NONLINEAR EFFECTS ASSOCIATED WITH THE PROPAGATION OF SHORT WAVELENGTH SOUND IN CONDUCTORS

Yu. M. GAL'PERIN and V. D. KAGAN

Institute of Semiconductors, USSR Academy of Sciences

Submitted May 6, 1970

Zh. Eksp. Teor. Fiz. 59, 1657–1666 (November, 1970)

The propagation in conductors of high-frequency sound whose wavelength is much smaller than the mean free path of the conduction electrons is considered. By a method of iterations with respect to the amplitude of the wave, developed in the present article, it is shown that the most important mechanisms of nonlinearity are heating of the electron gas by the effective field of the sound wave, and also the strong distortion of the distribution function of the electrons which are moving in phase with the wave. Estimates show that at the present stage of development of experimental techniques, the nonlinear effects under consideration are quite accessible to experimental investigation. A theory is developed for the propagation of a wave of finite amplitude in the case when the dominant nonlinear mechanism is distortion of the distribution function of the electrons moving in phase with the wave.

RECENTLY it has become possible to investigate the propagation in conductors of high-frequency sound whose wavelength $2\pi/q$ is much smaller than the mean free path l of the electrons,

$$ql \gg 1. \quad (1)$$

The propagation of such a sound wave of small amplitude has been well investigated theoretically[1] and experimentally.[2-5] The object of the present work is an investigation of nonlinear effects, which have been insufficiently studied in detail. The absorption of a beam of waves having a sufficiently broad spectrum

$$\frac{\Delta q}{q} \gg \frac{\bar{p}}{\hbar q}\frac{1}{ql} \quad (2)$$

(Δq is the width of the spectrum of the beam of waves, $\bar{p}$ is the characteristic momentum of an electron) is considered in the interesting article by Zil'berman[6] which appeared recently. We shall consider the opposite limiting case—nonlinear effects associated with the propagation of a monochromatic wave.

If the wavelength of the sound is much larger than the mean free path of the electrons ($ql \ll 1$), one can regard the sound wave as an external field, slowly varying in time and space, which acts on the conduction electrons. The interaction of such a wave with the electrons may be described in the hydrodynamical approximation, where the basic mechanism is concentration.[7] The concentration mechanism of nonlinearity consists in the capture of a portion of the conduction electrons by the potential wells of the effective periodic field associated with the sound wave. A decrease in the concentration of free electrons in turn has an effect on the electrical conductivity of the substance and changes the absorption and velocity of sound. It is obvious that the parameter which determines the effectiveness of such a nonlinear mechanism is the quantity $e\varphi_0/\bar{\epsilon}$, where φ_0 is the amplitude of the potential of the effective field associated with the wave and $\bar{\epsilon}$ is the characteristic electron energy.

If $ql \gg 1$, the sound wave effectively interacts only with the electrons which are moving in phase with the wave, for which

$$qv \sim \omega \quad (3)$$

(ω is the frequency of the sound). This leads to a substantial distortion of the electron distribution function in the region of velocities satisfying condition (3). The amount of nonlinearity is determined by this distortion, which depends both on the intensity of the sound wave and on the relaxation processes of the electron's momentum, which tend to make the electron distribution function isotorpic.

Another nonlinear mechanism is heating of the electron gas by the field of the sound wave, which arises in the case when the energy transferred to the electrons from the sound wave during the energy relaxation time τ_ϵ is of the order of the average energy of an electron (compare with [8]).

