}$–10^{19} W/cm^2, i.e., $I \gtrsim I_{rel}$ ($I_{rel} \approx 2.5 \times 10^{18}$ W/cm^2 for a neodymium-glass laser).

2. NONRELATIVISTIC INTENSITIES

In the considered irradiation-duration range $\tau \sim 10^{-11}$–10^{-12} sec, one can apparently disregard the possible plasma-heating mechanisms connected with excitation of collective oscillations in the plasma, and consider only the "single-particle" mechanisms—the inverse bremsstrahlung and Compton scattering of the optical radiation.

Let us start with the bremsstrahlung mechanism of the heating. The rate of electron heating in this mechanism is given by

$$(d\varepsilon_e/dt)^{br} = \varepsilon_{vib}\nu_{eff}, \tag{8}$$

where the effective electron-ion collision frequency ν_{eff} depends, generally speaking, both on the temperature T_e and on the radiation-field intensity I.[6,7] At weak radiation intensities, when $\epsilon_{vib} \ll \epsilon_e$, we have the usual expression for ν_{eff}:

$$\nu_{eff}(\varepsilon_e) = {}^4/_3\sqrt{2\pi}Z^2e^4n_iL/m^{1/2}T_e^{3/2}, \tag{9}$$

where n_i is the concentration of the ions with charge Ze, and L is the Coulomb logarithm. In the opposite case, when (5) is satisfied, ν_{eff} is given by[6]

$$\nu_{eff}(I) = 2\pi^{-3/2}\frac{Z^2em\omega^3c^{3/2}n_iL}{I^{3/2}}\left(1+\frac{1}{2}\ln\frac{\varepsilon_{vib}}{2T_e}\right). \tag{10}$$

Accurate to the logarithm term, this formula is obtained from (9) by replacing $3T_e/2$ in the latter by ϵ_{vib}. To estimate the value of ν_{eff} in the intermediate region $\epsilon_e \sim \epsilon_{vib}$, we can use, with sufficient accuracy, the interpolation formula

$$\nu_{eff}(\varepsilon_e, I) \approx 2\sqrt{3\pi}\frac{Z^2e^4n_iL}{m^{1/2}(\varepsilon_e+\varepsilon_{vib})^{3/2}}. \tag{10'}$$

We now turn to the heating mechanism connected with the Compton scattering of laser radiation by the plasma electrons. As to the spontaneous Compton effect, in the temperature region $T_e \gg \hbar\omega$ of interest to us the inverse Compton effect always predominates, and leads to cooling of the electrons at a rate[8]

$$\left(\frac{d\varepsilon_e}{dt}\right)^{aC} = -\frac{8}{3}\sigma_0\frac{\varepsilon_e}{mc^2}I, \tag{11}$$

where $\sigma_0 = 8\pi r_0^2/3$ ($r_0 = e^2/mc^2$) is the Thompson cross section.

Let us now consider the induced Compton effect, which, generally speaking, must be taken into account if

$$N_\lambda = 4\pi^3c^2I/\hbar\omega^3\Delta\omega\Omega_0 \gg 1,$$

where $\Delta\omega$ is the width of the frequency spectrum of radiation having an integral intensity I, and Ω_0 is the width of the solid angle in which the angular spectrum of the radiation is concentrated. We note that finite width of the frequency ($\Delta\omega$) and angular (Ω_0) radiation spectra is a necessary condition for stimulated Compton scattering with energy transfer to the electrons (i.e., with their heating). The impossibility of having stimulated scattering with energy transfer to the electron in a plane wave follows from the energy and momentum conservation laws, which lead to the following well-known relation between the frequencies ω and ω' of the incident and scattered photons and the unit vectors $\mathbf{q}$ and $\mathbf{q}'$, which determine the direction of motion of these photons:

$$\omega\left(1-\frac{\mathbf{vq}}{c}\right) = \omega'\left(1-\frac{\mathbf{vq}'}{c}\right)+\frac{\hbar\omega}{2mc^2}\omega'(\mathbf{q}-\mathbf{q}')^2, \tag{12}$$

