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MOTION OF PARTICLES INTERACTING WITH PHONONS AT FINITE TEMPERATURES AND IN STRONG FIELDS

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The magnitude of a current of ions interacting with phonons is calculated for the case of strong fields when the existence of a threshold for decay of an ion excitation into two is important, and at low temperatures, when scattering of ions by acoustic phonons is predominant. It is shown that, owing to the presence of a decay threshold, the dependence of the ion current on the field strength possesses a vertical tangent when the field strength is equal to the maximum friction force acting on the ion.

PARTICLES of small concentration (henceforth called ions) interacting with the phonon field at a temperature of absolute zero were considered previously by one of the authors.[1,2] The purpose of the present work is the extension of this model to finite temperatures, with account taken of the singularities in the spectrum of the ions produced by the presence of a terminal point, because of the decay of the ionic excitations into two such excitations.[1-3] We shall assume that the temperature is sufficiently small that the damping of the ionic excitations is small in comparison with their energies. So far as the electric field accelerating the ions is concerned, we shall assume that it is small in comparison with the atomic fields, but sufficiently large that, in spite of the presence of collisions with thermal phonons, it can accelerate the ions to values of the momentum $p \sim p_c$, where p_c is the threshold momentum for decay into two excitations. The temperature is assumed to be so small that only acoustic phonons are excited, the scattering by which determines the retarding force acting on the ion. As in [1] we shall assume for simplicity the phonons to be harmonic, and disregard their interaction with one another.

1. DERIVATION OF THE FOKKER-PLANCK EQUATION FOR IONS AT FINITE TEMPERATURES

It was shown in [1] that for p not too close to p_c, one should have a kinetic equation for the quasiparticle distribution function f(p), which is defined as the coefficient of $2\pi\delta(\omega - \epsilon_0(p) - \Sigma(p, \omega))$ in the expression for the Fourier transform of the ionic correlation function $G^+(x, x') = \langle\psi^+(x)\psi(x')\rangle$ ($\psi^+(x)$ and $\psi(x)$ are the creation and annihilation operators of the ions at the point x = (r, t)). We have introduced here $\epsilon_0(p)$—the energy of the free ions, and $\Sigma(p, \omega)$—the mass operator of the single-particle ion Green's function G. This equation has the form

$$\mathbf{E}\frac{\partial f}{\partial \mathbf{p}} = \left(1 - \frac{\partial \Sigma}{\partial \omega}\right)^{-1}_{\mathbf{p},\,\varepsilon(p)} \Big\{2f(\mathbf{p})\,\mathrm{Im}\,\Sigma(\mathbf{p}, \varepsilon(p)) - \int \frac{d^3q\,d\omega'}{(2\pi)^4}\gamma(\mathbf{p}, \varepsilon(p); \mathbf{q}, \omega') f(\mathbf{p}+\mathbf{q})\, 2\pi\delta(\varepsilon(p) + \omega' - \varepsilon_0(\mathbf{p}+\mathbf{q}) - \Sigma(\varepsilon(p)+\omega', \mathbf{p}+\mathbf{q}))\Big\}. \quad (1.1)$$

Here γ denotes the complete vertex of the interaction of two ions.

The right side is different from zero because of the presence of real phonons, which lead to the appearance of Im Σ for $p < p_c$ and to a non-zero value of the integral with γ on the trajectory of the pole $\omega = \epsilon(p)$ of the single-particle ion Green's function. Here, because of the smallness of the temperature, we can take into account the absorption of only a single acoustic phonon. This circumstance, however, makes it necessary to consider also the emission of a single acoustic phonon, since the integration over the phase volume of each phonon is limited to a region $\sim T^3$ (the energy of the emitted phonon is of the order of the energy of the absorbed phonon $\sim T$). For this reason, we must restrict ourselves in the calculation of Im Σ to the diagram shown in Fig. 1, where the shaded square denotes the complete vertex of interaction of the ion with the phonon, the crossed dashed line corresponds to the absorption of a real phonon and the uncrossed to emission. In γ, one is similarly restructed to the diagram of Fig. 2.