We see that the indicated nonlinear mechanisms are basic, where in a number of cases the first mechanism, which has not been considered in the existing literature, dominates. In this connection in the case $ql \gg 1$ the nonlinear effects appear at very much smaller intensities of the sound than the concentration nonlinearity in the case of small values of ql.[1)]

1. CORRECTIONS TO THE ABSORPTION AND VELOCITY OF SOUND

In this section we develop a method of iterations in the amplitude of the sound wave in order to determine the nonlinear corrections to the absorption and velocity of sound, and also in order to compare the effectiveness of the various nonlinear mechanisms. For definiteness, let us assume that the interaction of the electrons with sound is piezoelectric in nature. For simplicity let us consider the case of the simplest geometry, when the transverse or longitudinal sound wave

1) As a rule the nonlinear effects of lattice origin in conductors begin at much larger sound intensities than the nonlinear effects of electronic origin. [9]

is propagating along an axis of symmetry of not lower than third order (the x axis). Here we shall assume the energy spectrum of the electrons to be quadratic and isotropic; however, in order of magnitude the obtained results are valid for an arbitrary energy spectrum. The necessary generalizations to the case of a deformation potential and also to the case of many-valley conductors do not present any difficulties and will be carried out below.

Under the indicated assumptions, the complete system of equations for the electrons and for the sound wave has the form

$$\rho\frac{\partial^2 u}{\partial t^2}=c\frac{\partial^2 u}{\partial x^2}-\beta\frac{\partial^2\varphi}{\partial x^2}, \tag{4}$$

$$-\varepsilon_0\frac{\partial^2\varphi}{\partial x^2}-4\pi\beta\frac{\partial^2 u}{\partial x^2}=4\pi en, \tag{5}$$

$$\frac{\partial f}{\partial t}+v_x\frac{\partial f}{\partial x}-e\frac{\partial\varphi}{\partial x}\frac{\partial f}{\partial p_x}+\hat{I}f=0. \tag{6}$$

Here u is the lattice displacement, β is the piezoelectric modulus, ϵ_0 is the dielectric constant, ρ is the density of the crystal, c is the modulus of elasticity, n is the excess concentration of electrons, f is the distribution function of the electrons, φ is the potential of the electric field associated with the sound wave, and $\hat{I}$ is the collision operator of the electrons.

The conduction electrons are described by Boltzmann's kinetic equation (6). Such a classical description, as one can easily show, is possible upon fulfilment of the condition

$$\hbar^2 q^2/m<\hbar/\tau_p. \tag{7}$$

Condition (7) indicates the smallness of the corrections caused by taking the finiteness of the phonon's momentum into consideration in the law of energy conservation associated with an elementary interaction event in comparison with the uncertainty of the electron's energy.

In the linear approximation, assuming $u\sim\exp i(qx-\omega t)$, from Eqs. (4)–(6) we have the following dispersion equation:

$$\frac{q_0^2-q^2}{q^2}\left[1-\frac{4\pi e^2}{\varepsilon_0 q^2}K_q^0(\omega)\right]=\chi, \tag{8}$$

where $q_0=\omega/w_0$, $w_0=\sqrt{c/\rho}$, and $\chi=4\pi\beta^2/\epsilon_0 c$ is the electromechanical coupling constant. The function $K_q^0(\omega)$ is defined by the relation

$$n(q,\omega)=e\varphi(q,\omega)K_q^0(\omega) \tag{9}$$

and is calculated from the kinetic equation (6).

In general Eq. (8) has four solutions corresponding to two sound waves (forward and backward) and two electron density "waves" which are attenuated over distances of the order of the Debye radius. Since we are interested in the absorption and amplification of sound, let us assume that a periodic displacement having a frequency ω is created at the boundary of the crystal and there are no reflected waves. Neglecting the narrow region near the crystal's boundary, one can also disregard the electron density "wave". Equation (8) determines the wave vector of the sound wave, whose imaginary part describes the absorption of sound, and the difference between the real part and the value of q_0 is equal to the change in the velocity of sound due to the interaction with the electrons.