where $\mathbf{v}$ is the velocity of the electron prior to scattering (it is assumed that $v/c \ll 1$). For a plane wave, stimulated scattering is possible only when $\mathbf{q} = \mathbf{q}'$, but then it follows from (12) that $\omega = \omega'$. Formula (12) makes it possible to write a condition on the degree of nonmonochromaticity of the radiation $\Delta\omega/\omega$ at which stimulated scattering through a specified angle 2θ is possible[1)]

$$\frac{\Delta\omega}{\omega} \geqslant \frac{\mathbf{v}}{c}(\mathbf{q}-\mathbf{q}') = \left|2\frac{v_\parallel}{c}\sin\theta\right|; \tag{13}$$

Here $v_\parallel$ is the component of $\mathbf{v}$ in the direction of $\mathbf{q}-\mathbf{q}'$. At a laser-beam solid angle $\Omega_0 = 2\pi(1-\cos\theta_0)$ ($2\theta_0$ is the angular aperture of the beam), the scattering angle is $2\theta \le 2\theta_0$, and in order for almost all the beam photons and almost all the plasma electrons, with Maxwellian velocity distribution, to satisfy condition (13), it is necessary to have[8]

1) It is assumed that $v/c \gg \hbar\omega/mc^2$. Then (12) yields

$$\frac{\omega-\omega'}{\omega} = \frac{\mathbf{v}}{c}(\mathbf{q}-\mathbf{q}')+\frac{\mathbf{vq}'}{c}\frac{\mathbf{v}}{c}(\mathbf{q}-\mathbf{q}')+\frac{\hbar\omega}{2mc^2}(\mathbf{q}-\mathbf{q}')^2.$$

$$\frac{\Delta\omega}{\omega} \geqslant \frac{\Omega_0^{1/2}}{2}\sqrt{\frac{T_e}{mc^2}}. \quad (14)$$

When this condition is satisfied, the rate of heating of the electrons is determined by the formula[8]

$$\left(\frac{d\varepsilon_e}{dt}\right)^{sC} = \frac{3\pi}{4}\frac{\sigma_0}{m}\frac{\Omega_0}{\omega^3}\frac{\omega}{\Delta\omega}I^2. \quad (15)$$

On the other hand, if a condition inverse to (14) is satisfied, i.e.,

$$\mu \equiv \frac{2}{\Omega_0^{1/2}}\frac{\Delta\omega}{\omega}\sqrt{\frac{mc^2}{T_e}} \ll 1, \quad (16)$$

then the condition (13) is satisfied only by a small μ-th fraction of all the plasma electrons, and accordingly only this fraction is subject to the stimulated scattering process.[2)] Under these conditions the Compton mechanism of electron heating by a laser pulse can assume a role only if its duration τ is large compared with the relaxation time of the electron temperature $\tau_{ee} \sim 1/\nu_{eff}^{ee}$, i.e., when $\tau\nu_{eff}^{ee} \gg 1$ (ν_{eff}^{ee} is the effective frequency of the interelectron collisions). From this follows, by virtue of (9),[3)] a limitation on the heating temperature T_{e1}, up to which the Compton mechanism plays an important role (if (16) is satisfied). This limitation is given by ($Z = 1$, $n_i = n_e$)

$$T_e \ll T_{e1} \equiv 2\left(\frac{\pi}{9}\right)^{1/3}\left(\frac{\omega_p}{\omega}\right)^{4/3}\left(\frac{Lr_0}{\lambda}\right)^{2/3}(\omega\tau)^{2/3}mc^2 \quad (17)$$

($\omega_p^2 = 4\pi e^3 Z n_i/m$); for $\omega_p/\omega \approx 1$, $L \approx 10$, $\tau \approx 10^{-12}$ sec, and $\lambda = 1.06\,\mu$, this yields $T_e \ll 1$ keV. Thus, if the plasma heating takes place with condition (16) satisfied, then the Compton mechanism plays an important role only at sufficiently low heating temperatures satisfying (17) (obviously, the heating rate is determined here by the product of the right-hand side of (15) by μ). In particular, if the initial temperature $T_e(0)$ satisfies simultaneously the condition (16) and the condition $T_e(0) \gtrsim T_{e1}$, then when such a plasma is heated the Compton mechanism does not play any role at all, leading only to an insignificant deviation of the electron velocity distribution function from the isotropic form.[4)]