Since the vertex for emission or absorption of the acoustic phonon should vanish for a momentum of the phonon approaching zero, then, in contrast with [1], we shall assume that

$$D(\mathbf{q}, t) = \langle\varphi_\mathbf{q}(t)\varphi_{-\mathbf{q}}(0)\rangle = \frac{\omega(q)}{2}[n(q)e^{i\omega(q)t} + (1+n(q))e^{-i\omega(q)t}],$$
$$n(q) = [e^{\omega(q)/T} - 1]^{-1}, \quad (1.2)$$

as this is usually written, for example, for the Fröhlich model[4] (the factor $\omega(q)$ actually appears from the bare ion-phonon vertex). In such a determination of D, we can assume the bare vertex g to be constant, just as in [1]. Further simplification is associated with the fact that, in the calculation of the vertices, the momentum of the external phonons is small in comparison with the

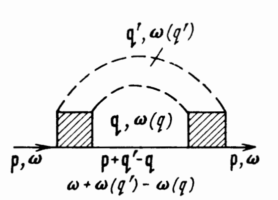

FIG. 1

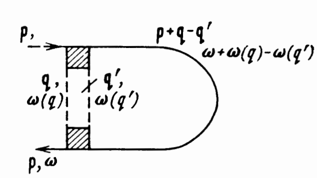

FIG. 2

characteristic momentum of the internal lines, which allows us to use the Ward identities.

We express the complete vertex of the interaction of the ion with a phonon in terms of the complete compact vertices (see Fig. 3):

$$\Gamma(\pi,\chi,\chi') = \Gamma_3(\pi,\pi-\chi)G(\pi-\chi)\Gamma_3(\pi-\chi,\pi-\chi+\chi') + \Gamma_3(\pi,\pi+\chi')G(\pi+\chi')\Gamma_3(\pi+\chi',\pi-\chi+\chi') + \Gamma_4(\pi,\chi,\chi'),$$
$$\pi = (\mathbf{p},\omega), \quad \chi = (\mathbf{q},\omega(q)). \tag{1.3}$$

Equation (1.3) must be known in zeroth order in χ and χ'. Using the Ward identity, we immediately obtain

$$\Gamma_4(\pi,\chi,\chi') = g^2 \frac{\partial^2\Sigma(\mathbf{p},\omega)}{\partial\omega^2}\Big|_{\omega=\varepsilon(p)}. \tag{1.4}$$

Since the Green's functions in (1.3) are considered close to the pole (χ and χ' small, $\omega = \epsilon(p)$), it is necessary to know Γ_3 with accuracy up to χ and χ', inclusively. Using the symmetry $\Gamma_3(\pi, \pi-\chi) = \Gamma_3(\pi-\chi, \pi)$, we can write

$$\Gamma_3(\pi,\pi-\chi) \approx \Gamma_3(\pi,\pi) + \frac{1}{2}\frac{\partial\Gamma_3(\pi,\pi)}{\partial\pi}(-\chi)$$
$$\approx g\left(1-\frac{\partial\Sigma}{\partial\omega}\right)^{1/2}_{\mathbf{p},\,\varepsilon(p)}\left(1-\frac{\partial\Sigma}{\partial\omega}\right)^{1/2}_{\mathbf{p}-\mathbf{q},\,\varepsilon(p)-\omega(q)}. \tag{1.5}$$

where we used again the Ward identity. Near the pole, G(p, ω) has the form

$$G(\mathbf{p},\omega) = \frac{(1-\partial\Sigma/\partial\omega)^{-1}_{\mathbf{p},\,\omega}}{\omega-\varepsilon(p)} + \frac{1}{2}\left(1-\frac{\partial\Sigma}{\partial\omega}\right)^{-2}_{\mathbf{p},\,\omega}\frac{\partial^2\Sigma}{\partial\omega^2}\Big|_{\mathbf{p},\,\omega}. \tag{1.6}$$

