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THEORY OF OSCILLATIONS OF THE INELASTIC DIFFERENTIAL CROSS SECTION AT HIGH ENERGIES

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A theory of the oscillations of the inelastic differential cross section near the threshold reduced angle $\tau_0 = E\vartheta_0$ is presented for transitions due to pseudointersection of the terms. The theory employs the high-energy approximation and the weakness of the coupling between the states. The latter requires that the energy E be larger than the value E_{max} at which the cross section is maximal. Application of the results in an analysis of the data of[2] on inelastic scattering in the He^+-Ne system shows good agreement with experiment.

An analysis of the differential cross sections of elastic and inelastic scattering of atoms or ions makes it possible to obtain extensive information on the electronic terms of the colliding pair of atoms. A splendid example of such an analysis is found in the investigations[1,2] of elastic and inelastic scattering of He^+ by the atoms Ne, Ar, and others. An interpretation of the anomalies of the differential cross sections points unequivocally[1,2] to the existence of term intersection that leads to excitation.

The theories[3,4] of oscillations of cross sections due to intersection of terms attribute them to interference of contributions from two trajectories calculated under the assumption that the transition occurs at the point of intersection (and not in some finite region). Yet such a representation cannot be used to describe the first periods of the cross-section oscillations following the threshold angle. Kotova[5] used the theory of transitions in the Landau-Zener model in its quantum variant[6] to investigate oscillations due to an inelastic process. The analysis given in[5] is valid for any energy. In this connection, the procedure for obtaining quantitative characteristics of the terms from the oscillations of the cross sections turns out to be quite complicated. One can count on obtaining appreciable simplifications of the theory in the high-energy limit. For single-channel elastic scattering, the use of even the first term of the expansion[7] of the scattering functions in powers of $1/E$ turned out to be quite useful for a unified reduction of the data of many experiments in terms of the reduced angle $\tau = E\theta$ and the reduced cross section $\rho(\tau) = \theta \sin \theta\sigma(\theta, E)$.

In the present paper we obtain, in just such a high-energy limit, an expression for the inelastic differential cross section near the threshold angles τ_0, for transitions due to term intersection. We consider the case of weak interaction between the states, corresponding to energies $E > E_{max}$, where E_{max} is the energy at which the cross section of the inelastic process reaches the maximum value. The developed theory is applicable further to an analysis of data[2] on inelastic scattering in the He^+-Ne system with excitation of an Ne atom in the state $2p^5 3s$.

Let us consider the problem of electronic transitions in the simplest variant of two electronic adiabatic states φ_1 and φ_2. In the well known high-energy approximation or in the "impact-parameter" approximation[8,9], the wave function of the system is written in the form ($\hbar = 1$):

$$\psi = c_1(b, z)\varphi_1 \exp\left(ikz - i\int_{-\infty}^{z} V_1(r)\,dz/u\right) + c_2(b, z)\varphi_2 \exp\left(ikz - i\int_{-\infty}^{z} V_2(r)\,dz/u\right), \quad (1)$$

where b is the impact parameter, $r = \sqrt{b^2 + z^2}$, $k = \sqrt{2mE}$ the relative momentum of the system, $u = \sqrt{2E/m}$ is the velocity, and V_1 and V_2 are the corresponding potential energies.

The coefficients $c_i(z)$ satisfy the equation

$$\frac{idc_i(z)}{dz} = \frac{1}{u} V_{ij}(r) \exp\left(-i\int_{-\infty}^{z} (V_i - V_j)\frac{dz}{u}\right) c_j, \quad i \neq j, \quad j, i = 1, 2 \quad (2)$$

with boundary conditions $c_1(-\infty) = 1$, $c_2(-\infty) = 0$, where $V_{12} = V_{21}^*$ is the matrix element of the interaction between the states. Since we are interested only in the zeroth term of the expansion of the quasi-classical phases in terms of V_i/E, we shall not discuss the difficulties that arise in the next-higher orders of the expansion as a result of the differences between the velocities at each term (see, for example,[10]). It will suffice henceforth to concern ourselves only with the representation of the quasiclassical phases in a convergent form, for which purpose it is necessary to put

$$\eta_i = \frac{1}{u}\int_0^{\infty} V_i\,dz = \frac{1}{u}\int_{-\infty}^{0} V_i\,dz = \frac{1}{u}\int_0^{\infty} \frac{1}{r}\frac{dV_i}{dr} z^2\,dz.$$

