COHERENT STATES AND GREEN'S FUNCTION OF A CHARGED PARTICLE IN VARIABLE ELECTRIC AND MAGNETIC FIELDS

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New integrals of motion are obtained, coherent states and the Green's functions are constructed, and the amplitudes of the transition between the Landau levels are calculated in closed form for a nonrelativistic charged particle moving in homogeneous variable electric and magnetic fields. The problem of an oscillator with variable frequency in such fields is solved. Linear adiabatic invariants are discussed.

1. INTRODUCTION

It is known that to describe a quantum system with a Hamiltonian that is quadratic in the coordinate and momentum operators it is very convenient to use the method of integration over the Feynman classical trajectories[1]. For systems with such Hamiltonians, the Green's function is of the form

$$F(t_2, t_1)\exp(iS_{\mathrm{cl}}/\hbar).$$

It is also known that Glauber's representation of coherent states[2], which has been widely used of late, is very convenient, from the point of view of operating with pure classical concepts, for quantum systems with quadratic Hamiltonians. The calculation of the transition probabilities by using such states as generating functions was carried out by Schwinger[3]. For a one-dimensional oscillator with a constant frequency, nonspreading packets moving along classical trajectories, which in essence are coherent states, were considered by Schrödinger[4]. The representation of coherent states is closely connected with the Fock-Bargmann representation[5,6].

Until recently, coherent states were introduced mainly for systems having a quadratic Hamiltonian independent of the time, although the problem of constructing and using the standard properties of coherent states for nonstationary systems is of considerable interest. Coherent states for a charged particle in a uniform constant magnetic field, and also in constant uniform electric and magnetic fields ($\mathbf{E}\cdot\mathbf{H} = 0$) were introduced in[7] (both the Schrödinger equation and the Dirac equation were considered). These coherent states are closely connected with the classical packets constructed by Darwin[8] and by Kennard[9]. The coherent states for an n-dimensional oscillator with frequencies that depend arbitrarily on the time, and also for a nonrelativistic charge in a uniform variable magnetic field were constructed and used in[10] to calculate the Green's function and the probabilities of the transitions between energy levels. Coherent states were introduced and used in[11] for an n-dimensional oscillator with variable frequencies, acted upon by driving forces with an arbitrary time dependence, and in[12] for a charged particle moving in a uniform constant magnetic field and in a variable electric field perpendicular to it and arbitrarily dependent on the time.

The purpose of the present study is to find all the independent linear integrals of motion for a nonrelativistic charged particle moving in electric and magnetic fields having an arbitrary time dependence and satisfying the quasistationarity condition, and also to find the exact solution of the Schrödinger equation of this problem, which is a generalization of the Landau solution for a charge in a constant uniform magnetic field[13] to the case of variable fields. We introduce coherent states for the problem in question and use them to calculate the green's function in explicit form. For the case of electric and magnetic fields such that in the remote past and in the future the magnetic field is constant and the electric field is equal to zero, we calculate exactly (not by perturbation theory) the amplitudes of the transition between the levels with given energy and angular momentum; these amplitudes are expressed in terms of Hermite's polynomials of four variables. We calculate the amplitudes of the transition between the coherent states. We also discuss the adiabatic invariants for the problem in question. We solve completely the problem of a charged oscillator situated in variable uniform electric and magnetic fields.

2. THE INVARIANTS

The Schrödinger equation for a charged particle moving in a variable electromagnetic field is

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2}\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y}\right) + \frac{\omega^2}{8}(x^2+y^2)\psi + \frac{i\omega}{2}\left(x\frac{\partial\psi}{\partial y} - y\frac{\partial\psi}{\partial x}\right) - (E_1x + E_2y)\psi. \tag{1}$$

We used a system of units in which $\hbar = c = 1$, and the charge and mass are set equal to unity for simplicity. The external electromagnetic field is given by the potentials*

$$\mathbf{A} = {}^1/_2[\mathbf{H}(t)\mathbf{r}], \quad \varphi = -\mathbf{E}(t)\mathbf{r},$$

where $\mathbf{H}(t) = (0, 0, \omega(t))$, $\mathbf{E}(t) = (E_1(t), E_2(t), 0)$. For simplicity we consider the case $\mathbf{E}\cdot\mathbf{H} = 0$.