Let us determine the region of values of the intensity of the optical radiation I for which the Compton heating mechanism is fundamental if the condition (14) is satisfied. According to (8) and (15), this region is determined from the inequality

$$\varepsilon_{vib}\nu_{eff} < \frac{3\pi}{4}\frac{\sigma_0}{m}\frac{\Omega_0}{\omega^3}\frac{\omega}{\Delta\omega}I^2.$$

Substituting here expression (10′) for ν_{eff}, we obtain the following condition for the temperature T_e and intensity I:

$$\varepsilon_e > \varepsilon_{vib}[(I_{Compt}/I)^{5/3} - 1], \quad (18)$$

where

$$I_{Compt} = 2^{6/5}\left(\frac{\omega_p}{\omega}\right)^{4/5}\left(\frac{\Delta\omega}{\omega\Omega_0}\right)^{3/5}\left(\frac{LZr_0}{\lambda}\right)^{3/5}I_{rel}. \quad (19)$$

From (18) we see that at $I \geq I_{Compt}$ the Compton heating mechanism is the fundamental one at any electron temperature, and the role of this mechanism increases with increasing T_e. However, the intensity starting with which the Compton mechanism becomes dominant has a weak temperature dependence, and therefore the value of I_{Compt} can be taken to be the sought intensity boundary in the entire possible range of T_e. When $\omega_p/\omega \approx 1$, $L \approx 10$, $Z = 1$, $\Delta\omega/\omega \approx 10^{-3}$, $\lambda = 1.06\,\mu$ and $\Omega_0 \approx 6\times10^{-5}$ sr[5)] we have $I_{Compt} \approx 10^{-2} I_{rel}$. For large values of Ω_0 (lenses with shorter focal lengths) the values of I_{Compt} decrease.

We shall show now that in the picosecond range of irradiation durations τ and for a sufficiently dense plasma the cooling of the plasma electrons during the time of the pulse can be neglected. The cooling due to the expansion of the plasma in this range of τ is obviously always negligibly small, and it is therefore necessary to consider only the energy loss to elastic collisions of electrons with ions, bremsstrahlung, and energy loss due to the inverse Compton effect.[6)] It is easily seen, first, that if the ions are not too heavy the bremsstrahlung loss is always small compared with the loss to elastic collisions. Indeed, the rate of the latter is

$$\left(\frac{d\varepsilon_e}{dt}\right)^{el} = -\frac{2m}{AM}\varepsilon_e\nu_{eff} \quad (20)$$

(M is the proton mass and A is the atomic weight of the ion), whereas the rate of loss to bremsstrahlung can be represented by the formula (see, for example, [9])

$$\left(\frac{d\varepsilon_e}{dt}\right) = -\frac{16}{9\pi}\frac{\alpha}{mc^2}\varepsilon_e^2\nu_{eff} \quad \left(\alpha = \frac{e^2}{\hbar c}\right). \quad (21)$$

From a comparison of (20) and (21) it follows that when

$$A \ll mc^2/4\varepsilon_e \quad (22)$$

the bremsstrahlung loss can be neglected. On the other hand, it is likewise easy to show that the loss mechanisms due to elastic collisions and the inverse Compton effect compete with each other in the considered regions of the temperature T_e and intensity I: at sufficiently high values of the temperature $T_e \ll mc^2$ and intensity $I \ll I_{rel}$ the decisive mechanism is the inverse Compton effect. It turns out, however, that under our conditions these two are likewise insignificant. As to the inverse Compton effect, its contribution is always small compared with the stimulated Compton effect if condition (14) is satisfied. The condition for this, on the basis of (11) and (15), is

2) It is easily seen that these are the electrons whose velocities make sufficiently small angles with the laser-beam axis.

3) It is obvious that when $\tau\nu_{eff}^{ee} \gg 1$, the energy is $\epsilon_e(\tau) > \epsilon_{vib}$, and it is therefore necessary to use formula (9) for ν_{eff}.