If we now take it into account that Im Σ appears only when the internal line in the diagram of Fig. 1 is replaced by

$$\operatorname{Im} G = -\pi\left(1-\frac{\partial\Sigma}{\partial\omega}\right)^{-1}_{\mathbf{p},\,\varepsilon(p)}\delta(\varepsilon(p)+\omega(q')-\omega(q)-\varepsilon(\mathbf{p}+\mathbf{q}'-\mathbf{q})),$$

then we get, for the values of **p**, **q**, and **q**$'$ causing the argument of the δ function to vanish, by substituting (1.4)–(1.6) in (1.3),

$$\Gamma(\mathbf{p},\mathbf{q},\mathbf{q}') = g^2\left(1-\frac{\partial\Sigma}{\partial\omega}\right)_{\mathbf{p},\,\varepsilon(p)}\left(\omega(q')-\frac{\partial\varepsilon}{\partial\mathbf{p}}\mathbf{q}'\right)^{-1}\left(\omega(q)-\frac{\partial\varepsilon}{\partial\mathbf{p}}\mathbf{q}\right)^{-1}$$
$$\times\frac{\partial^2\varepsilon}{\partial p_i\,\partial p_k}q_i q_k'. \tag{1.7}$$

In this expression, terms of higher order in q and q$'$ are discarded.

By using Eq. (1.7), it is not difficult to show that the kinetic equation (1.1) has the following form:

$$\mathbf{E}\frac{\partial f}{\partial\mathbf{p}} = -\int\frac{d^3q\,d^3q'}{(2\pi)^6}A^2(\mathbf{p},\mathbf{q},\mathbf{q}')\,[n(q')(1+n(q))f(p)$$
$$-n(q)(1+n(q'))f(\mathbf{p}+\mathbf{q}'-\mathbf{q})]\,2\pi\delta(\varepsilon(p)+\omega(q')$$
$$-\omega(q)-\varepsilon(\mathbf{p}+\mathbf{q}'-\mathbf{q})), \tag{1.8}$$

where the total phonon-ion, A(**p**, **q**, **q**$'$), scattering amplitude is equal to

$$A(\mathbf{p},\mathbf{q},\mathbf{q}') = \frac{1}{2}g^2c(qq')^{1/2}\left(cq-\frac{\partial\varepsilon}{\partial\mathbf{p}}\mathbf{q}\right)^{-1}\left(cq'-\frac{\partial\varepsilon}{\partial\mathbf{p}}\mathbf{q}'\right)^{-1}\frac{\partial^2\varepsilon}{\partial p_i\,\partial p_k}q_i q_k', \tag{1.9}$$

c is the sound velocity. The result (1.9) for the scattering amplitude corresponds to consideration of only diagrams with bare vertices with simultaneous replacement of the G lines by the purely polar part $\tilde{G} = 1/(\omega - \epsilon(p))$, with a residue equal to unity. Thus, the long-wave phonons are scattered by dressed particles in exactly the same way as is given by the formulas of second order perturbation theory. A similar result is well known in quantum electrodynamics for the scattering of long-wave photons by an electron (see, for example, [5]). The same result was obtained for foreign particles in He II at low particle momenta.[6]

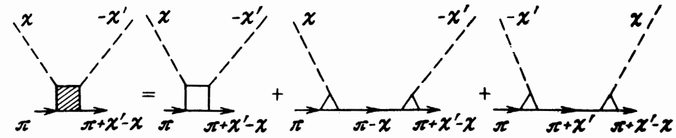

FIG. 3

For the momenta $p \sim p_c$ of interest to us, the condition $p \gg q \sim T$ is satisfied. Therefore, the kinetic equation (1.8) goes over into the Fokker-Planck equation (see, for example, [7])

$$\frac{\partial}{\partial\mathbf{p}}\left[\left(\mathbf{E}-\frac{1}{\mu(p)}\frac{\partial\varepsilon}{\partial\mathbf{p}}\right)f(\mathbf{p})-\frac{T}{\mu(p)}\frac{\partial f(\mathbf{p})}{\partial\mathbf{p}}\right]=0, \tag{1.10}$$

where $T/\mu(p)$, in accord with (1.8), is given by the expression

$$\frac{T}{\mu(p)} = \frac{1}{6}\int\frac{d^3q\,d^3q'}{(2\pi)^6}A^2(\mathbf{p},\mathbf{q},\mathbf{q}')\,n(q')(1+n(q))$$
$$\times 2\pi(\mathbf{q}-\mathbf{q}')^2\delta(\varepsilon(p)+\omega(q')-\omega(q)-\varepsilon(\mathbf{p}+\mathbf{q}'-\mathbf{q})). \tag{1.11}$$