In the case of constant potentials, Eq. (1) describes the motion of a particle in crossed constant fields[7,12].

*$[\mathbf{H}(t)\mathbf{r}] \equiv \mathbf{H}(t)\times\mathbf{r}$.

The problem of a particle having a spin and moving in such fields reduces to the solution of Eq. (1) by replacement of the components of the wave function

$$\psi_{s_z}=\psi_{s_z}{}^0\exp\left[i\mu_0\int\limits_0^t\omega(\tau)\,d\tau\cdot s_z\right],$$

corresponding to states with given spin projections s_z on the magnetic field. In Eq. (1) we took into account that part of the wave function which depends on the variables x and y, since the motion along the magnetic field is free. The chosen field potentials satisfy Maxwell's equations in the quasistationary-field approximation.

We introduce the variables z(t) corresponding to a transition to a moving coordinate frame[10],

$$z(t)=-2^{-1/2}(x+iy)\exp\left[i\int\limits_0^t\omega(\tau)\,d\tau/2\right]. \tag{2}$$

In Terms of these variables, the Schrödinger equation (1) assumes the form

$$i\frac{\partial\psi}{\partial t}=-\frac{\partial^2\psi}{\partial z\,\partial z^*}+\frac{\omega^2}{4}|z|^2\psi+(Fz^*+F^*z)\psi, \tag{3}$$

where we have introduced the complex force

$$F=2^{-1/2}(E_1+iE_2)\exp\left[i\int\limits_0^t\omega(\tau)\,d\tau/2\right].$$

As noted in[10], according to the correspondence principle, any quantum system should possess 2n real (or n complex) integrals of motion, where n is the number of degrees of freedom. These integrals of motion can be chosen to correspond to the initial data on the classical trajectory of the system in phase space. The integrals of motion can be determined by the method used in[10]. It is easy to verify directly that the operators

$$\begin{aligned}A&=2^{-1/2}[\dot{\varepsilon}z+i\varepsilon\partial/\partial z^*+\dot{\varepsilon}z_0-\varepsilon\dot{z}_0],\\ B&=2^{-1/2}[i\dot{\varepsilon}z^*-\varepsilon\partial/\partial z+i\dot{\varepsilon}z_0^*-i\varepsilon\dot{z}_0^*]\end{aligned} \tag{4}$$

are integrals of the motion (they commute with the operator $i\partial/\partial t-\hat{H}$). The time-dependent functions $\epsilon(t)$ and $z_0(t)$ are solutions of the equations

$$\ddot{\varepsilon}+\omega^2\varepsilon/4=0,\quad \ddot{z}_0+\omega^2 z_0/4=0, \tag{5}$$

with

$$\varepsilon=|\varepsilon|\exp\left[i\int\limits_0|\varepsilon|^{-2}d\tau\right].$$

The operators (4) satisfy the commutation relations of the Bose creation and annihilation operators

$$[A,A^+]=[B,B^+]=1,\quad [A,B]=[A,B^+]=0.$$

In the case of a zero electric field ($z_0=0$), these operators go over into the integrals of motion constructed in[10]. If we put in this case

$$\varepsilon=\varepsilon_c\equiv 2^{1/2}\omega_c^{-1/2}\exp(i\omega_c t/2), \tag{6}$$

where ω_c is a constant frequency, then the invariants (4) go over into the operators constructed in[7] for a charge moving in a uniform constant magnetic field.

The physical meaning of the integrals of motion (4), which are linear functions of the coordinate and momentum operators, lies in the fact that their real and imaginary parts specify the initial point on the classical trajectory. In the case of a constant magnetic field the operator B specifies the center of the circle of classical motion, and the operator A specifies the initial coordinates of the motion on a circle about this center. According to the Ehrenfest theorem, the mean values of the coordinate and momentum operators move on classical trajectories. The physical meaning of the operators A and B in (4) is also illustrated by the fact that their complex eigenvalues specify precisely the initial mean values of the coordinates and the momenta. All other integrals of motion, particularly quadratic ones, can be constructed from the linear invariants (4).