Using the solution of (2) in first order of perturbation theory in terms of the interaction between the states V_{12}, we express the amplitude of the inelastic scattering through an angle θ in the form

$$f_{12}(\theta) = \frac{-1}{mu}\frac{1}{\sqrt{2\pi \sin\theta}}\int_0^{\infty} \sqrt{l + 1/2}\,dl \int_{-\infty}^{\infty} V_{12}(r)\frac{dz}{u} e^{iS(b,z,\tau)}, \quad (3)$$

where the angular momentum l is connected with the impact parameter: $l = mub$, and the action S is equal to

$$S(b, z, \tau) = \frac{1}{u}\left\{ \int_{-\infty}^{0} \frac{1}{r}\frac{dV_1}{dr} z^2\,dz + \int_{0}^{\infty} \frac{1}{r}\frac{dV_2}{dr} z^2\,dz - \int_{0}^{z} \Delta V(r)\,dz - 2ib\tau \right\}. \tag{4}$$

Here $\Delta V = V_1(r) - V_2(r)$ and $\tau = E\theta$ is the reduced angle. At $V_1 = V_2$, the action $S = S(b, \tau)$ coincides with the first term of the expansion in V/E at small values of θ[7].

We shall carry out the integration in (3), following[5], first with respect to the impact parameter b (or l), assuming that the saddle-point method with respect to the variable b is valid for all values of z. Thus, from the condition

$$\left.\frac{\partial S(bz\tau)}{\partial b}\right|_{b=b(z,\tau)} = 0 \tag{5}$$

we obtain for each z and τ the saddle-point value of $b(z, \tau)$ satisfying the equation

$$-\frac{b}{2}\left.\left(\int_{-\infty}^{z} \frac{1}{r}\frac{dV_1}{dr}dz + \int_{z}^{\infty} \frac{1}{r}\frac{dV_2}{dr}dz \right)\right|_{b=b(z\tau)} = \tau. \tag{6}$$

The value of the action at $b = b(z\tau)$ is conveniently represented in the form

$$S(b(z\tau)z) = S(b(z_0\tau), z_0) - \frac{1}{u}\int_{z_0}^{z} \Delta V(b(z', \tau)z')\,dz'. \tag{7}$$

The latter can easily be verified by differentiating $S(b(z\tau)z)$ with respect to z with (5) taken into account

$$\frac{d}{dz}S(b(z\tau)z) = -\frac{1}{u}\Delta V(b(z, \tau)z). \tag{8}$$

Carrying out the indicated integration with respect to l (or with respect to b) in (3), we obtain for the reduced inelastic cross section $\rho(\tau, E) = 0 \sin\theta\,|f_{12}(\theta)|^2$ the expression

$$\rho(\tau, E) = \left| \frac{1}{u}\int_{-\infty}^{\infty} V_{12}(b(z, \tau), z)\sqrt{\tilde{\rho}(z\tau)}\exp\left\{ -\frac{i}{u}\int_{z_0}^{z} \Delta V(b(z', \tau)z')\,dz' \right\} \right|^2, \tag{9}$$

$$\tilde{\rho}(z, \tau) = \tau b(z\tau)\frac{db(z\tau)}{d\tau} = \frac{1}{2}\frac{db^2(z\tau)}{d\ln\tau}, \tag{10}$$

where the function $b(z, \tau)$ is determined by Eq. (6). We have thus reduced the problem to a form analogous to the problem of transitions under the influence of a small perturbation while moving along the trajectories $z = ut$ and $r(z) = \sqrt{b^2(z, \tau) + z^2}$.