In the particular case $\partial\epsilon/\partial p \ll c$, this expression gives the following formula for $\mu(p)$:

$$\mu^{-1}(p) = \frac{g^4T^8}{c^{12}}\frac{(2\pi)^5}{3^4\cdot 20}\left[\left(\frac{\partial^2\varepsilon}{\partial p^2}\right)^2+\frac{2}{p^2}\left(\frac{\partial\varepsilon}{\partial p}\right)^2\right]. \tag{1.12}$$

It must be noted that the kinetic equation (1.1) for the conditions $p \gg q$, q' always goes over into the Fokker-Planck equation, although the scattering can also take place from optical phonons (if the temperatures are sufficiently high) or from impurities. Here, however, $\mu(p)$ will obviously not be so simply connected with the ion spectrum as in formula (1.12).

2. SOLUTION OF THE FOKKER-PLANCK EQUATIONS AND CALCULATION OF THE CURRENT

Equation (1.10) is valid for the distribution function of ionic quasiparticles far from the points where the decay process is important, i.e., the process of spontaneous radiation of an optical phonon, in accord with the conservation law

$$\varepsilon(p) = \varepsilon(\mathbf{p}-\mathbf{q}) + \omega(q). \tag{2.1}$$

Near p_c, the ion, absorbing the existing acoustic phonon, can fall into the region

$$\varepsilon(p) + cq' \geqslant \tilde{\varepsilon}(\mathbf{p}+\mathbf{q}'), \tag{2.2}$$

where $\tilde{\epsilon}(p)$ is the minimum of the right side of (2.1). According to [3], near p_c, we have $\tilde{\epsilon}(p) - \epsilon(p) \approx a(p_c - p)^2$, whence it follows that the radiation of optical phonons becomes important for $p_c - p \sim T^{1/2}$. In exactly the same way, near the point $p = p_m$, to which the ions fall as a result of the radiation of an optical phonon, there is a region with the dimensions $|p - p_m| \sim T^{1/2}$ where the arrival from the vicinity of $p = p_c$ is important. In these special regions, the Fokker-Planck equation is inapplicable. However, we can take into account the presence of these regions with the help of the boundary condition for the Fokker-Planck equation. At low temperatures, the dimensions of these regions are small. Therefore, they can be replaced by an absorbing wall for $p = p_c$ and by a source of particles for $p = p_m$. The number of particles absorbed by the wall per unit time and created by the source at p_m can be assumed to be proportional to the distribution function at $p = p_c$. Thus, for the particle flux in momentum space

$$\mathbf{J}(\mathbf{p}) = \left(\mathbf{E} - \frac{1}{\mu(p)}\frac{\partial\varepsilon}{\partial\mathbf{p}} - \frac{T}{\mu(p)}\frac{\partial}{\partial\mathbf{p}}\right) f(\mathbf{p}) \tag{2.3}$$

the following conditions hold:

$$J = 0 \quad \text{for} \quad p < p_m, \quad p > p_c,$$
$$\mathbf{J}(\mathbf{p})\frac{\mathbf{p}}{p}\bigg|_{p=p_c-0} = \mathbf{J}(\mathbf{p})\frac{\mathbf{p}}{p}\bigg|_{p=p_m+0}\left(\frac{p_m}{p_c}\right)^2 = \alpha f(\mathbf{p}_c). \tag{2.4}$$

Furthermore, in the range of fields of interest to us

$$E \sim \frac{1}{\mu(p)}\frac{\partial\varepsilon}{\partial p}\bigg|_{p\sim p_c}$$

the problem can be reduced to a one-dimensional one. For this, we note that the form of the distribution function for $p_\perp \perp \mathbf{E}$ is determined by the processes in the special regions and by diffusion, in accord with the Fokker-Planck equation. It is easy to see that the scatter with respect to $p_\perp$, due to the processes in the special regions, is of the order of $T^{1/2}$, since this is the value of the scatter of the momentum of the radiated optical phonons. Diffusion in the perpendicular direction does not change the value of this scatter. Thus $p_\perp \ll p_x \parallel \mathbf{E}$, and we can integrate Eq. (1.10) over $d^2p_\perp$, assuming that all the quantities depend only on p_x. Using the condition (2.4), we get

$$\left(E - \frac{1}{\mu(p_x)}\frac{\partial\varepsilon}{\partial p_x} - \frac{T}{\mu(p_x)}\frac{\partial}{\partial p_x}\right)\tilde{f}(p_x) = \alpha\tilde{f}(p_c)\Theta(p_x - p_m), \tag{2.5}$$

where Θ is the step function, and

$$\tilde{f}(p_x) = \int f(p_x, \mathbf{p}_\perp)\frac{d^2p_\perp}{(2\pi)^2}, \quad \tilde{f}(p_x > p_c) = 0.$$

