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EFFECT OF IMPURITIES ON THE SPONTANEOUS MAGNETIZATION OF A TRANSFORMED ISING LATTICE

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A formula is obtained for the spontaneous magnetization $m_c(T)$ in an exactly-solvable two-dimensional transformed Ising model containing impurities which are in thermodynamic equilibrium with the lattice. It is shown that for $I_1 < 0$ (I_1 is the new exchange integral, $|I_1| \ll I$) an anomalous decrease of the spontaneous moment ($dm_c/dT > 0$) occurs in the limit $T \to 0$, where this decrease is associated with the formation of complexes of "reversed" spins.

1. Kosevich and the author[1] formulated a simple dislocation model of ferromagnetism in nonmagnetic crystals, in which the ferromagnetism is due to the exchange interaction of electron spins along the dislocation lines. Here the two-dimensional Ising model was used, and it was shown that the investigation of the phase transition and spontaneous magnetization of such a system reduces to a problem first solved by Onsager[2] and Yang.[3] However in contrast to[2,3], in[1] certain effective quantities depending on the temperature T and on the external magnetic field H play the role of the exchange integral I and the magnetic moment μ of a site, i.e., the model considered in[1] is a special case of the so-called "transformed" Ising lattice (see, for example,[4]). Pokrovskiĭ[5] has pointed out that electrons settled on the dislocations may lead to a breaking of the exchange interaction in the corresponding spin chains. In terms of the model discussed in[1], this means that the effective exchange integral I is replaced by a new value I_1 (in particular, $I_1 = 0$ for complete breaking of the exchange interaction), and the effective magnetic moment μ of a site is replaced by a new value μ_1.

We shall assume that one can qualitatively discuss the question of the effect of impurities on the spontaneous magnetization in the dislocation model of ferromagnetism, having first considered the following model problem which has an exact solution in the two-dimensional case (for an arbitrary concentration of impurities). There exists a "transformed" (in the sense indicated above) Ising lattice containing impurities which, by intruding between neighboring sites, change I to I_1 and μ to μ_1. In order to solve this problem we shall use the method and results of Lushnikov's work[6], in which a similar problem was solved in the absence of any magnetic field.

2. The partition function of a pure lattice in a magnetic field can obviously be written in the form

$$Z = \sum_r x^{N-r} y^r \sum_m g(R, m) f^{R-m} t^m \equiv \sum x^{N-r} y^r f^R g_R\left(\frac{t}{f}\right),$$

$$g_R(z) \equiv \sum g(R, m) z^m, \tag{1}$$

where $x = y^{-1} = \exp(\beta I)$ (for a ferromagnetic substance $I > 0$), $f = t^{-1} = \exp(2\beta\mu H/q)$ where q is the number of nearest neighbors, and μ is the magnetic moment of a single site; N is the total number of bonds in the lattice, and $R \equiv N - r$ is the number of positive bonds which join sites having identical spin directions (just as in[6], we denote the number of negative bonds by r); m denotes the number of positive bonds in which both spins are directed opposite to the magnetic field (in what follows we shall denote them by ↓↓). Accordingly, R − m is the number of bonds in which both spins are directed "along the field" (we denote them by ↑↑); g(R, m) is the number of configurations of the spins of the lattice with positive bonds, out of which there are m bonds of the type ↓↓.

Let us consider any spin configuration (R, m) and let us arrange s impurities in all possible ways between the atoms of the lattice in such a way that there turn out to be p impurities on the positive bonds. The number of ways of distributing p impurities on R positive bonds, where k impurities are distributed on R − m bonds of the type ↑↑ and p − k on m bonds of the type ↓↓, is given by $C^k_{R-m} C^{p-k}_m$. The introduction of impurities leads to the replacement of the energy factors f and t, associated with the presence of the magnetic field, by w and l, respectively: $w = l^{-1} = \exp(2\beta\mu_1 H/q)$ where μ_1 is the new magnetic moment. Here we note that the negative bonds do not give any contribution to the energy associated with the presence of the magnetic field.