We shall find it useful to investigate the integral in (9) by the saddle-point method. The saddle points $z_i(\tau)$ of the exponential in (9) are determined from the condition for the vanishing of the derivative (8):

$$\Delta V(b(z, \tau), z)\,|_{z_i(\tau)} = 0.$$

Consequently, the points $z_i(\tau)$ are the points of intersection of the curve $b = b(z, \tau)$ and the circle $b^2 + z^2 = r_0^2$ (r_0 is the term-intersection radius). It is obvious that when τ exceeds a certain τ_0, the circle and b $b(z, \tau)$ will cross each other twice. However, there exists a value of τ_0 such that the $b(z, \tau_0)$ curve is tangent to the circle at the point $z_0 b_0$, and the saddle point is not valid in the vicinity of this value. Outside this vicinity (see formula (26) below) we can use the usual saddle-point method for the calculation of (9), and we obtain

$$\rho(\tau, E) = \left| \sqrt{\tilde{\rho}(z_1\tau)2\pi V_{12}^2\left| u^2\frac{d^2S(z_1)}{dz^2}\right|^{-1}}\, e^{iS_1 - i\pi/4} - \sqrt{\tilde{\rho}(z_2\tau)2\pi V_{12}^2\left| u^2\frac{d^2S(z_2)}{dz^2}\right|^{-1}}\, e^{iS_2 + i\pi/4} \right|^2. \tag{11}$$

Here $z_i = z_i(\tau)$, $\tilde{\rho}(z, \tau)$ is given by formula (10), and $V_{12} = V_{12}(r_0)$. The difference between the actions $\Delta S = S_1 - S_2$ is equal to

$$S(b(z_1, \tau), \tau) - S(b(z_2, \tau), \tau) = -\frac{1}{u}\int_{z_1}^{z_2} \Delta V(b(z, \tau), z)\,dz, \tag{12}$$

and the phases $\pm\pi/4$ correspond to $\Delta F = d\Delta V/dr|_{r_0} < 0$, when $d^2S/dz^2 \gtrless 0$ for $z_i \lessgtr 0$.

Expression (11) has the usual form[3,4] of a sum of two interfering terms, and the actions S_1 and S_2 correspond to transition to another term at the points where the terms $z_1(\tau)$ and $z_2(\tau)$ intersect:

$$S_i(\tau, E) = \frac{1}{u}\left\{ \int_{-\infty}^{0} \frac{1}{r}\frac{dV_1}{dr}z^2\,dz + \int_{0}^{\infty} \frac{1}{r}\frac{dV_2}{dr}z^2\,dz - \int_{0}^{z_i(\tau)} \Delta V(r)\,dz + 2b_i\tau \right\},$$

$$r = \sqrt{b_i^2 + z^2}, \quad b_i = b(z_i, \tau). \tag{13}$$

As always[2], the difference between the two impact parameters, which contribute to the interference at definite values of τ, is

$$\Delta b = b_1 - b_2 = \frac{d}{d\tau}[S_1(\tau, E) - S_2(\tau, E)]. \tag{14}$$

The second derivative of the action, which enters in (10), is, in accordance with (8),

$$\left.\frac{d^2S(b(z, \tau)z)}{dz^2}\right|_{z_i} = -\frac{\Delta F}{u}\left.\left(\frac{z}{r_0} - \frac{b}{r_0}\frac{\partial b(z\tau)}{\partial z}\right)\right|_{z_i}, \quad \Delta F = \left.\frac{d\Delta V}{dr}\right|_{r_0}, \tag{15}$$

and the derivative of $b(z, \tau)$ with respect to z can be connected, on the basis of (6), with the derivative of $b(z, \tau)$ with respect to τ:

$$\frac{\partial b(z\tau)}{\partial z} = \frac{b}{2r}\frac{d\Delta V}{dr}\frac{\partial b(z\tau)}{\partial \tau}. \tag{16}$$

The quantity $\partial b/\partial\tau$, which is the reciprocal of $d\tau/db$, is expressed at a fixed value of z in terms of the potentials V_1 and V_2 with the aid of (6).