The solution of Eq. (2.5) has the following form:

$$\tilde{f}(p_x) = \tilde{f}(p_c)e^{-\varphi(p_x)}\left(1 - \frac{\alpha E}{T}\int_{p_c}^{p_x} e^{\varphi(p)}\mu(p)\Theta(p - p_m)\,dp\right). \tag{2.6}$$

where

$$\varphi(p) = \frac{\varepsilon(p) - \varepsilon(p_c)}{T} - \frac{E}{T}\int_{p_c}^{p}\mu(p')\,dp'.$$

Equation (2.6) allows us to consider the change in the distribution function with increase in the field, and to compute the current. The function φ has an extremum at the points where

$$F(p) \equiv \frac{1}{\mu(p)}\frac{\partial\varepsilon}{\partial p} = E.$$

For sufficiently small E, there is only a single point p(E) corresponding to minimum φ. With increasing E, the value of p(E) increases. However, it can be established that F(p) has a maximum as a function of p. Actually we have from (1.12)

$$\frac{\partial F}{\partial p} \equiv \frac{\partial}{\partial p}\left(\frac{1}{\mu(p)}\frac{\partial\varepsilon}{\partial p}\right) = \text{const}\cdot\left[\left(\frac{\partial^2\varepsilon}{\partial p^2}\right)^3 + \frac{\partial^3\varepsilon}{\partial p^3}\frac{\partial^2\varepsilon}{\partial p^2}\frac{\partial\varepsilon}{\partial p} + \frac{6}{p^2}\left(\frac{\partial\varepsilon}{\partial p}\right)^2\frac{\partial^2\varepsilon}{\partial p^2} - \frac{4}{p^3}\left(\frac{\partial\varepsilon}{\partial p}\right)^3\right]. \tag{2.7}$$

According to [3], $\partial\epsilon/\partial p\,|_{p_c} = \partial\epsilon/\partial p\,|_{p_m}$; consequently, $\partial\epsilon/\partial p$ has a maximum at some point p^* between p_m and p_c. But then at the point $p = p^*$ we have $|\partial F/\partial p|_{p^*} < 0$, while for small p we have $\partial F/\partial p > 0$. Thus, the friction force F(p) acting on the ion reaches a maximum for $p_1 < p^* < p_c$. It then follows that $\varphi(p)$ has a single minimum for $E < E_1$ ($E_1 = F(p_1)$), which changes to a point of inflection for $E = E_1$. Since the distribution function is concentrated near the minimum of φ, then, for $E < E_1$, the current per particle is equal to

$$j(E) = \partial\varepsilon/\partial p\,|_{p=p(E)}. \tag{2.8}$$

Near the critical field E_1, we can expand the current into $p_1 - p(E)$ and obtain the following result:

$$j(E) = \frac{\partial\varepsilon}{\partial p}\bigg|_{p_1} + \frac{\partial^2\varepsilon}{\partial p^2}\bigg|_{p_1}(p(E) - p_1) = j(E_1) - (E_1 - E)^{1/2}\frac{\partial^2\varepsilon}{\partial p^2}\bigg|_{p_1}\left(-\frac{1}{2}\frac{\partial^2}{\partial p^2}F(p)\right)_{p_1}^{-1/2}. \tag{2.9}$$

For $E = E_1$, there is a discontinuity in the solution. The function $\varphi(p)$ becomes monotonic and the principal contribution to the integral (2.6) is made in the vicinity of the point p_c. Discarding the exponentially small terms, we get the following expression for $\tilde{f}$ in the range $E > E_1$:

$$\tilde{f}(p) = \text{const}\cdot(E - F(p))^{-1}. \tag{2.10}$$

For E close to E_1, the function $\tilde{f}(p)$ is concentrated near p_1 as before. For the calculation of the dependence of the current on the field in this case, we write the current in the following form:

$$j(E) = \frac{\partial\varepsilon}{\partial p}\bigg|_{p_1} + \left(\int\tilde{f}(p)\,dp\right)^{-1}\int\left(\frac{\partial\varepsilon}{\partial p} - \frac{\partial\varepsilon}{\partial p}\bigg|_{p_1}\right)\tilde{f}(p)\,dp.$$