The contribution to the partition function, associated with the presence of the magnetic field, coming from a single spin configuration containing R positive bonds (out of these, there are m bonds of the type ↓↓) on which p impurities are distributed, is given by

$$f^{R-m} t^m \sum_k C^k_{R-m} C^{p-k}_m w^k f^{-k} l^{p-k} t^{k-p} = \frac{1}{2\pi i} \oint \frac{d\zeta}{\zeta^{p+1}} (f + \zeta w)^{R-m} (t + \zeta l)^m,$$

where the integral is taken around the circumference of a circle whose center is at the point $\zeta = 0$. Now let us calculate the total contribution to the partition function Z_s coming from one spin configuration containing R positive bonds, also taking the exchange interaction into account. The number of ways of distributing the remaining s − p impurities on r negative bonds is C^{s-p}_r, and then the desired contribution (for fixed values of R and m) is given by

$$x^{N-r} y^r \sum_p C^{s-p}_r u^p v^{s-p} x^{-p} y^{p-s} \frac{1}{2\pi i} \oint \frac{d\zeta}{\zeta^{p+1}} (f + \zeta w)^{R-m} (t + \zeta l)^m.$$

It is essential that the operation of "averaging" the initial partition function over all possible configurations Q of the s impurities commutes with the "averaging" of the quantity $\exp\{-\beta E(Q, [\sigma])\}$ over all possible spin configurations $[\sigma]$, i.e.,

$$Z_s = \sum_Q \sum_{[\sigma]} \exp\{-\beta E(Q,[\sigma])\} = \sum_{[\sigma]} \sum_Q \exp\{-\beta E(Q,[\sigma])\}.$$

Taking this into account, Z_s has the form

$$Z_s = \sum_r x^{N-r} y^r \sum_p G_p(R,f) C_r^{s-p} u^p v^{s-p} x^{-p} y^{p-s}, \tag{2}$$

where

$$G_p(R,f) \equiv \frac{1}{2\pi i} \oint \frac{d\zeta}{\zeta^{p+1}} \sum g(R,m)(f+\zeta w)^{R-m}(t+\zeta l)^m = \frac{1}{2\pi i}\oint \frac{d\zeta}{\zeta^{p+1}} (f+\zeta w)^R g_R\left(\frac{t+\zeta l}{f+\zeta w}\right).$$

For the following calculations we make the change of variable $\zeta wt(1 + \zeta wt) = \lambda$. Then

$$G_p(R,f) = \frac{f^{R-p} w^p}{2\pi i} \int_{a-i\infty}^{a+i\infty} \frac{g_R[t^2(1-\lambda)+l^2\lambda]\, d\lambda}{\lambda^{p+1}(1-\lambda)^{R-p+1}}, \tag{3}$$

where $0 < a < 1$. We note that formula (2) together with relation (3) expresses the partition function of a lattice containing impurities in an arbitrary magnetic field in terms of the corresponding function for an ideal lattice. For $H = 0$ the result of Lushnikov[6] is obtained in a natural way. The function $g_R(z)$ in an arbitrary magnetic field H is known only in the one-dimensional case, although formula (3) itself is valid for a lattice of any number of dimensions.

Now let the size of the system increase: $N \to \infty$, $s \to \infty$, and $c = s/N$ (c denotes the impurity concentration). Since the magnetic moment of the system is a self-averaging quantity, we have

$$\lim_{N,s\to\infty} \frac{p}{R} = c^*,$$

where c^* has a simple physical meaning—it is the average (in the thermodynamic sense) value of the ratio of the number p of impurities rupturing the positive bonds to the total number $R \equiv N - r$ of positive bonds associated with a given temperature. If the partition function for the lattice containing impurities appearing in the work of Lushnikov[6] is written in the form

$$Z_s = \sum_r g_N(r)\Gamma(N,s,r),$$

where

$$\Gamma(N,s,r) \equiv \sum_p C_{N-r}^p C_r^{s-p} x^{N-r-p} y^{r-s+p} u^p v^{s-p} = \frac{1}{2\pi i}\oint \frac{dz}{z^{s+1}} (x+zu)^{N-r}(y+zv)^r,$$

then in the absence of a magnetic field the quantity of interest to us is given by

$$c_0^* = \frac{1}{Z_s}\sum_r g_N(r) \sum_p \frac{p}{N-r} C_{N-r}^p C_r^{s-p} x^{N-r-p} y^{r-s+p} u^p v^{s-p}.$$

Taking the relation

$$\frac{p}{N-r} C_{N-r}^p = C_{N-r-1}^{p-1}$$

into account, it is easy to show that the summation over p which appears in the expression for c_0^* is identically equal to

$$u\Gamma(N-1,s-1,r) = \frac{u}{2\pi i}\oint \frac{dz}{z^s}(x+zu)^{N-r-1}(y+zv)^r.$$