The iterations are carried out in the following way. The equilibrium distribution function F_0 is used to the lowest order in the amplitude of the sound wave in the term $-e(\partial\varphi/\partial x)(\partial f/\partial p_x)$ of Eq. (6). This gives

$$f^{(1)}=\frac{1}{\mathbf{qv}-\omega-i\hat{I}}e\varphi^{(1)}\mathbf{q}\frac{\partial F_0}{\partial\mathbf{p}}. \tag{10}$$

By virtue of condition (1) the function $f^{(1)}$ is large only in a narrow range of values of v ($\mathbf{q}\cdot\mathbf{v}\sim\omega$). Therefore, in connection with the action of the collision operator on it, in this region only the part $\hat{I}f^{(1)}=\nu_p f^{(1)}$ is essential, where $\nu_p=\sum_{p'}W_{pp'}$ and $W_{pp'}$ is the probability for an electron transition from the state p to p′. For $K_q^0(\omega)$ we have the expression[1]

$$K_q^0(\omega)=\frac{2}{(2\pi\hbar)^3}\int d^3p\frac{1}{\mathbf{qv}-\omega-i\nu_p}\mathbf{q}\frac{\partial F_0}{\partial\mathbf{p}}, \tag{11}$$

which one can easily evaluate:

$$-\frac{4\pi e^2K_q^0(\omega)}{\varepsilon_0 q^2}=\frac{\varkappa^2}{q^2}\left(1+i\xi\frac{w}{\bar{v}}\right). \tag{12}$$

Here $\bar{v}$ is the characteristic electron velocity ($\bar{v}=\sqrt{2T/m}$ for Boltzmann statistics and $v=v_F$ for Fermi statistics), κ is a quantity which is the reciprocal of the Debye radius, $\xi=\pi$ for Fermi statistics, and $\xi=2\sqrt{\pi}$ for Boltzmann statistics. It is clear that expression (12) does not depend on the collision time; collisions only influence the corrections of order $(ql)^{-1}$. This is the case of so-called collisionless absorption, which is analogous to Landau damping in a plasma.[10] By virtue of the smallness of the coupling constant χ for conductors, in contrast to a plasma, the situation when Im $ql\ll 1$ is typical. We shall assume that this condition is satisfied.

To second order in the amplitude of the wave, due to the term

$$-2\,\mathrm{Re}\,e\frac{\partial\varphi^{(1)}}{\partial x}\left[\frac{\partial f^{(1)}}{\partial p_x}\right]^*$$

of Eq. (6) there arises a correction to the part ($\sim\exp(-\Gamma x)$, $\Gamma=\mathrm{Im}\,q$) of the distribution function $F=F_0+\Delta F$ which is slowly changing in space. Correct to terms proportional to $(e\varphi_0)^2$ the kinetic equation for ΔF has the form

$$-2[e\tilde{\varphi}_0]^2e^{-2\Gamma x}\mathbf{q}\frac{\partial}{\partial\mathbf{p}}\frac{\nu_p}{(\mathbf{qv}-\omega)^2+\nu_p^2}\mathbf{q}\frac{\partial F_0}{\partial\mathbf{p}}+\hat{I}\Delta F=0, \tag{13}$$

where $\tilde{\varphi}_0$ is the amplitude of the potential at the boundary of the crystal.

The function $\Delta F(p)$ is an extremely anisotropic function of p. Its average over a surface of constant energy determines the transfer of energy from the sound wave to the electrons, giving rise to heating of the electron gas. Averaging (13) over a surface of constant energy and taking into consideration that $\hat{I}\Delta F\sim\Delta F/\tau_\epsilon$, one can easily obtain the parameter characterizing the heating of the electrons, which determines the nonlinear corrections to the absorption and velocity of sound. For the corrections to the function $K_q^0(\omega)$ we have

$$\frac{\Delta K_q^0(\omega)}{K_q^0(\omega)}\sim\left[\frac{e\tilde{\varphi}_0}{\bar{\varepsilon}}\right]^2e^{-2\Gamma x}\begin{cases}\dfrac{\tau_\varepsilon}{\tau_p}, & \dfrac{w}{\bar{v}}\omega\tau_p\ll 1;\\[2mm] \dfrac{\tau_\varepsilon}{\tau_p}(ql)\left(\dfrac{w}{\bar{v}}\right)^2, & \dfrac{w}{\bar{v}}\omega\tau_p\gg 1.\end{cases} \tag{14}$$

Expression (14) is valid for Boltzmann statistics. For Fermi statistics, as one can easily verify, the correction is quadratic with respect to the parameter appearing on the right-hand side of (14). Within the framework of the iteration method, such a correction would be taken into account directly.