It is easy to see that the numerator of the second term is determined by the region to the left of p_1, where f (p) can be considered independent of $E - E_1$, while the denominator is determined by the region $|p - p_1| \sim (E - E_1)^{1/2}$, where $\tilde{f}(p)$ is large ($\tilde{f}(p) \sim 1/(E - E_1)$). Therefore the current is equal to

$$j(E) = j(E_1) + \text{const}\cdot(E - E_1)^{1/2}. \tag{2.11}$$

The sign of the coefficient in front of the square root cannot be determined from general considerations, since it depends on the specific form of the spectrum $\epsilon(p)$. Therefore, the form of j(E) near the critical field E_1 can be that of either Fig. 4a or Fig. 4b. With further increase of the field, the solution $\tilde{f}$ approaches a constant, and for $E \gg F(p_1)$ the current approaches

$$j_0 = \frac{\varepsilon(p_c) - \varepsilon(p_m)}{p_c - p_m}. \tag{2.12}$$

However, generally speaking, it is not possible to say whether this approach will be monotonic or not.

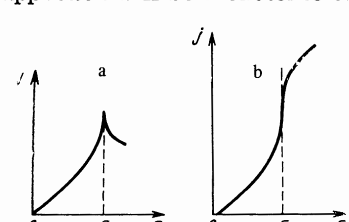

FIG. 4

Finally, we must ascertain the field values for which the quantum effects associated with the decay threshold become important. These effects lead to a nonanalytic contribution $\delta j \sim E^{1/3}$ to the current j_0.[2] For this, we consider in more detail those processes which take place in the special region $p_c - p \sim T^{1/2}$. In accord with the results of [2], the kinetic equation can be used in the region $p_c - p \gg E^{1/3}$. Assuming the condition $E \gg T^{3/2}$ to be satisfied, we write down the kinetic equation (1.1) in the following form:

$$\frac{\partial}{\partial \mathbf{p}}\left[\left(\mathbf{E}-\frac{1}{\mu(p)}\frac{\partial \varepsilon}{\partial \mathbf{p}}\right)f-\frac{T}{\mu(p)}\frac{\partial f}{\partial \mathbf{p}}\right]=-\frac{f(p)}{\tau_{0c}(p)}, \quad (2.13)$$

where

$$\frac{1}{\tau_{0c}(p)}=\frac{2\operatorname{Im}\Sigma_{0c}}{1-\partial\Sigma/\partial\omega}\bigg|_{\mathbf{p},\,\varepsilon(p)}$$

results from the process of absorption of an acoustic and the emission of an optical phonon. The principal contribution to $\operatorname{Im}\Sigma_{0c}$ is made by the diagram of Fig. 5 (the optical phonon is indicated by the wavy line), since, besides the singularity in the vertex $\Gamma_3 \approx g(1 - \partial\Sigma/\partial\omega)_{p,\epsilon(p)}$, it contains the square of the Green's function $G(p + q')$ near the pole (q' small). If we use the fact that

$$\frac{\partial\Sigma}{\partial\omega}\bigg|_{\mathbf{p}}\sim\frac{1}{p-p_c}\sim T^{-1/2},\quad G(\mathbf{p}+\mathbf{q}')\sim T^{-1}\left(1-\frac{\partial\Sigma}{\partial\omega}\right)^{-1}_{\mathbf{p}},$$

$$\frac{G^{-1}(\mathbf{p}-\mathbf{q})}{(1-\partial\Sigma/\partial\omega)_{\mathbf{p}_m}}\approx\varepsilon(p)-\varepsilon(\mathbf{p}-\mathbf{q})-\omega(q)\approx\varepsilon(p)-\tilde{\varepsilon}(p)-\beta(\mathbf{q}-\mathbf{p}_c+\mathbf{p}_m)^2,$$

then it is not difficult to show that this diagram gives

$$\frac{1}{\tau_{0c}(p)}\sim T^3\exp\left\{-\frac{a(p_c-p)^2}{T}\right\}. \quad (2.14)$$