The foregoing statement also holds for other quantum systems (see, for example,[10,12]), but for nonquadratic Hamiltonians it is a more complicated matter to construct explicitly linear integrals of motion whose complex eigenvalues specify the initial mean values of the coordinate and momentum operators. Actually, this is equivalent to the problem of constructing coherent states and calculating the Feynman path integral for nonquadratic Hamiltonians.

2. REDUCTION TO THE CASE $\varphi = 0$ AND COHERENT STATES

Let us consider the connection between the solutions of Eqs. (1) and (3) and the solutions of the Schrödinger equations when there is no electric field. It is easy to verify directly that the unitary operator (compare with[17])

$$D=\exp\{-i(z\dot{z}_0^*+z^*\dot{z}_0)\}\exp\left\{z_0\frac{\partial}{\partial z}+z_0^*\frac{\partial}{\partial z^*}+i\int\limits_0^t\left(\frac{\omega^2}{4}|z_0|^2-|\dot{z}_0|^2\right)d\tau\right\} \tag{7}$$

establishes a one-to-one correspondence between the solutions ψ of Eqs. (1) and (3) and the solutions ψ_0 for the charge in a variable magnetic field at a zero electric field, i.e., $\psi=D\psi_0$. It is therefore easy to transfer all the results obtained for a charge in the magnetic field[7,10] to the here-considered general case.

Let us construct the coherent states $|\alpha\beta\rangle$ given by the explicit expression

$$\begin{aligned}|\alpha\beta\rangle=\varepsilon^{-1}\pi^{-1/2}\exp\Big\{&i\frac{\dot{\varepsilon}}{\varepsilon}(|z|^2+|z_0|^2)-\frac{1}{2}(|\alpha|^2+|\beta|^2)\\ &-\left(i\dot{z}_0^*-i\frac{\dot{\varepsilon}}{\varepsilon}z_0^*\right)z-\left(i\dot{z}_0-i\frac{\dot{\varepsilon}}{\varepsilon}z_0\right)z^*\\ &-i\varepsilon^{-1}2^{+1/2}[\alpha(z^*+z_0^*)-i\beta(z+z_0)]-i\alpha\beta\frac{\varepsilon^*}{\varepsilon}+i\int\limits_0^t\left(\frac{\omega^2}{4}|z_0|^2-|\dot{z}_0|^2\right)d\tau\Big\}.\end{aligned} \tag{8}$$

It is easy to verify that the normalized wave function $|\alpha\beta\rangle$ is a solution of Eqs. (1) and (3) and satisfies the conditions $A|\alpha\beta\rangle=\alpha|\alpha\beta\rangle$ and $B|\alpha\beta\rangle=\beta|\alpha\beta\rangle$. α and β are constant complex numbers. The coherent states $|\alpha\beta\rangle$ (8) can be obtained from the ground state with the aid of unitary Weyl shift operators[2]

$$D(\alpha)=\exp(\alpha A^+-\alpha^*A),\quad D(\beta)=\exp(\beta B^+-\beta^*B),$$

in accordance with the easily verified relation $D(\alpha)D(\beta)|00\rangle=|\alpha\beta\rangle$.

The coherent states (8) comprise a complete system of functions, with $|\alpha\beta\rangle^+=\langle\alpha\beta|$ and

$$\pi^{-2}\int|\alpha\beta\rangle\langle\alpha\beta|\,d^2\alpha\,d^2\beta=1,\quad d^2\alpha=d\,\mathrm{Re}\,\alpha\,d\,\mathrm{Im}\,\alpha.$$

The scalar product of two coherent states is given by

$$\langle\gamma\delta|\alpha\beta\rangle = \exp\{-{}^1/_2[|\alpha|^2+|\beta|^2+|\gamma|^2+|\delta|^2]+\gamma^*\alpha+\delta^*\beta\}. \quad (9)$$