Then, writing Z_s in integral form, and then carrying out the change of the integration variable indicated in[6], we obtain the following relations for c_0^*

$$c_0^* = u \oint \frac{dz}{z^s}(x+zu)^{N-1} S_N\left(\frac{y+zv}{x+zu}\right) \Big/ \oint \frac{dz}{z^{s+1}}(x-zu)^N S_N\left(\frac{y+zv}{x+zu}\right)$$

$$= \int_{a-i\infty}^{a+i\infty} t\, \frac{S_N[y^2(1-t)+v^2t]\,dt}{t^{s+1}(1-t)^{N-s+1}} \Big/ \int_{a-i\infty}^{a+i\infty} \frac{S_N[y^2(1-t)+v^2t]\,dt}{t^{s+1}(1-t)^{N-s+1}}.$$

Just as in article [6], one can evaluate these integrals by using the method of steepest descents. In this connection the smooth factor t in the integral, standing in the numerator of the fraction, is taken outside of the integral sign at the saddle point ξ. Then the final calculation of c^* in the case $H = 0$ leads to a simple result: $c_0^* = \xi$, where ξ is related to η by the relation $\eta = (1 - \xi)y^2 + \xi v^2$, and η is determined from the corresponding equation in[6]. Thereby the physical meaning of the variable ξ $(0 < \xi < 1)$, which appears in the basic formulas of Lushnikov's work[6], is clarified. Namely, $\xi = c_0^*(T, c)$ is the concentration of the impurities distributed on the positive bonds.

Let us return to the calculation of the function $G_p(R, f)$. Evaluating the integral on the right-hand side of formula (3) by the method of steepest descent, we obtain

$$G_p(R,f) = \tilde{f}^R \exp[R\{\ln g(\omega) - c^*\ln\chi - (1-c^*)\ln(1-\chi)\}].$$

Here χ denotes the saddle point, $\omega \equiv (1-\chi)t^2 + \chi l^2$, and $\tilde{f}$ differs from $f = \exp(2\beta\mu H/q)$ by the replacement of μ by $\tilde{\mu} = \mu_1 c^* + \mu(1 - c^*)$. The saddle point χ is determined from the equation which has the form:

$$-\frac{c^*}{\omega - t^2} + \frac{1-c^*}{l^2-\omega} + \frac{d}{d\omega}\ln g(\omega) = 0. \tag{4}$$

It is possible to obtain the solution of Eq. (4) in explicit form only in the case of a weak magnetic field $(\mu H/kT \ll 1)$. Then the root of Eq. (4) which reduces, for $H = 0$, to the partition function calculated by Lushnikov[6] has the form

$$\omega = 1 - \frac{2\beta H}{q}[\mu_1 c_0^* + \mu(1-c_0^*)] + O(H^2),$$

i.e., in this approximation ω does not depend on the specific form of the function $\ln g(\omega) \equiv R^{-1}\ln g_R(\omega)$. In this connection

$$c^*\ln\chi + (1-c^*)\ln(1-\chi) = c_0^*\ln c_0^* + (1-c_0^*)\ln(1-c_0^*) + O(H^2).$$

Then taking into consideration the obvious relation

$$\exp[R\{-c_0^*\ln c_0^* - (1-c_0^*)\ln(1-c_0^*)\}] = C_R^p$$

and having omitted the terms $\sim O(H^2)$, we obtain the result that $G_p(R, f) = C_R^p \tilde{f}^R \tilde{g}_R$, where a tilde above the symbol for a function means that in its argument μ is replaced by $\tilde{\mu} = \mu_1\xi + \mu(1 - \xi)$. Thus, the problem of the calculation of the partition function of a lattice containing impurities which change the magnetic moment μ of a site to a new value μ_1 reduces, in the presence of a weak magnetic field $\mu H/kT \ll 1$, to a calculation of the partition function for a lattice having a certain effective magnetic moment per site, $\tilde{\mu} = \mu_1\xi + \mu(1 - \xi)$, in which the impurities only change the value of I to I_1.

3. For a problem in which the impurities do not change the magnetic moment of a site, by again using

the methods of article[6] we can indicate a solution in the case of an arbitrary (and not just for the case of a weak) magnetic field, which may apparently be of independent interest. Thus, let us calculate the partition function of a lattice containing impurities which change I to I_1 (but do not change the magnetic moment) in an arbitrary magnetic field. With Eq. (2) taken into account we have

$$Z_s = \sum_r x^{N-r} y^r f^R g_R(f) \sum_p C^p_{N-r} C^{s-p}_r u^p v^{s-p} x^{-p} y^{p-s} = \frac{1}{2\pi i} \oint \frac{dz}{z^{s+1}} (x+zu)^N S_N\left(\frac{y+zv}{x+zu}, f\right),$$

where the function $S_N(y^2, f)$ is defined in terms of the partition function for a perfect lattice by the formula $Z = x^N S_N(y^2, f)$. Evaluation of this integral is carried out in the same way as in article[6], with only one difference—the position of the saddle point ξ is now also determined by the magnitude of the field H: $\xi = \xi(H)$.