We now estimate the rate of decay of the distribution function in the special region due to this process. For this, we consider the range of fields $E \gg T^6$. For these fields, we can retain in the left side of (2.13), in the special region, only the term $\mathbf{E}\,\partial f/\partial \mathbf{p}$. Then the characteristic momentum at which $f(p)$ essentially falls into the special region, $\Delta p \sim E\tau_{0c} \sim E/T^3$. If Δp is smaller than the dimension of the special region $T^{1/2}$, i.e., $E < T^{1/2}$, then the damping of the distribution function is completely accounted for by the process of absorption of an acoustic phonon with emission of an optical phonon. The particle flux $\alpha\bar{f}(p_c)$ into the absorbing wall (see (2.5)) is then expressed in terms of the exact distribution function

$$\alpha\tilde{f}(p_c)=\int\frac{1}{\tau_{0c}(p)}f(\mathbf{p})\frac{d^3p}{(2\pi)^2}.$$

For $E > T^{7/2}$, this process is no longer able to assure the damping of $f(p)$. Therefore, the damping takes place as a result of the spontaneous emission of an optical phonon in the range $p_c - p \sim E^{1/3}$. Consequently, the results of [2] are applicable in the region of fields $E > T^{1/2}$.

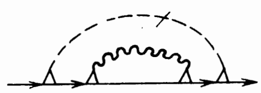

FIG. 5

3. DISCUSSION OF RESULTS AND POSSIBLE GENERALIZATIONS

The present model can be used for an estimate of the behavior of ions in He II in that temperature range in which the scattering by phonons is predominant.[1)] To find the vertex of interaction of ions with phonons, one can use the results and the methods of Saam,[6] in which the validity of the phenomenological description of the interaction of the ion with phonons within the framework of the quantum hydrodynamics is proved[9,10] for such momenta of the ion for which its spectrum can be assumed to be quadratic. In our case, the ion spectrum is essentially nonquadratic, in view of the presence of the decay threshold. However, by repeating the calculations of [6] with account of the nonquadratic nature of the spectrum, we can see that in this case also the phenomenological description is valid if only the condition $\partial\epsilon/\partial p \ll c$ is satisfied. For the calculation of the interaction vertex of the ion with the phonon, it is necessary to write the initial formula (9) in [9] for the transformation of the energy of the ion in the moving He II in the following form

$$\varepsilon_{\mathbf{p}}[\mathbf{v}_s]=\varepsilon(\mathbf{p}-m\mathbf{v}_s)+(\mathbf{p}-m\mathbf{v}_s)\mathbf{v}_s+m\mathbf{v}_s^2/2\approx\varepsilon(\mathbf{p})+\left(\mathbf{p}-m\frac{\partial\varepsilon}{\partial\mathbf{p}}\right)\mathbf{v}_s-\frac{m}{2}\left(v_s^2-m\frac{\partial^2\varepsilon}{\partial p_i\partial p_k}v_{si}v_{sk}\right), \quad (3.1)$$

where m is the mass of the bare ion and v_s the superfluid velocity of He II.

The phonon-ion scattering amplitude obtained with the help of (3.1) differs from formula (24) in [9] and has the following form:

$$A(\mathbf{p},\mathbf{q},\mathbf{q}')=\frac{c(qq')^{1/2}}{2n_4}\left[n_4\frac{da}{dn_4}+m_4\frac{\partial^2\varepsilon}{\partial p_i\partial p_k}\frac{q_iq_k'}{qq'}\times\left(1+a-\frac{m}{m_4}\right)^2+\frac{\mathbf{q}\mathbf{q}'}{qq'}\left(1+a-\frac{m}{m_4}\right)\right], \quad (3.2)$$

where m_4 is the mass of He^4, n_4 the density of He II,

$$(a+1)\frac{m_4c^2}{n_4}=\frac{\partial\varepsilon(p,n_4)}{\partial n_4}\bigg|_{p=0}.$$