4) This effect is due to the fact tat in stimulated Compton scattering, the directions of the recoil-rlectron momenta lie near a plane perpendicular to the laser-beam axis. When the indicated two conditions are satisfied, the Compton mechanism leads to a rapid "erosion" of the distribution function along the beam axis (see footnote[2)]).

5) At this value of Ω_0, condition (14) is satisfied up to energies $\epsilon_e = 0.1mc^2$. We note also that the numerical values of the parameters ω_p/ω, L, Z, $\Delta\omega/\omega$, and, let us add, $\tau' \approx 10^{-12}$ sec, assumed here are typical for laser plasma experiments in the picosecond range, and will be used below without any stipulations.

6) At the temperatures of interest to us, $T_e \gtrsim 1$ kV, the recombination radiation of the plasma plays no role.

$$I \gg 2^7 \frac{T_e}{mc^2}\frac{e^2}{\lambda\Delta}\frac{\Delta\omega}{\omega\Omega_0} I_{cr}. \qquad (23)$$

When $\epsilon_e/mc^2 \approx 10^{-1}$, $e^2/\lambda\Delta \approx 10^{-4}$, $\Omega_0 \approx 6\times 10^{-5}$ rad, this yields $I > 10^{-2} I_{cr}$. On the other hand, if condition (14) is not satisfied and the Compton heating mechanism does not play any role, then the condition for neglecting the inverse Compton effect takes, on the basis of (8), (11), and (10′), the form

$$\varepsilon_e(\varepsilon_e + \varepsilon_{vib})^{3/2} \ll \frac{ZL}{7}\left(\frac{\omega_p}{\omega}\right)^2 (mc^2)^{5/2}.$$

For a dense plasma, when $(ZL/7)(\omega_p/\omega)^2 \gtrsim 1$, this condition is likewise always satisfied.

The energy loss to elastic collisions can be appreciable (at sufficiently long heating times), in principle, only when there is no Compton heating mechanism. In the presence of the latter, i.e., when (14) is satisfied and $I > I_{Compt}$, the rate of these losses (20) can be readily shown to be always small compared with the heating rate (15). On the other hand, if the pure bremsstrahlung heating mechanism is in operation, then the loss to elastic collisions should be taken into account only for heating times:

$$\tau \gtrsim \delta^{-5/3}\nu_{eff}^{-1}(I), \qquad (24)$$

where $\delta = 2m/AM$, and $\nu_{eff}(I)$ is determined by formula (10).[7] In the picosecond range of τ, however, and when $I > I_{cr}$, the inverse condition is always satisfied, and can be written in the form

$$I \gg \left(\frac{2m}{AM}\right)^{10/9}(ZL)^{2/3}\left(\frac{\omega_p}{\omega}\right)^{4/3}\left(\frac{\omega}{\omega_a}\right)^{2/3}(\omega\tau)^{2/3} I_{cr}. \qquad (25)$$

When $A = 1$ and $\omega/\omega_a \sim 10^{-1}$, this yields $I \gg 10^{-1} I_{cr}$.

Let us consider directly the process of heating plasma electrons by a laser pulse. We are interested in the final value of the average energy $\epsilon_e(\tau, I)$ as a function of the pulse intensity I, which is assumed to be uniform. At sufficiently low intensities, when $I < I_{Compt}$, we have the pure bremsstrahlung heating mechanism. In this region of I, on the basis of the solution of (8) with allowance for expression (10′) for ν_{eff}, we obtain

$$\varepsilon_e(\tau, I) = \varepsilon_{vib}\left\{\left[\left(1+\frac{\varepsilon_e(0)}{\varepsilon_{vib}}\right)^{5/2} + {}^5/_2\tau\nu_{eff}(I)\right]^{2/5} - 1\right\}, \qquad (26)$$