Just as on $f^{(1)}$, the effect of the collision operator on the anisotropic part ΔF reduces to multiplication by ν_p. Therefore, by determining the anisotropic part ΔF from Eq. (13) and calculating $\Delta K_q(\omega)$ with its aid, we have

$$\Delta K_q(\omega)=[e\tilde{\varphi}_0]^2 e^{-2\Gamma x}\frac{(2m)^{3/2}}{2\pi^3\hbar^3}\int \varepsilon^{-3/2}\frac{dF_0}{d\varepsilon}d\varepsilon \times\int_{-1}^{1}\left(x-\frac{w}{v}-\frac{i}{ql}\right)^{-4}\left(x-\frac{w}{v}+\frac{i}{ql}\right)^{-1}dx. \quad (15)$$

If collisions are neglected expression (15) tends to infinity, which is a consequence of the interaction of the sound wave with the electrons moving in phase with it. In the case of Fermi statistics, and also for Boltzmann statistics, if $s > -\frac{1}{3}$ ($\tau_p(\epsilon) = \tau_p(T)(\epsilon/T)^s$), the lower limit on the integration over the energy is unimportant. Evaluation of expression (15) gives

$$\frac{\Delta K_q(\omega)}{|K_q^0(\omega)|}=\pi\left[\frac{e\tilde{\varphi}_0}{\bar{\varepsilon}}\right]^2 e^{-2\Gamma x}[ql(\bar{\varepsilon})]^4\left[\gamma_1\frac{1}{ql}+i\gamma_2\frac{w}{\bar{v}}\right]; \quad (16)$$

$\gamma_1 = \gamma_2 = 1$ for Fermi statistics; however in the case of Boltzmann statistics $\gamma_1 = \Gamma(3s+1)/\Gamma(\frac{3}{2})$, $\gamma_2 = \Gamma(4s+1)/\Gamma(\frac{5}{2})$. If $s < -\frac{1}{3}$ (scattering by acoustic phonons) one can show that the region of small energies also gives a small contribution, and expression (16) is valid in order of magnitude.

Now let us compare corrections (14) and (16).

A. In the case of Fermi statistics the corrections (16) are fundamental:

$$\frac{\Gamma-\Gamma_0}{\Gamma_0}=-\gamma_2\left[\frac{e\tilde{\varphi}_0}{\bar{\varepsilon}}\right]^2 e^{-2\Gamma x}(ql)^4, \quad (17)$$

$$\frac{\Delta(w-w_0)}{w-w_0}\left[\frac{e\tilde{\varphi}_0}{\bar{\varepsilon}}\right]^2 e^{-2\Gamma x}(ql)^3\begin{cases}\gamma_1, & \omega\tau_p\ll 1;\\ \gamma_2\omega\tau_p\dfrac{\varkappa^2}{q^2+\varkappa^2}, & \omega\tau_p\gg 1.\end{cases} \quad (18)$$

B. In the case of Boltzmann statistics the following alternatives are possible:

1) $w\omega\tau_p/\bar{v} \gg 1$. Here expressions (17) and (18) are valid.

2) $w\omega\tau_p/\bar{v} \ll 1$, $\tau_\epsilon/\tau_p > (ql)^4$. Here the heating of the electrons plays the major role:

$$\frac{\Gamma-\Gamma_0}{\Gamma_0}\sim\left[\frac{e\tilde{\varphi}_0}{\bar{\varepsilon}}\right]^2 e^{-2\Gamma x}\frac{\tau_\varepsilon}{\tau_p}\frac{\varkappa^2-q^2}{\varkappa^2+q^2}. \quad (19)$$

One can qualitatively explain this dependence in the following way: for $q \ll \kappa$ the interaction is strongly screened; consequently an increase of the average electron energy, leading to a decrease in the screening, in turn leads to an increase in the coefficient of absorption. For $q > \kappa$ the screening is unimportant, but the number of electrons interacting with the wave decreases as a consequence of the increase of the average energy, thus leading to a decrease of the absorption. For the correction to the velocity of sound we have

$$\frac{\Delta(w-w_0)}{w-w_0}\sim\left[\frac{e\tilde{\varphi}_0}{\bar{\varepsilon}}\right]^2 e^{-2\Gamma x}\frac{\tau_\varepsilon}{\tau_p}\frac{\varkappa^2}{q^2+\varkappa^2}. \quad (20)$$

The increase in the velocity of sound is also related to the decrease of the screening.