It is now easy to write down an equation determining the point $\tau = \tau_0$ when $z_1(\tau_0) = z_2(\tau_0) = z_0$, i.e., when the curve $b(z, \tau_0)$ is tangent to the circle $b^2 + z^2 = r_0^2$. Since the point z_0 is a multiple zero of the function (8), it follows that the second derivative of the action $S(b(z, \tau)$ with respect to z should vanish at this point; with allowance for (15) and (16), this is written in the form

$$\left.\frac{d^2S(b(z, \tau)z)}{dz^2}\right|_{z_0} = -\frac{\Delta F}{u}\left(\frac{z_0}{r_0} + \frac{b^2\Delta F}{2r_0}\left.\frac{\partial b(z_0\tau)}{\partial\tau}\right|_{\tau_0}\right) = 0. \tag{17}$$

Thus, the values of z_0 and τ_0 should satisfy the equations

$$\frac{z_0}{b(z_0\tau_0)} = -\frac{\Delta F}{2r_0}b(z_0\tau)\left.\frac{\partial b(z_0\tau)}{\partial\tau}\right|_{\tau_0},$$

$$b^2(z_0\tau_0) + z_0^2 = r_0^2. \tag{18}$$

At τ close to the value τ_0, which we shall henceforth call the threshold value, expression (11) for the cross section does not hold. We shall therefore calculate the integral in (9) in the vicinity of τ_0 without using the saddle-point method. To this end it suffices to take the pre-exponential factor in (9) at the point z_0, τ_0, and to expand the argument of the exponential in (9) in a series in $\Delta z = z - z_0$ and $\Delta\tau = \tau - \tau_0$. We represent the function $b(z, \tau)$ in the vicinity of (z_0, τ_0) in the form

$$b(z,\tau) = b_0 + \beta(\tau - \tau_0) + \varkappa(z - z_0) + \frac{\gamma}{2b_0}(z - z_0)^2. \quad (19)$$

We have introduced here the notation

$$b_0 = b(z_0, \tau_0), \quad \beta = \left.\frac{\partial b(z_0\tau)}{\partial\tau}\right|_{\tau=\tau_0},$$
$$\varkappa = \left.\frac{\partial b(z\tau_0)}{\partial z}\right|_{z_0} = -\frac{z_0}{b_0}, \quad \gamma = b_0\left.\frac{\partial^2 b(z,\tau_0)}{\partial z^2}\right|_{z_0}. \quad (20)$$

In the expansion (19) we have already taken into account the fact that the region Δz, which contributes to the integral, is of the order of $\sim\sqrt{\beta b_0 \Delta\tau}$. Using for $\Delta V(r)$ a linear approximation in the vicinity of the point of the intersection $\Delta V(r) = \Delta F(r - r_0)$, and taking (19) into account, we obtain for z and τ close to z_0 and τ_0 the following expression for the exponential in (9):

$$S(b(z,\tau), z) = S_0 - \frac{\Delta F}{u}\left[\frac{\beta b_0}{2r_0}\Delta\tau(z - z_0) + \frac{q}{2r_0}\frac{(z - z_0)^3}{3}\right]. \quad (21)$$

Here $S_0 = S(b_0, z_0, \tau_0)$ and

$$q = 1 + \varkappa^2 + \gamma \quad (22)$$

q is a number of the order of unity. The term quadratic in $(z - z_0)$ in (21) vanishes when account is taken of equations (18), which determine the point $\tau_0 z_0$.