The coherent states specify the generating function for states that are eigenstates of the quadratic integrals of motion A^+A and B^+B, in accordance with the well-known relations (see[2,7])

$$|\alpha\beta\rangle = \exp\left[-\frac{1}{2}(|\alpha|^2+|\beta|^2)\right]\sum_{n_1,n_2=0}^{\infty}\frac{\alpha^{n_1}\beta^{n_2}}{\sqrt{n_1!\,n_2!}}|n_1n_2\rangle, \quad (10)$$

where the wave functions $|n_1n_2\rangle$ satisfy the Schrödinger equation (1) and the orthogonality relations

$$\langle m_1m_2|n_1n_2\rangle = \delta_{m_1n_1}\delta_{m_2,n_2},$$

with

$$A^+A|n_1n_2\rangle = n_1|n_1n_2\rangle, \quad B^+B|n_1n_2\rangle = n_2|n_1n_2\rangle.$$

It is easy to calculate the explicit form of the functions

$$|n_1n_2\rangle = |00\rangle\left(\frac{p!}{(p+|n_1-n_2|)!}\right)^{1/2} i^{n_1}(-1)^{(n_1+n_2+|n_1-n_2|)/2}$$
$$\times\left(\frac{z_0+z}{z_0^*+z^*}\right)^{(n_2-n_1)/2}\left(\frac{\varepsilon^*}{\varepsilon}\right)^{(n_1+n_2)/2}\left(\frac{\sqrt{2}z_0+\sqrt{2}z}{\varepsilon}\right)^{|n_1-n_2|}L_p^{|n_1-n_2|}\left(2\left|\frac{z_0+z}{\varepsilon}\right|^2\right), \quad (11)$$

where $p = {}^1/_2(n_1+n_2-|n_1-n_2|)$, $L_p^m(x)$ is a generalized Laguerre polynomial[14] and the quantity $|00\rangle$ is given by formula (8), in which $\alpha = \beta = 0$. In the case of a constant magnetic field and a zero electric field, the solution (11) goes over, as can be readily verified, into the known wave functions[13] corresponding to states with given energy and angular momentum $l_z = n_2 - n_1$. Thus, although neither the energy nor the angular momentum is conserved in this problem, there exist quantum numbers that correspond to the initial angular momentum and the initial energy and also determine the solutions completely at subsequent instants of time. We have thereby shown that the quantum numbers in this problem are exact invariants.

It is easy to verify that in the coherent state (8) the distributions with respect to the quantum numbers n_1 and n_2 are Poisson distributions:

$$|\langle n_1n_2|\alpha\beta\rangle|^2 = \exp\{-(|\alpha|^2+|\beta|^2)\}\frac{|\alpha|^{2n_1}|\beta|^{2n_2}}{n_1!\,n_2!}. \quad (12)$$

Since we know for this problem the exact solutions (8) and (11), which constitute a complete system of functions, we can find an expression for the Green's function, which can be obtained from the relation

$$G(2,1) = \pi^{-2}\int d^2\alpha\, d^2\beta\,|\alpha\beta,2\rangle\langle\alpha\beta,1|,$$

or by using the operator (7) and the known Green's function for the problem without the electric field[10]. Leaving out the known factor[1] corresponding to free motion along the magnetic field, we have the following expression for the Green's function of Eqs. (1) and (3):

$$G(x_2y_2t_2x_1y_1t_1) = [2\pi i|\varepsilon(t_1)\varepsilon(t_2)|\sin\gamma]^{-1}\cdot$$
$$\times\exp\left\{\frac{i}{2}\left[\frac{R_2^2}{2}\frac{d|\varepsilon|^2}{dt_2}-\frac{R_1^2}{2}\frac{d|\varepsilon|^2}{dt_1}+(\mathbf{R}_2-\mathbf{R}_1)^2\operatorname{ctg}\gamma+2[\mathbf{R}_1\mathbf{R}_2]\right]\right.$$
$$\left.+i\int_{t_1}^{t_2}\left(\frac{\omega^2}{4}|z_0|^2-|\dot z_0|^2\right)d\tau - i[z_2\dot z_0^*(t_2)+z_2^*\dot z_0(t_2)-z_1\dot z_0^*(t_1)-z_1^*\dot z_0(t_1)]\right\} \quad (13)$$