where $\nu_{eff}(I)$ is determined by formula (10). Let $\epsilon_e(0) \ll \epsilon_{vib}$ (for a self-forming plasma $\epsilon_e(0) \sim \Delta$). Then for $\tau\nu_{eff}(I) \gg 1$ the energy is $\epsilon_e(\tau) > \epsilon_{vib}$ and increases slowly with I, like $I^{2/5}$; for $\tau\nu_{eff}(I) \ll 1$ the energy $\epsilon_e(\tau) \ll \epsilon_{vib}$ and decreases with I like $I^{-1/2}$. At a specified τ, there exists an intensity

$$I^* = 2^{1/3}\left(\frac{\omega_p}{\omega}\right)^{4/3}\left(\frac{ZLr_0}{\lambda}\right)^{2/3}(\omega\tau)^{2/3} I_{rel}, \qquad (27)$$

such that $\epsilon_e(\tau, I^*) = \epsilon_{vib}$ (then $\nu\tau_{eff}(I^*) = 1.86$). If $I \le I^*$, then the electrons have time to become heated by the pure bremsstrahlung mechanism during the course of the laser pulse, and if $I > I^*$ they do not. Under typical conditions $I^* \approx 3\times 10^{-2} I_{rel}$. Besides having the indicated meaning, the intensity I^* almost coincides with the point I_{max} that determines the position of the maximum on the plot of $\epsilon_e(\tau, I)$ against I at a fixed τ, namely, at the point I_{max} we have $\tau\nu_{eff}(I_{max}) = 1.72$, i.e., $I_{max} = 1.05 I^*$.

Thus, the maximum energy up to which the electrons can be heated in the pure bremsstrahlung mechanism is

$$\varepsilon_{e\,max}^{\tau}(\tau) \approx \varepsilon_{vib}(I^*) = 2^{1/3}\left(\frac{\omega_p}{\omega}\right)^{4/3}\left(\frac{ZLr_0}{\lambda}\right)^{2/3}(\omega\tau)^{2/3} mc^2, \qquad (28)$$

which under typical conditions amounts to 0.9 keV.

A qualitative plot of the solution (26) against I is shown in the figure (curve 2).

To ascertain how the Compton mechanism changes the described heating picture when $I > I_{Compt}$, let us consider separately two cases of laser-pulse focusing, which correspond conditionally to short-focus and long-focus lenses. We introduce the parameter

$$\xi(\Omega_0) = \frac{I^*}{I_{Compt}} \approx \left(\frac{\omega_p}{\omega}\right)^{8/15}\left(\frac{ZLr_0}{2\lambda}\right)^{4/15}(\omega\tau)^{2/3}\left(\frac{\Omega_0\omega}{\Delta\omega}\right)^{2/5}. \qquad (29)$$

Then the short-focus lens, by definition, corresponds to the condition $\xi > 1$, and the long-focus lens to $\xi < 1$, with the solid angles Ω_0^{sf} and Ω_0^{lf} satisfying, according to (29), the conditions

$$\begin{matrix}\Omega_0^{sf} \gg \\ \Omega_0^{lf} \ll\end{matrix}\left\{\frac{\Delta\omega}{\omega}\left(\frac{\omega}{\omega_p}\right)^{4/3}\left(\frac{2\lambda}{ZLr_0}\right)^{2/3}(\omega\tau)^{-5/3}\right\}. \qquad (30)$$

Under typical conditions this means that $\Omega_0^{sf} \gg 10^{-3}$ sr (or $2\theta_0^{sf} \gg 3.5\times 10^{-2}$ rad) and $\Omega_0^{lf} \ll 10^{-3}$ sr.

a) Short-focus Lens ($\xi > 1$)

In this case $I_{Compt} < I^*$ and consequently the influence of the Compton mechanism begins to be felt before the electron energy reaches the maximum value it attains by bremsstrahlung heating $\epsilon_{e\,max}^{br}(\tau)$. The deviation from the solution (26) begins at an approximate heating energy

$$\varepsilon_e(\tau, I_{Compt}^{sf}) = (1.85\xi^{3/5} - 1)\,\varepsilon_{vib}(I_{Compt}^{sf}) = \frac{1.85\xi^{3/5} - 1}{\xi}\varepsilon_{e\,max}^{\tau}(\tau). \qquad (31)$$