Substituting (21) in (9), we obtain for $\rho(\tau)$ near the threshold angle τ_0 the expression

$$\rho(\tau, E) = r_0^2 A\left(\frac{E}{E_1}\right)^{-2/3}\Phi^2\left(B\frac{\Delta\tau}{\tau_0}\left(\frac{E}{E_1}\right)^{-1/3}\right). \quad (23)$$

Here $\Phi(x)$ is the Airy function, and A and B are dimensionless parameters independent of τ and E, with values

$$A = 2\left(\frac{\tilde{\rho}_0}{r_0^2}\right)\left(\frac{2\pi V_{12}^2}{\hbar u_1 \Delta F}\right)\left(\frac{\Delta F r_0^2}{\hbar u_1}\right)^{1/3}\left(\frac{2}{q}\right)^{2/3}, \quad (24)$$

$$B = \left(\frac{\tilde{\rho}_0}{r_0^2}\right)\left(\frac{\Delta F r_0^2}{\hbar u_1}\right)^{1/3}\left(\frac{2}{q}\right)^{1/3}. \quad (25)$$

Here

$$\tilde{\rho}_0 = \frac{1}{2}\left.\frac{db^2(z_0\tau)}{d\ln\tau}\right|_{\tau=\tau_0}, \quad u_1 = \sqrt{\frac{2E_1}{m}},$$

q is a number of the order of unity, defined by formula (22).

In the reduction of the data it is convenient to put $E_1 = 1$ eV, so that E/E_1 in (23) is simply the energy E expressed in electron volts, and for the He^+-Ne pair $u_1 = u_1(1\text{ eV}) = 0.346 \times 10^{-2}$ atomic units.

At

$$|\Delta\tau| > 2\tau_0\left(\frac{E}{E_1}\right)^{1/3}\frac{1}{B} = \delta\tau, \quad (26)$$

when the argument x of the function $\Phi(x)$ in (23) has a modulus larger than 2, a sufficiently good approximation of $\Phi(x)$ can be obtained from the asymptotic formulas that give the following expressions for the cross section:

$$\rho(\tau, E) = r_0^2 C\left(\frac{E}{E_1}\frac{|\Delta\tau|}{\tau_0}\right)^{-1/2}\exp\left\{-D\left(\frac{E}{E_1}\right)^{-1/2}\left(\frac{\Delta\tau}{\tau_0}\right)^{3/2}\right\}, \quad \tau < \tau_0, \quad (27)$$

and

$$\rho(\tau E) = r_0^2 C\left(\frac{E}{E_1}\frac{\Delta\tau}{\tau_0}\right)^{-1/2}\left|1 - \exp\left\{iD\left(\frac{E}{E_1}\right)^{-1/2}\left(\frac{\Delta\tau}{\tau_0}\right)^{3/2} + \frac{i\pi}{2}\right\}\right|^2, \quad \tau > \tau_0 + \delta\tau, \quad (28)$$

where

$$C = \frac{\tilde{\rho}_0 q}{2r_0^2}\frac{V_{12}^2}{\hbar u_1 \Delta F}, \quad D = \frac{4}{3}\sqrt{\frac{2}{q}}\frac{\Delta F r_0^2}{\hbar u_1}\left(\frac{\tilde{\rho}_0}{r_0^2}\right)^{3/2}. \quad (29)$$

The asymptotic formula (28) coincides with the expression (11) obtained by the saddle-point method, provided we substitute in the latter in explicit form the values of $z_i(\tau)$ in the vicinity of $\tau \sim \tau_0$, namely

$$z_{1,2}(\tau) = z_0 \pm \sqrt{\frac{2b_0}{q}|\beta\Delta\tau|}. \quad (30)$$

Formula (30) can easily be obtained from the equation $b^2(z_i(\tau)\tau) + z_i^2(\tau) = r_0^2$ when account is taken of the function $b(z, \tau)$ in the vicinity of z_0, τ_0.

Thus, the behavior of the cross section of the inelastic process due to the intersection of the terms is described near the threshold ($\tau \sim \tau_0$) by formula (23).