The free energy $\mathcal{F}_c$ of a lattice containing impurities in a magnetic field, calculated per bond, is expressed in terms of

$$F(y^2, f) \equiv \lim_{N\to\infty} \frac{\ln S_N(y^2, f)}{N}$$

in the following way:

$$-\beta\mathcal{F}_c = \hat{F}(\eta, f) - c\ln\xi - (1-c)\ln(1-\xi) + c\beta I_1 + (1-c)\beta I. \quad (5)$$

Here $\eta = (1-\xi)y^2 + \xi v^2$, and the saddle point ξ is determined from the condition $\partial(\beta\mathcal{F}_c)/\partial\xi = 0$, which it is convenient to write in the form

$$-\frac{c}{\eta - y^2} + \frac{1-c}{v^2-\eta} + F'(\eta) = 0. \quad (6)$$

by using the variable η.

We note that for $H = 0$ one can write Eq. (6) for the determination of the quantity η, that is, the corresponding Lushnikov equation[6], in the form

$$-\frac{c}{\eta - y^2} + \frac{1-c}{v^2-\eta} + \frac{\nu(\eta)}{\eta} = 0. \quad (7)$$

Here the quantity $\nu(\eta) \equiv \eta F'(\eta)$ has a simple physical meaning, namely: for an ideal lattice $\nu(y^2)$ is the concentration of negative bonds associated with a given temperature T. Actually, the quantity $F(y^2)$ is by definition related to the free energy per bond of an ideal lattice, $\mathcal{F}(y^2)$, by the relation $-\beta\mathcal{F}(y^2) = -(1/2)\ln y^2 + F(y^2)$. Let us further take into consideration that the energy per bond of a perfect lattice is given by $\mathcal{E}(y^2) = d[\beta\mathcal{F}(y^2)]/d\beta$. Then one can easily show that $y^2F'(y^2) = [\mathcal{E}(y^2) + I]/2I$. The numerator of the last fraction, namely the quantity $\mathcal{E}(y^2) + I$, is the energy per bond, measured from the ground state. Since the quantity 2I is equal to the change of energy associated with the resplacement of one positive bond by a negative bond, then this fraction coincides with the concentration of negative bonds in the lattice.

Now let us calculate the magnetic moment of the system (per site)

$$M_c \equiv -\frac{\partial\mathcal{F}_c}{\partial H} = -\left(\frac{\partial\mathcal{F}_c}{\partial\eta}\right)_H \frac{\partial\eta}{\partial H} - \left(\frac{\partial\mathcal{F}_c}{\partial H}\right). \quad (8)$$

One can easily show that with Eq. (6) taken into account the first term in (8) vanishes. Then if the magnetic moment of a pure lattice in a magnetic field has the form $M = M(y^2, f)$, then it is obvious from Eq. (8) that the magnetic moment of a lattice containing impurities is given by $M_C = m(\eta, f)$, i.e., it differs only by the replacement of the quantity $y^2 \equiv e^{-2\beta I}$ by $\eta(H)$. Then the spontaneous magnetization is given by

$$M_c^0 \equiv \lim_{H\to 0} M(\eta, f) = \mu m(\eta),$$

where $\mu m(y^2)$ is the spontaneous moment of the lattice without any impurities, and η is determined from Eq. (7).

4. Let us return to the problem originally formulated, concerning the determination of the spontaneous magnetization of an Ising lattice containing impurities which change the exchange integral I to the new value I_1 and change the magnetic moment μ of a site to the new value μ_1. Since in order to calculate the spontaneous magnetization M_C^0 it is sufficient to confine one's attention to a vanishingly small magnetic field, then one can use the assertion formulated at the end of Sec. 2. We recall that this assertion makes it possible to formally use the calculation of the magnetic properties for a lattice containing impurities which do not change the magnetic moment μ of a site. Then the spontaneous moment M_C^0 of this system has the following natural form:

$$M_c^0 = [\mu_1\xi + \mu(1-\xi)]m(\eta), \quad (9)$$

where $\mu m(y^2)$ is the spontaneous moment of an ideal lattice, and η and ξ are determined from Eq. (7).