At $\Omega_0^{sf} = 3\times 10^{-2}$ sr, we have $\xi \approx 4$, and this energy amounts to approximately $0.8\,\epsilon_{e\,max}^{br}(\tau) \approx 0.7$ keV. An important fact is that in the case of short-focus lenses the energy $\epsilon_e(\tau, I_{Compt}^{sf})$ never satisfies the conditions (14) and (17). As to the former condition, under typical conditions at $\Omega_0^{sf} \gg 10^{-2}$ sr, the right-hand side of (14) exceeds 1.6×10^{-3}.[8] The second condition is satisfied only at angles that are unrealistically large for laser experiments ($\Omega_0 \sim 1$ sr).

This fact means, in turn, that when $I > I_{Compt}^{sf}$ the Compton mechanism alone is incapable of heating the electrons to a higher temperature than that given by (31). This does not mean, of course, that when $I > I_{Compt}^{sf}$ the Compton heating mechanism does not play any role at all and the electrons cannot be heated

[7] The condition (24) follows from the solution of the heating equation $d\epsilon_e/dt = \epsilon_{vib}\nu_{eff} - \delta\epsilon_e\nu_{eff}$. When $\tau \gtrsim \delta^{-1}\nu_{eff}^{-1}(\epsilon_{vib}/\delta) = \delta^{-5/3}\nu_{eff}^{-1}(I)$ the energy is $\epsilon_e(\tau) \sim \epsilon_{vib}/\delta$ and it is necessary to take into account the losses to elastic collisions.

[8] The right-hand side of (14) increases with Ω_0 approximately like $\Omega_0^{1/3}$.

more strongly than by the bremsstrahlung mechanism. The point is that when $I > I^{sf}_{Compt}$ the initial stage of heating, so long as condition (14) is satisfied, is due entirely to the Compton mechanism. The rate of heating is then proportional to I^2, and for short-focus lenses at $I \gtrsim I^{sf}_{Compt}$, according to (15) and (19), amounts to approximately 10^{13} keV/sec and more. This means that in the region $I \gtrsim I^{sf}_{Compt}$ the heating of the electrons to energies on the order of $\epsilon_e(\tau, I^{sf}_{Compt})$ is due to the Compton mechanism, and furthermore it is quite fast (within a time $\lesssim 0.1\tau$). During the rest of the time, the heating is principally a result of the bremsstrahlung mechanism. This mechanism, however, turns out to be effective under the given conditions only if $\epsilon_e(\tau, I^{sf}_{Compt}) \gtrsim \epsilon_{vib}(I)$ or, since we have, according to (31), $\epsilon_e(\tau, I^{sf}_{Compt}) \approx \epsilon^{br}_{e\,max} \approx \epsilon_{vib}(I^*)$ for not too large Ω_0^{sf} (for example, $\lesssim 10^{-1}$ sr), when $I < I^*$. This condition follows from the solution (26), which describes the subsequent heating of the electrons due to the bremsstrahlung mechanism, if we substitute in it for the initial energy $\epsilon_e(0)$ the quantity $\epsilon_e(\tau, I^{sf}_{Compt}) \approx \epsilon_{vib}(I^*)$. When $I \gg I^*$, the value of the initial energy $\epsilon_{vib}(I^*)$ in (26) does not play any role and formula (26) leads to energy values $\epsilon_e(\tau, I), < \epsilon_{vib}(I)$, thus indicating that the bremsstrahlung heating mechanism is ineffective. To the contrary, when $I \lesssim I^*$, formula (26) leads to energy values $\epsilon_e(\tau, I) > \epsilon_{vib}(I^*) = \epsilon^{br}_{e\,max}$. The heating energy in the interval $I^{sf}_{Compt} < I \lesssim I^*$ can be roughly estimated, in accord with (26), by means of the formula

$$\varepsilon_e(\tau, I) \approx \frac{\varepsilon^{\tau}_{e\,max}(\tau)}{\xi}\{[(1+\xi)^{5/2} + 4.65\xi^{3/2}]^{2/5} - 1\}, \qquad (32)$$

which yields approximately $(1.5-2)\epsilon^{br}_{e\,max}(\tau)$ (see the figure, curve 3).