We now turn to the experimental data of[2] on the scattering of He^+ by Ne with excitation of an Ne atom in the state $2p^5 3s$. A typical plot of $\rho(\tau, E)$ at fixed energy consists of an exponential decrease of the cross section when $\tau < \tau_0 \sim 10^3$ eV-deg, regular oscillations when $\tau > \tau_0$; the period of the oscillations changes from 400 to 800 eV-deg when the energy E changes from 70 to 500 eV (for details see[2]). The amplitude, positions, and periods of the oscillations reveal a methodic modulation due to secondary long-wave oscillations. In[2] they are attributed to interference in the dissociation region, a process independent of the transition in the intersection region. In the present reduction of the experimental data of[2] we shall therefore not explain the nature of these slow oscillations. We shall show only that the experimental characteristics of the rapid oscillations as functions of the energy near the threshold $\tau \sim \tau_0$ are described much better by formula (23) than by the classical formulas[3,4] such as (11).

According to (23), the relative positions τ_i of the maxima and minima of the cross section $\rho(\tau, E)$ ($i = 1, 3, \ldots$ number the maxima, starting with $i = 1$ for the first one; $i = 2, 4, \ldots$ number the minima) are given by the formula ($E_1 = 1$ eV):

$$|\tau_i - \tau_j| = \frac{\tau_0}{B}E^{1/3}|x_i - x_j| = f_{ij}E^{1/3}, \quad (31)$$

where x_i ($i = 1, 2, 3, \ldots$) give successively the positions of all the extrema and zeroes of the Airy function. We shall need subsequently the values $x_1 = -1.019$, $x_1 - x_3 = 2.23$, $x_1 - x_2 = 1.32$. The experimental values of $\tau_3 - \tau_1$ and $\tau_2 - \tau_1$ shown in Fig. 1 were taken by us from Fig. 7 of[2] and are plotted as functions of $E^{1/3}$, where E is the energy in eV. We see that both functions can be approximated by the straight lines (31) with slopes

$$f_{13} = 96.4 \text{ eV}^{2/3}\text{-deg} \quad f_{12} = 57.0 \text{ eV}^{2/3}\text{-deg} \qquad (32)$$

the ratio of which agrees exactly with the predicted $f_{13}/f_{12} = |x_1 - x_3|/|x_1 - x_2| = 1.69$. We note that the deviation of the points from the straight line has a perfectly regular character and is undoubtedly due to the same mechanism that causes modulation of the amplitude.

From f_{13} (or f_{12}) we can find the parameter $\tau_0/B = f_{ij}|x_i - x_j|$. To find the threshold value τ_0 we use the formula

$$\tau_1(E) = \tau_0 + \frac{\tau_0}{B} E^{1/3}|x_1| = \tau_0 + f_1 E^{1/3}. \qquad (33')$$

Figure 2 shows the experimental values τ_1 of the position of the first maximum as a function of $E^{1/3}$. The best straight line (33) with slope $f_1 = |x_1/(x_1 - x_3)| f_{13} = 44 \text{ eV}^{2/3}$-deg, drawn through the point, has an intercept $\tau_0 = 930$ eV-deg at $E = 0$. The same mean value of τ_0 is obtained if one plots $\tau_1 - (\tau_3 - \tau_1)x_1/(x_1 - x_3)$ as a function of E. The value $\tau_0 = 930$ eV-deg is in full agreement with the fact that $x = 0$ (the zero of the argument of the Airy function) corresponds to a point on the exponential decrease of the first peak, at a level corresponding to ~ 0.45 of the height of the peak (see, for example, Fig. 2 of[2]). The calculated values of f_{ij} and τ_0 make it possible to find the value of $B = 21.6$ for the dimensionless parameter (24).