We note that the general expression (9) obtained for the spontaneous magnetic moment in an arbitrarily transformed Ising lattice includes the corresponding result which was obtained earlier by Essam and Garelick[7] for the case of a model of dilute ferromagnetism. This model was proposed by Syozi in[8] and then was investigated in detail in the absence of a magnetic field by Syozi and Miyazima.[9] Using the notation introduced in[7], let us indicate how to pass from the quantities I and I_1 introduced by us, and also μ and μ_1, to the corresponding quantities in[7]. Namely: $I = (2\beta)^{-1}\ln\cosh 2K$, $I_1 = 0$, $\mu = 1 + (1/2)q\tanh 2K$, and $\mu_1 = 1$. Thus, the Syozi model is a special case of the transformed Ising model considered by us, and in addition it is very specific in the sense that the introduction of impurities leads to a complete breaking of the exchange interaction between the corresponding lattice sites ($I_1 = 0$). The model considered by us makes it possible to also analyze the case $I_1 \neq 0$.

5. Now let us apply the results obtained in Sec. 2 and 3 to a planar square lattice in which the impurities change I to I_1, but they do not change the value of the magnetic moment μ of a site, and in addition for simplicity we set $\mu = 1$. Then the spontaneous moment M_C^0 for such a lattice is calculated according to the formula $M_C^0 = m(\eta)$, where $m(z)$ is the well-known expression obtained by Yang[3] for the spontaneous magnetization of a planar square Ising lattice (per spin) below the temperature T_λ of the phase transition:

$$m(z) = \frac{(1+z^2)^{1/4}(1-6z^2+z^4)^{1/8}}{(1-z^2)^{1/2}}, \quad z \equiv e^{-2\beta I}, \quad (10)$$

and η, as already indicated, is determined from Eq.

(7). Using the results of article[6], let us present formulas for the spontaneous magnetization in the neighborhood of the phase transition point in the case of a total breaking of the bonds ($I_1 = 0$) and a small concentration of impurities, $c \ll 1$. In the vicinity of T_λ the spontaneous magnetization per spin has the form

$$m(\eta) = A\delta^{1/8}, \tag{11}$$

where $A = (8 + 4\sqrt{2})^{1/2}$ and δ is the deviation of η from its value η_0 at the critical point ($\eta = \eta_0 - \delta$). According to[6],

$$\delta \approx \frac{\alpha\delta_0}{\alpha + c\ln c - c\ln|\delta_0|}. \tag{12}$$

Here $\alpha = \pi(\sqrt{2} + 1)^3/2$, and δ_0 is the deviation of the quantity y^2 from $y_\lambda^2 \equiv \exp\{-2I/kT_\lambda\}$ ($y^2 = y_\lambda^2 + \delta_0$, $\delta_0 \ll 1$). In order to see the dependence on the temperature T in Eq. (11), it is necessary to take into consideration that

$$\delta_0 = (1 - \sqrt{2})2I\tau / T_\lambda, \quad \tau = (T_\lambda - T) / T_\lambda.$$

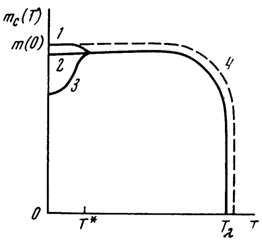
It is also of interest to discuss the behavior of the spontaneous magnetization in the limit $T \to 0$. As we shall see below, depending on the sign of the new exchange integral I_1, $\lim_{T\to 0} M_c^0(T)$ exhibits a qualitatively different dependence on the impurity concentration, where as before we shall regard the concentration as small ($c \ll 1$).

Thus, let us consider the solution of Eq. (7) as $T \to 0$ in the case $|I_1| \ll J$. Using the expression for the energy of a square lattice (see, for example,[10]), one can easily show that for $\eta \ll 1$ the expansion $\nu(\eta) = 2\eta^4 + O(\eta^6)$ is valid. In the case of a complete breaking of the bonds ($I_1 \equiv 0$ and $v = 1$) $y^2 \equiv e^{-2\beta I} \to 0$ as $T \to 0$, and for $T = 0$ Eq. (7) degenerates into the relation $c = \eta + (1 - \eta)\nu(\eta)$. Having substituted here the expansion of $\nu(\eta)$ for $\eta \ll 1$ and inverting the resulting power series in η, we find that $\eta = c + O(c^4)$ for $T = 0$. Having this solution of Eq. (7) to the zero order approximation with respect to the temperature, one can easily obtain the temperature dependence of η by assuming $y^2 \ll 1$ to be a very small parameter for the problem. It turns out that $\eta = c + \gamma e^{-2\beta I}$, where $\gamma \sim 1$.