We thus arrive at the conclusion that when plasma electrons with critical density ($\omega_p \approx \omega$) are heated by picosecond laser pulses with intensity $I \ll I_{rel}$, the use of short-focus lenses does not make it possible to obtain a temperature $T_e = 2\epsilon_e/3$ larger than 1–1.5 keV. Higher temperatures can be attained only by using a superdense plasma ($\omega_p > \omega$), when the heating can occur in a thin skin layer. Then, according to (28), the maximum temperature is $(1-1.5)(\omega_p/\omega)^{4/3}$ keV. We emphasize once more that the maximum temperature is reached in this case only in a definite, rather narrow intensity interval $I^{sf}_{Compt} \lesssim I \lesssim I^*$ or under typical conditions at $\Omega_0^{sf} \approx 3 \times 10^{-2}$ sr, $2 \times 10^{15} \lesssim I \lesssim 8 \times 10^{15}$ W/cm^2. For $I \gg I^*$ ($\approx 8 \times 10^{15}$ W/cm^2) and up to "relativistic intensities," the heating temperature stays at an approximate level of 1 keV. In this region of values of I we always have $\epsilon_{vib} > \epsilon_e(\tau)$, and consequently the spectrum of the hard bremsstrahlung of the plasma corresponds to the vibrational energy of the electrons, and not to their temperature T_e.

b) Long-focus Lens ($\xi < 1$)

In this case $I_{Compt} > I^*$ and consequently the influence of the Compton mechanism occurs after the electron energy reaches its maximum value in the bremsstrahlung heating mechanism $\epsilon^{br}_{e\,max}(\tau)$. Deviations from the solution (26) begin at an approximate heating energy

$$\varepsilon_e(\tau, I^{lf}_{Compt}) = [(1+4{,}65\xi^{3/2})^{2/5} - 1]\,\varepsilon_{кол}(I^{lf}_{Compt}) = \frac{(1+4{,}65\xi^{3/2})^{2/5} - 1}{\xi}\varepsilon^{\tau}_{e\,max}(\tau). \qquad (33)$$

for $\Omega_0^{lf} = 6 \times 10^{-5}$ sr, we have $\xi \approx 0.33$ and this energy amounts to approximately $0.9\epsilon^{br}_{e\,max}(\tau) \approx 0.8$ keV.

In the intensity region $I > I^{lf}_{Compt}$, the main heating mechanism is the Compton effect, since condition (14) remains satisfied for long-focus lenses. In this range of I, the energy $\epsilon_e(\tau, I)$, according to (15) can be represented by the formula

$$\varepsilon_e(\tau, I) = \varepsilon_{vib}\frac{\omega\tau}{4}\frac{\Omega_0{}^{lf}\omega}{\Delta\omega}\frac{I}{I_{rel}}, \qquad (34)$$

and increases with I like I^2. However, for long-focus lenses this increase still remains slow, owing to the smallness of Ω_0^{lf}. Let us determine the intensity I^{**} at which the electron energy again becomes equal to $\epsilon_{vib}(I^{**})$. From (34) we get

$$I^{**} = \frac{\Delta\omega}{\omega\Omega_0{}^{lf}}\frac{4}{\omega\tau}I_{rel}. \qquad (35)$$

At $\Omega_0^{lf} = 6 \times 10^{-5}$ sr, this yields $I^{**} \approx 10^{17}$ W/cm^2 and $\epsilon_e(\tau, I^{**}) = \epsilon_{vib}(I^{**}) \approx 10$ keV (see the figure, curve 4).

Thus, the use of long-focus lenses makes it possible, in principle, to obtain electrons with higher temperatures ($T_e \approx 7$ keV) than in the case of short-focus lenses. For intensities $I < I^*$ and $I > I^{**}$, the spectrum of the hard bremsstrahlung of the plasma is determined by its electron temperature, and when $I^* < I < I^{**}$ it is determined by the vibrational energy ϵ_{vib}.