A confirmation of the indicated analysis may be the dependence of the function $N(\tau, E)$, introduced in[2], on the energy and on τ. The function $N(\tau, E)$ assumes the integer values 0, 1, 2, ... at the points τ_1, τ_3, ...—the maxima—and half-integer values at the points of the minima. At large τ ($N(\tau) \geq 2$), when formula (11) is valid, we have

$$N(\tau, E) = \frac{1}{2\pi u} \int_{z_1(\tau)}^{z_2(\tau)} \Delta V(b(z, \tau), z)\, dz - \frac{1}{4},$$

so that $d[E^{1/2}N(\tau, E)]/d\tau$ does not depend on the energy. Indeed, if the slopes of the curves $E^{1/2}N(\tau, E)$ as functions of τ, shown in Fig. 7 of[2], are determined by starting from the points $\tau_i \geq \tau_5$ ($N \geq 2$), then $d(E^{1/2}N)/d\tau$ is independent of the energy with a good degree of accuracy. Yet the slopes of the $E^{1/2}N(\tau E)$ curves as functions of τ, obtained in[2] from the first points τ_i ($i = 1, 2, 3$), should depend on the energy, in accordance with (31) and the definition of $N(\tau, E)$ in the following manner:

$$\frac{d}{d\tau}[E^{1/2}N(\tau, E)] = E^{1/2}\frac{1}{\tau_3 - \tau_1} = 1.04 \cdot 10^{-2} E^{1/6} \text{ eV}^{-1/2}\text{-deg}^{-1}$$

Figure 3 duplicates the points of Fig. 8 of[2]. The curve representing the function (33′) describes the experimental points better than the constant value 0.026 assumed in[2].

The theoretical dependences of the absolute values of the cross section on the energy agree with experiment somewhat less closely than the data on the periods of the oscillations. It is possible that this is connected with the poorer resolution with respect to τ at high energies[2]. According to (23), the cross section at the first maximum is equal to

$$\rho(\tau_1, E) = 0.9\, r_0^2 A E^{-2/3}. \qquad (34)$$

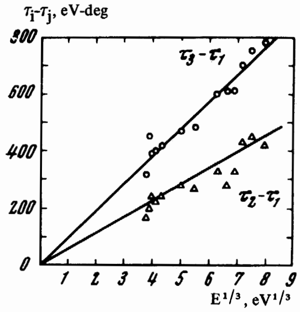

FIG. 1

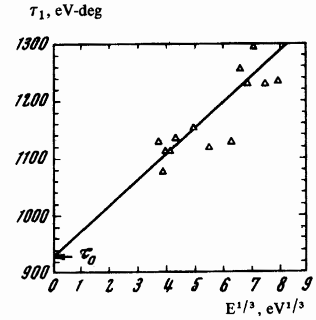

FIG. 2

FIG. 1. Energy dependence of the period of the oscillations near the threshold. The points were obtained from the data of [2].

FIG. 2. Determination of the threshold value of the angle τ_0 from the dependence of the position τ_1 of the first peak on $E^{1/3}$. The straight line with slope $f_1 = 44$ eV$^{2/3}$-deg, calculated from formula (33), has an intercept at $\tau_0 = 930$ eV-deg. The points were taken from [2].

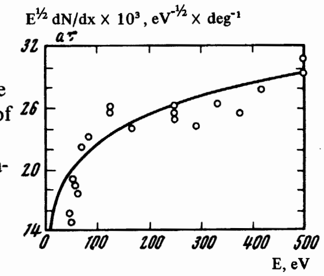

FIG. 3. Energy dependence of the function $d[E^{1/2}N(\tau, E)]/d\tau$ at values of τ close to the threshold τ_0. Points—from [2], solid curve—theoretical relation (formula (33′)).

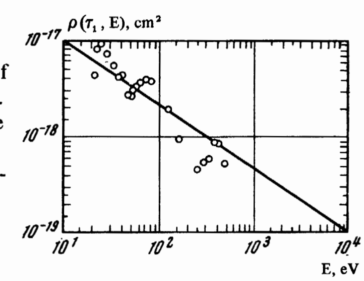

FIG. 4. Energy dependence of the inelastic-scattering amplitude. Points—from [2]; the straight line corresponding to the theoretical relation (35) is used as an approximation of $\rho(\tau_1 E)$ to determine the constant A.