where x_i, y_i, and t are expressed in terms of z_i by formulas (2), and the lengths of the vectors $\mathbf{R}_i$ and the quantity γ are given by the equations

$$R_i = \sqrt{2}|z_i+z_0(t_i)|/|\varepsilon(t_i)|, \quad \gamma = \int_{t_1}^{t_2}|\varepsilon|^2 d\tau,$$

while the difference between the angles of the vectors $\mathbf{R}_1$ and $\mathbf{R}_2$ is

$$\varphi_{\mathbf{R}_2}-\varphi_{\mathbf{R}_1} = \arg[z_2+z_0(t_2)]-\arg[z_1+z_0(t_1)]-\gamma.$$

It is easy to verify that at a zero electric field ($z_0 = 0$) formula (13) goes over into the expression obtained in[10] for the Green's function (we note that in[10] there are misprints in the expression for the Green's function), and in the case of a constant magnetic field $\epsilon = \epsilon_c$ (see (6)) formula (13) goes over into the well-known expression obtained in[15] (see also[1]).

4. TRANSITION AMPLITUDES AND GENERATING FUNCTION FOR THE TRANSITION PROBABILITIES

We now consider the question of calculating the matrix elements of the S matrix for the case when a constant magnetic field ω_i existed prior to the zero instant of time, at which a variable electric field was turned on and the magnetic field started to vary in magnitude, and in the remote future the electric field is turned off and the magnetic field becomes constant and equal to ω_f. It is easy to calculate the S matrix at any instant of time t. The S matrix is expressed in terms of the parameters $\xi(t)$, $\eta(t)$, $\delta_a(t)$ and $\delta_b(t)$, which are closely connected with the solutions of Eqs. (5). The parameters $\xi(t)$ and $\eta(t)$ are expressed in terms of the solution $\epsilon(t)$ and its derivative $\dot\epsilon(t)$:

$$\varepsilon = (2/\omega_f)^{1/2}[\xi\exp(i\omega_f t/2) - i\eta\exp(-i\omega_f t/2)],$$
$$\dot\varepsilon = i(\omega_f/2)^{1/2}[\xi\exp(i\omega_f t/2) + i\eta\exp(-i\omega_f t/2)],$$

with

$$\varepsilon^*\dot\varepsilon - \varepsilon\dot\varepsilon^* = 2i, \quad |\xi|^2-|\eta|^2 = 1.$$

$\epsilon(t)$ goes over into $(2/\omega_i)^{1/2}\exp(i\omega_i t/2)$ as $t\to-\infty$, and $\xi(t)$ and $\eta(t)$ become constant complex numbers as $t\to\infty$.

The solution of the inhomogeneous equation (5) is

$$z_0 = -2^{-1/2}(\delta_a\varepsilon^* + \delta_b^*\varepsilon),$$
$$\delta_a = -2^{-1/2}\int_0^t\varepsilon(\tau)F(\tau)d\tau, \quad \delta_b = -i2^{-1/2}\int_0^t\varepsilon(\tau)F^*(\tau)d\tau. \quad (14)$$

When $t<0$ we have $F(t) = 0$ and $z_0 = 0$, and as $t\to\infty$ the force $F(t) = 0$ and $\delta_a(t)$ and $\delta_b(t)$ become constant numbers. The final states $|\mu\nu, f\rangle$ describe the coherent states of the particle in a constant field with frequency ω_f. These states are generated from the vacuum $|00, f\rangle$ in the usual manner:

$$|\mu\nu, f\rangle = D(\mu)D(\nu)|00,f\rangle,$$

where

$$D(\mu) = \exp(\mu A_f^+ - \mu^* A_f), \quad D(\nu) = \exp(\nu B_f^+ - \nu^* B_f).$$

(We note that in[10] the vacuum differs from that introduced in the present article by a phase factor.) The operators A_f and B_f are given by formula (4), and the explicit form of these states is given by formula (7); in (4) and (8) it is then necessary to put

$$z_0 = 0, \quad \varepsilon(t) = (2/\omega_f)^{1/2}\exp(i\omega_f t/2).$$

The operators A and B are connected with A_f and B_f by the relations

$$A = \xi(t)A_f + \eta(t)B_f^+ + \delta_a(t), \quad B = \xi(t)B_f + \eta(t)A_f^+ + \delta_b(t). \quad (15)$$