3) $(ql)^2 < \tau_\epsilon/\tau_p < (ql)^4$. In this connection the change in the velocity of sound is described by expression (20), and the absorption of sound is described by formula (17).

4) $\tau_\epsilon/\tau_p < (ql)^2$, $w\omega\tau_p/\bar{v} \ll 1$. Here formulas (17) and (18) are valid.

Together with the correction to the slowly varying part of the distribution function in second order with respect to $e\varphi_0$ there appears an induced wave of the second harmonic, $e\varphi^{(2)} \sim \exp 2i(qx-\omega t)$, which is caused by the term $-e(\partial\varphi^{(1)}/\partial x)(\partial f^{(1)}/\partial p_x)$ of Eq. (6), and also a free wave of doubled frequency which is the solution of the linearized system of Eqs. (4)–(6). Its amplitude is determined from the boundary condition—the sum of the free and induced waves of doubled frequency should be equal to zero on the crystal boundary. Therefore, the amplitudes of these waves are of the same order of magnitude, and in order to determine the nonlinear corrections below we consider only the induced wave.

The generated second-harmonic wave creates a wave of variable concentration

$$n^{(2)}=e\varphi^{(2)}K_{2q}^0(2\omega)+[e\varphi^{(1)}]^2K_q^1(\omega), \quad (21)$$

where

$$K_q^1(\omega)=\frac{2}{(2\pi\hbar)^3}\int d^3p\,\Xi^{-1}(2\mathbf{q},2\omega)\mathbf{q}\frac{\partial}{\partial\mathbf{p}}\left[\Xi^{-1}(\mathbf{q},\omega)\mathbf{q}\frac{\partial F_0}{\partial\mathbf{p}}\right], \quad (22)$$

$$\Xi(\mathbf{q},\omega)=\mathbf{qv}-\omega-i\nu_p. \quad (23)$$

The first term in Eq. (21) represents a linear redistribution of the concentration of electrons in the second-harmonic wave, and the second term is a consequence of the nonlinearity of the kinetic equation. From Eqs. (21) and (6) we have

$$e\varphi^{(2)}=[e\varphi^{(1)}]^2\frac{K_q^1(\omega)}{4K_q^0(\omega)-K_{2q}^0(2\omega)}. \quad (24)$$

In third order with respect to the amplitude of the sound, the generated second-harmonic wave, interacting with the fundamental wave (the term $-e(\partial\varphi^{(2)}/\partial x) \times [\partial f^{(1)}/\partial p_x]^*$), gives rise to a wave having the frequency of the fundamental harmonic (secular terms). The secular terms renormalize the wave vector of the fundamental wave, which can be taken into account by calculating the nonlinear corrections to $K_q^0(\omega)$. We have

$$\Delta K_q(\omega)=[e\tilde{\varphi}_0]^2e^{-2\Gamma x}\frac{2}{(2\pi\hbar)^3}\int d^3p\,\Xi^{-1}(\mathbf{q},\omega)\mathbf{q}\frac{\partial}{\partial\mathbf{p}}\times\Big\{\Xi^{-1}(2\mathbf{q},2\omega)\left[\mathbf{q}\frac{\partial}{\partial\mathbf{p}}\Xi^{-1}(\mathbf{q},\omega)+\frac{2K_q^1(\omega)}{4K_q^0(\omega)-K_{2q}^0(2\omega)}\right] -\frac{2K_q^1(\omega)}{4K_q^0(\omega)-K_{2q}^0(2\omega)}\mathbf{q}\frac{\partial}{\partial\mathbf{p}}[\Xi^{-1}(\mathbf{q},\omega)]^*\Big\}\mathbf{q}\frac{\partial F_0}{\partial\mathbf{p}}. \quad (25)$$