Figure 4 shows the experimental plot (Fig. 10 of[2]) of $\rho(\tau_1, E)$ against E in a logarithmic scale. The straight line with the slope $-2/3$, which represents (34), corresponds to

$$\rho(\tau_1, E) = 10^{-17}\left(\frac{10}{E}\right)^{2/3} [\text{cm}^2]. \qquad (35)$$

From a comparison of (34) and (35) we find the value of A. Thus, the dimensionless parameters (24) and (25), which determine the threshold behavior of the cross section, are equal to

$$A = 0.514, \quad B = 21.6. \qquad (36)$$

However, the values of A and B depend strongly on the three dimensionless parameters $\tilde{\rho}_0/r_0^2$, $\Delta F r_0^2/\hbar u_1$, and $\delta = 2\pi V_{12}^2/\hbar u_1 \Delta F$. The number q determined in (22) is close to unity. Among the three indicated quantities, we can estimate independently the order of magnitude of $\tilde{\rho}_0/r_0^2$. Indeed, the lower bound of

$$\tilde{\rho}_0 = \frac{1}{2}\frac{\partial b^2(z_0\tau)}{\partial \ln \tau}\bigg|_{\tau=\tau_0}$$

is approximately the maximum value of the cross section $\rho_{max} \sim 10^{-17}$ cm^2, attained at $E_{max} = 25$ eV[2]. The equality $\tilde{\rho}_0 = \rho_{max}$ corresponds to a unity probability of the transition at a given τ and E (at E = 25 eV, of course, both the high-energy approximation and perturbation theory in V_{12} are contradicted). On the other hand, one can compare $\tilde{\rho}_0$ with the elastic cross section

$$\rho_{el} = \frac{1}{2}\frac{d b_{el}^2(\tau)}{d \ln \tau},$$

which equals $\sim 3 \times 10^{-17}$ cm^2 in the region of appearance of the first oscillations of the elastic cross section of He^+-Ne[1]. From the picture of the potentials of the He^+-Ne system, discussed in[2], we can expect $\tilde{\rho}_0 < \rho_{el}$. Thus, we can assume that $10^{-17} \le \tilde{\rho}_0 < 3 \times 10^{-17}$ cm^2.

The table lists the characteristics of the ΔF and V_{12} terms in the intersection region and the value of $\kappa = z_0/b_0 = \tilde{\rho}_0 \Delta F/2r_0\tau_0$, all calculated from (36) at three values of $\tilde{\rho}_0$. Here $q \sim 1$. The obtained values are close to the estimates $\Delta F \sim F_1 = -26.2$ eV/at.un. and $V_{12} \sim 0.3$ eV, given in[2].

Our analysis shows that the high-energy approximation works well even at energies $E \ge 60$ eV at potentials $V_i(r_0) \sim 13.3$ eV at the intersection point[2]. It would undoubtedly be of interest, however, to obtain the next higher terms of the expansion in V/E, so as to determine uniquely the characteristics of the terms. The theory must also be extended to include a description of the anomalies, due to the intersection of the terms, in the elastic channel.

$10^{17}\tilde{\rho}_0$, cm^2	$-\Delta F$, eV/at. un.	V_{12}, eV	$\kappa = -z_0/b_0$
1	$57{,}5\sqrt{q}$	$0{,}327q$	$0{,}34\sqrt{q}$
1,5	$31{,}4\sqrt{q}$	$0{,}216q$	$0{,}27\sqrt{q}$
2	$19{,}8\sqrt{q}$	$0{,}162q$	$0{,}23\sqrt{q}$

Here $q = 1 + \kappa^2 + \gamma \approx 1$; $\gamma = b_0 \partial^2 b(z\tau_0)/dz^2|_{z_0}$.

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