We note that the operator (7) differs by a phase factor from the Weyl shift operator

$$D = \exp\left[i\int_0^t (Fz_0^* + F^*z_0)\,d\tau + \delta_a A^+ - \delta_a^* A + \delta_b B^+ - \delta_b^* B\right]. \quad (16)$$

The amplitude of the transition from the initial coherent state into the final coherent state can be obtained either by taking the integral or by using the properties of the Weyl-shift operator; it takes the form

$$\langle\mu\nu, f|\alpha\beta, i\rangle = \xi^{-1}\exp\Big\{\delta_a\delta_b\frac{\eta^*}{\xi} - (|\delta_a|^2 + |\delta_b|^2 + |\alpha|^2 + |\beta|^2 + |\mu|^2 + |\nu|^2) + \frac{1}{\xi}[(\alpha-\delta_a)\mu^* + (\beta-\delta_b)\nu^* - \eta\mu^*\nu^* + \eta^*\alpha\beta + \alpha(\xi\delta_a^* - \eta^*\delta_b) + \beta(\xi\delta_b^* - \eta^*\delta_a)] + i\int_0^t (Fz_0^* + F^*z_0)\,d\tau\Big\}. \quad (17)$$

For the probabilities of the transitions between the Landau levels, we can calculate the generating function

$$f(z_1z_2y_1y_2) = \sum_0^\infty w_{m_1m_2n_1n_2} z_1^{n_1} z_2^{n_2} y_1^{m_1} y_2^{m_2},$$

in the form

$$f = \Delta^{-1}\exp\Big\{\frac{1}{2\Delta}[sz_1 + tz_2 + uy_1 + wy_2 + (s+t+u+w)y_1y_2z_1z_2 - sy_1y_2z_1 - uz_1z_2y_2 - wz_1z_2y_1 - ty_1y_2z_1 - qy_1y_2 - (2u-q)y_1z_1 - (2w-q)y_2z_2 - (2s-2u+q)z_1z_2] - (s+t+u+w)/2|\xi|^2\Big\}, \quad (18)$$

where

$$\Delta = |\eta|^2(y_1y_2 - 1)(z_1z_2 - 1) + (y_1z_1 - 1)(y_2z_2 - 1),$$
$$u = |\delta_a|^2, \quad w = |\delta_b|^2, \quad s = |\xi|^2u + |\eta|^2w - 2\mathrm{Re}\,(\xi\eta\delta_a^*\delta_b^*),$$
$$t = s + w - u, \quad q = u - s + (s+w)R, \quad R = |\eta|^2/|\xi|^2.$$

The transition amplitude (17) is independent of the variable t at large values of the time, as can be readily verified by differentiation. The complex constants ξ, η, δ_a, and δ_b, which determine the asymptotic forms of the trajectories (14), determine completely the transition amplitude (17). Since the amplitude (17) determines the generating function for the amplitudes of the transition between states with given values of the energy and of the angular momentum $\langle m_1m_2, f | n_1n_2, i\rangle$, these amplitudes can easily be calculated by the usual differentiation. We have

$$\langle m_1m_2, f|n_1n_2, i\rangle = \frac{H_{n_1n_2m_1m_2}(x_1x_2x_3x_4)}{(m_1!\,m_2!\,n_1!\,n_2!)^{1/2}}\langle 00, f|00, i\rangle. \quad (19)$$

Here $H_{n_1n_2m_1m_2}$ is a Hermite polynomial of four variables, with

$$x_1 = \delta_a - \delta_b^*\eta/\xi^*, \quad x_2 = \delta_b - \delta_a^*\eta/\xi^*,$$
$$x_3 = -\delta_a^*/\xi^*, \quad x_4 = -\delta_b^*/\xi^*,$$

and the symmetric quadratic form $\varphi = a_{ik}x_ix_k$, which determines these Hermite polynomials, has four nonvanishing coefficients $a_{12} = -\eta^*/\xi$, $a_{34} = \eta/\xi$, $a_{13} = a_{24} = -1/\xi$. For the properties of Hermite polynomials of many variables see, for example,[16].