The friction force acting on the ion is then (see (1.11))

$$F(p)=\text{const}\cdot\frac{\partial\varepsilon}{\partial p}\left\{\left[m_4\left(a+1-\frac{m}{m_4}\right)\frac{\partial^2\varepsilon}{\partial p^2}+a+1-\frac{m}{m_4}\right]^2+2\left[a+1-\frac{m}{m_4}+m_4\left(a+1-\frac{m}{m_4}\right)^2\frac{1}{p}\frac{\partial\varepsilon}{\partial p}\right]^2+3n_4\frac{da}{dn_4}\right\}. \quad (3.3)$$

It is not difficult to see that the friction force has a maximum at some point $p_1 < p^*$ (p^* corresponds to the maximum of the ion velocity). Actually,

$$\frac{\partial F}{\partial p}\bigg|_{p^*}=\text{const}\cdot\left\{\frac{\partial\varepsilon}{\partial p}\frac{\partial^3\varepsilon}{\partial p^3}-\left[2+m_4\left(a+1-\frac{m}{m_4}\right)^2\times\frac{1}{p}\frac{\partial\varepsilon}{\partial p}\right]\frac{1}{p^2}\left(\frac{\partial\varepsilon}{\partial p}\right)^2\right\}_{p=p^*}<0,$$

since $\partial^3\epsilon/\partial p^3|_{p^*} < 0$ at the point of maximum velocity. Therefore, the qualitative considerations of Sec. 2 concerning the j(E) dependence remain valid for ions in He II, provided the critical velocity of the ion is small in comparison with the sound velocity.

According to the experimental data,[11-15] at low tem-

[1)] In He II, this temperature range lies between 0.45 and 0.55°. The lower limit is connected with the presence of impurities of He^3 (see [6]).

peratures the ion can move together with the vortex ring in He II, forming a bound state—a charged vortex ring. Two critical fields are observed[16] at which the ion current changes materially. It is assumed[16] that the lower field corresponds to the acceleration of charged vortex rings already existing in the He II (rings formed, for example, by the ion source). The higher field is assumed to correspond to the beginning of vortex ring formation in the motion of the ion in the field.

There is no known experimental ion spectrum in He II in the intermediate momentum region. It could be assumed that there is a continuous curve $\epsilon(p)$ which goes over from the parabolic law $p^2/2m^*$ to the dispersion law of the vortex ring $\epsilon \sim p^{1/2}$;[11] it must be assumed that the decay conditions are not satisfied anywhere. In this case, the velocity $\partial\epsilon/\partial p$ should also have a maximum at some point $p = p^*$. As we have shown earlier, the phonon friction force has a maximum in this case for some $p_1 < p^*$. Since the friction force for the vortex ring increases with increase in p like ln p,[11] the friction force F should have a minimum for some $p_2 > p^*$. Because of our assumption of the nonsatisfaction of the decay conditions, the Fokker-Planck equation (1.10) will be valid for any p, and therefore the form of the solution (1.10) in fields larger than E_1 will correspond to the new equilibrium position for $p > p_3$ (see Fig. 6).

Thus the current j(E) will change by a jump, and the jump will be negative because of the small velocity of the vortex rings, as is shown in Fig. 7. The dashed line in Fig. 7 corresponds to the acceleration of the already existing bare vortex rings. However, upon increase in the temperature, either the extrema of the curve F are preserved, and in this case the current j(E) will, as before, have a vertical tangent dj/dE, or they dissolve and disappear, in which case the current j(E) will have no singularities. Both these possibilities contradict the existing experimental data for j(E) at proton temperatures (for $T \sim 0.7°$, the current j(E) has a horizontal tangent before the jump). Therefore, the assumption as to the continuous nature of the spectrum $\epsilon(p)$ must be discarded.

On the basis of these considerations, it can be asserted that the experimental discovery of critical points of the type E_1 (see Fig. 4) on the curve j(E) at low temperatures (when scattering from acoustic phonons predominates) would indicate the presence of a termination point of the ion spectrum.

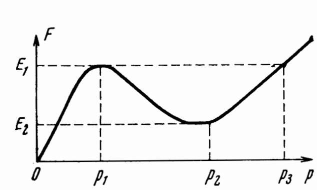

FIG. 6

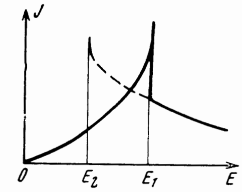

FIG. 7

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250