Let us point out the principal difficulties in obtaining high optical-radiation intensities when long-focuses lenses are used. The intensity I in the focal region is connected with the intensity I_l on the focusing lens (mirror) by the relation

$$I\Omega_l = I_l\Omega_0, \qquad (36)$$

where $\Omega_l = \pi(\varphi/2)^2$ is the solid angle of the laser beam incident on the focusing element (φ—total divergence angle of the beam). Independently of the total power of the employed laser, the intensity I_l is bounded from above by the limited endurance of the material of the focusing element. At picosecond durations τ, the limiting intensity is $I_l^{lim} \sim 10^{12}-10^{13}$ W/cm^2. On the other hand, the solid angle Ω_l is bounded from below by the diffraction divergence of the laser beam. At $\lambda \sim 1\,\mu$ and at a laser output aperture d = 5 cm, the diffraction angle is $\Omega_l^{dif} = 3 \times 10^{-10}$ sr. Thus, the maximum radiation intensity that can be reached in the focus is $\sim (\Omega_0/\Omega_l^{dif})I_l^{lim}$, which yields $2\,(10^{17}-10^{18})$ W/cm^2 at $\Omega_0 \approx 6 \times 10^{-5}$ sr. Under experimental conditions, however, such intensities will not be reached in practice with long-focus lenses in the nearest future, since the real divergences of the laser beams greatly exceed 3×10^{-10} sr.

3. CASE OF RELATIVISTIC INTENSITIES

This case corresponds to such high intensities that the electron energy in the field of the wave becomes comparable with its rest energy. For example, for a circularly-polarized plane wave the total energy of an electron initially at rest is[10]

$$\varepsilon = mc^2(1 + {}^1/_2 I / I_{\text{rel}}) = mc^2 + \varepsilon_{\text{vib}} \qquad (37)$$

When such strong radiation interacts with free electrons, several photons are absorbed immediately from the laser field in each elementary act, i.e., the processes acquire a many-quantum character. (For the bremsstrahlung effect the many-quantum character becomes manifest much sooner, when $\epsilon_{\text{vib}} \gtrsim T_e$.[7])

For $I \gtrsim I_{\text{rel}}$, the bremsstrahlung heating mechanism apparently is of no significance, since, according to (10), $\nu_{\text{eff}}(I)$ decreases rapidly with I already in the nonrelativistic region, and according to the results of [11] it continues to decrease also in the relativistic region. Leading to the heating of the electrons in this region of I is the (many-quantum) spontaneous Compton effect[5, 10] (in contrast to the non-relativistic region, where it is always inverse when $\hbar\omega < T_e$; see (11)). However, a computer calculation of this process[12] has shown that at $I = I_{\text{rel}}$, $10 I_{\text{rel}}$ and $10^2 I_{\text{rel}}$ the electron gas is heated in one picosecond pulse to temperatures $T_e = 1$, 6×10, and 2×10^3 eV, respectively, i.e., much below the vibrational energy $\epsilon_{\text{vib}}(I)$.

If this heating mechanism is the dominant one in the indicated intensity region, then the spectrum of the hard plasma bremsstrahlung will be determined during the time of action of the pulse by the energy ϵ_{vib}, in accordance with (37), and after the termination of the pulse by a much lower energy, corresponding to the temperature T_e. It follows from (37) that when $I \geq 2 I_{\text{rel}}$ the bremsstrahlung spectrum of the electrons contains hard quanta with energy $\geq mc^2$. Calculation shows that in the case of neodymium-laser radiation focused into a volume $\sim 10^{-7}$ cm^3 of a plasma with density $n_e = n_i = 10^{21}$ cm^{-3} and $Z = 1$, there will be radiated in one picosecond pulse, as a result of the bremsstrahlung mechanism, 6×10^6, 0.8×10^8, and 5×10^8 hard quanta at I/I_{rel} equal to 4, 2.5×10, and 10^2, respectively.

There are grounds for assuming, however, that at intensities $\geq I_{\text{rel}}$ an appreciable role in the plasma heating will be played by the stimulated (many-quantum) Compton effect. This circumstance may alter the conclusions both concerning the heating of the electrons in this intensity region and concerning the character of the hard γ radiation.

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