It is interesting to note that the amplitude obtained in (11) for the transition between energy levels of an oscillator with a variable frequency acted upon by a variable driving force can also be calculated exactly and expressed in terms of Hermite polynomials of two variables. The transition amplitudes for such an oscillator were first calculated by the generating-function method in[17] (see also[1,3]) and in the adiabatic approximation in[18]. These amplitudes were also constructed in[19] by using quadratic invariants that depend explicitly on the time. The overlap integrals, which give the transition probabilities for the oscillator, were investigated in[20].

It is of interest to consider the amplitude of the transition of the initial coherent state $|\alpha\beta, i\rangle$ to the Landau levels of the final state $|m_1m_2, f\rangle$. To this end it is necessary to differentiate the generating function given by formula (17) with respect to the parameters μ^* and ν^*. We obtain as a result

$$\langle m_1m_2, f|\alpha\beta, i\rangle = \langle 00, f|\alpha\beta, i\rangle\frac{\sqrt{m_1!\,m_2!}}{(\frac{1}{2}(m_1+m_2+|m_1-m_2|))!} \times\left(-\frac{\eta}{\xi}\right)^p\left(\frac{\alpha-\delta_a}{\xi}\right)^{(m_1-m_2+|m_1-m_2|)/2}\left(\frac{\beta-\delta_b}{\xi}\right)^{(m_2-m_1+|m_1-m_2|)/2} L_p(s),$$
$$p = \frac{m_1+m_2-|m_1-m_2|}{2}, \quad s = \frac{(\alpha-\delta_a)(\beta-\delta_b)}{\xi\eta} \quad (20)$$

where $L_p^{|m_1m_2|}$ is a generalized Laguerre polynomial[14] and the amplitude $\langle 00, f|\alpha\beta, i\rangle$ is given by formula (17).

5. CHARGED OSCILLATOR IN VARIABLE FIELDS

The results can be easily transferred to the case of a charged oscillator having equal variable oscillation frequencies $\Omega(t)$ along the x and y axes, having a frequency $\omega_3(t)$ along the z axis, and moving in the uniform fields

$$\mathbf{A} = \tfrac{1}{2}[\mathbf{Hr}], \quad \varphi = -\mathbf{E}(t)\mathbf{r},$$
$$\mathbf{E}(t) = (E_1(t), E_2(t), E_3(t)).$$

The wave function breaks up in this case into a product of two functions $\psi_1(x, y, t)\psi_2(z, t)$. The function, which depends on the variables x and y, satisfies the equation

$$i\frac{\partial\psi_1}{\partial t} = -\frac{\partial^2\psi_1}{\partial z\,\partial z^*} + \frac{\omega^2}{4}|z|^2\psi_1 + \Omega^2|z|^2\psi_1 + (Fz^* + F^*z)\psi_1, \quad (21)$$

where z is given by formula (2). Equation (21) reduces to Eq. (3) by making the substitution $\omega(t) \to \sqrt{\omega^2(t) + 4\Omega^2(t)}$. The function $\psi_2(z, t)$, which describes the motion of an oscillator along a magnetic field, satisfies the Schrödinger equation for an oscillator with variable frequency $\omega_3(t)$, moving under the action of a driving force; this equation was solved in[11,17]. The transition amplitudes, which are analogous to (19), factor out in this case and are expressed in terms of products of Hermite polynomials of two and four variables.

We have thus solved completely the general problem of constructing exact solutions, coherent states, the Green's function, and the transition amplitudes for a charged oscillator with variable frequency in variable electric and magnetic fields. The general connection between the coherent states and the noncompact groups U(2, 1) and U(3, 1), which are dynamic groups of the problems in question and the group of magnetic translations[21], makes it possible to introduce for our problem the quasimomentum representation and to construct Bloch functions that describe the states with a quantized electromagnetic flux. The coherent states are also convenient for an analysis of the density matrix.

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