Taking into consideration what has been said above, one can easily show that the low-temperature expansion of the relative change of the magnetic moment, $\Delta m \equiv [m_c(T) - m(0)]/m(0)$, (calculated per spin) will be given by

$$\Delta m = -2\eta^4 = -2c^4(1 + 4\gamma e^{2\beta I}/c).$$

We note that the decrease of the magnetic moment is proportional to the fourth power of the concentration. One can give an intuitive physical interpretation to this result. In fact, the magnetic moment of the lattice for $T = 0$ may decrease in comparison with the normal value only at the expense of the isolation of a group of sites, bounded by closed contours, along which occurs the simultaneous breaking of the bonds with the remaining part of the lattice. The smallest group of such a type is an individual spin, all four of whose bonds with the nearest neighbors are broken. For small concentrations, formation of precisely such groups is most probable. It is obvious that the number of such individual reversed spins is proportional to c^4.



Schematic graph showing the dependence of the spontaneous magnetic moment on temperature for a square lattice in the case of a small impurity concentration, $c \ll 1$, and $|I_1| \ll I$. For curve 1, $I_1 > 0$; for curve 2, $I_1 = 0$; for curve 3, $I_1 < 0$; and for curve 4, $I_1 = I$.

Now let $I_1 \ll I$ where $I_1 > 0$. In this case it is obvious that for an arbitrary concentration of impurities (even for $1/2 \le c \le 1$) there is a temperature $kT^* \sim I_1$ below which the lattice is essentially completely polarized (for $T = 0$ the spontaneous moment in this case is equal to the nominal value). Solving Eq. (7) in this case, one can easily show that $\eta = ce^{-2\beta I}$, i.e., η vanishes at $T = 0$.

Now let us consider the low-temperature behavior of the magnetic moment for $I_1 < 0$ and $|I_1| \ll I$. In this case for $T = 0$ the quantity $1/v^2 = 0$ and, as before, $y^2 = 0$. Then Eq. (7) degenerates into the relation $c = \nu(\eta)$. Assuming $c \ll 1$, one can easily obtain $\eta = (c/2)^{1/4}\{1 - O(\sqrt{c})\}$. Then the temperature dependence is given by the formula $\eta = (c/2)^{1/4} - |B| e^{-\beta|I_1|}$ where b is a constant which only depends on c, and the corresponding decrease of the magnetic moment (in comparison with the nominal value) is given by

$$\Delta m = -c[1 - 4|b|(2/c)^{1/4}e^{-2\beta|I_1|}].$$

Thus, in contrast to the case of complete breaking of the bonds Δm turns out to be proportional to the first power of the concentration. Such a result in the case $I_1 < 0$ is associated with the completely different structure of the ground state of the system ($T = 0$) in comparison with the previous two cases, $I_1 = 0$ and $I_1 > 0$. Namely, for $I_1 < 0$ the state having the smallest energy for $T = 0$ is the one such that all impurities "sit" on negative bonds. It is easy to figure out that the most favorable clusters are those which, being arranged along a closed contour, change the sign of the exchange interaction on all of the bonds belonging to such a contour. In this connection all of the spins inside the contour have the same direction, opposite to the nominal direction. It is natural that the complexes in which all four bonds of a single reversed spin have new exchange integrals $I_1 < 0$ are most probable. Since the formation of such complexes calls for favorable energy relations, their number is proportional to c if $c \ll 1$.

We further note that for $I_1 < 0$ the case of not too small values of c ($c \lesssim 1/2$) requires (even qualitatively) special consideration. Formally this is already clear from the fact that Eq. (7), which determines the dependence of the critical temperature T_λ on the impurity concentration c for $\eta = \eta_0 = \sqrt{2} - 1$, has a solution $T_\lambda = 0$ for $c = c_0 \equiv (2 - \sqrt{2})/4$ independently of the value of the exchange integral $|I_1|$.

It is helpful to represent the results of the analysis of the behavior of the magnetic moment for a temperature near 0°K in the case $c \ll 1$ and $|I_1| \ll 1$ in the form of a schematic graph (see the accompanying figure), where for purposes of comparison the curve of the spontaneous magnetization of an ideal lattice is also shown.

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