The third term inside the curly brackets in (25) gives the largest contribution, since upon integration over the cosine of the angle between q and v only in it do the poles turn out to be in different half-planes. An estimate tives

$$\frac{\Delta K_q(\omega)}{|K_q^0(\omega)|}\sim\left[\frac{e\tilde{\varphi}_0}{\bar{\varepsilon}}\right]^2e^{-2\Gamma x}(ql)^2\left(1+\frac{w}{\bar{v}}\right). \quad (26)$$

We note that the correction due to generation of the second harmonic is (ql) times smaller than the correction due to distortion of the slowly varying part of the distribution function. This is the essential difference from the hydrodynamical situation, where all of the corrections are of the same order $(e\varphi_0/\bar{\epsilon})$.

In the case of a deformation potential the system of equations for the electrons and for the sound wave can be obtained from Eqs. (4)–(6) by taking $\beta = 0$ and also by adding the terms $-\Lambda \partial n/\partial x$ in Eq. (4) and $\Lambda(\partial^2 u/\partial x^2) \times (\partial f/\partial p_x)$ in Eq. (6) (Λ is the deformation-potential constant). The dispersion equation takes the form

$$\frac{q_0^2-q^2}{q^2}=\eta K_q(\omega)\left[1-\frac{4\pi e^2}{\varepsilon_0 q^2}K_q(\omega)\right]^{-1}, \qquad (27)$$

where $\eta = \Lambda^2/c$.

If the spectrum consists of j nonintersecting ellipsoids and the time for intravalley relaxation is much smaller than the time for intervalley relaxation processes, then instead of Eq. (27) one can easily obtain (by a method analogous to the one discussed in [11])

$$\frac{q_0^2-q^2}{q^2}=\frac{1}{c}\sum_j K_q^j(\omega)\Lambda^j\left(\Lambda^j-\overline{\Lambda}+\overline{\Lambda}\left[1-\frac{4\pi e^2}{\varepsilon_0 q^2}\bar{K}_q(\omega)\right]^{-1}\right), \qquad (28)$$

where

$$\bar{K}_q(\omega)=\sum_j K_q^j(\omega), \quad \overline{\Lambda}=\left[\sum_j \Lambda^j K_q^j(\omega)\right]\Big/\bar{K}_q(\omega),$$

and j denotes the number of the ellipsoid. For $q \ll \kappa$ from (28) we have

$$\frac{q_0^2-q^2}{q^2}=\frac{1}{c}\sum_j K_q^j(\omega)\,|\Lambda^j-\overline{\Lambda}|^2. \qquad (29)$$

Thus, as a consequence of the redistribution of the electrons between the valleys, the screening is unimportant in many-valley conductors, and the corrections are determined by only the imaginary part of $K_q^j(\omega)$.

Let us present estimates of the possibility of observing nonlinear effects. In the piezoelectric semiconductor n-InSb at T = 77°K, $n = 10^{14}$ cm^{-1}, $\mu = 6 \times 10^5$ cm^2/V·sec, f = 1500 MHz, and $ql \sim 8$. Since $\tau_\epsilon/\tau_p \sim 10^4$, the most important nonlinearity is that due to heating, which begins to manifest itself when the sound intensity is of the order of 10^{-4} watts/cm^2. In the many-valley semi-metal bismuth at liquid helium temperatures and for $q \sim 10^4$ cm^{-1}, the quantity $ql \sim 10^2$, and the nonlinearity of type (17) is most important. It begins to manifest itself starting with intensities of the order of 10^{-6} to 10^{-5} watts/cm^2. Consequently, for $ql > 1$ nonlinear effects are quite accessible to experimental observation.

2. THE ROLE OF AN EXTERNAL ELECTRIC FIELD

It is easy to verify that for not too strong electric fields ($v_{dr} \sim w \ll \bar{v}$, $v_{dr} = \sum_p vF$ is the average drift velocity of the electrons), taking account of an external field reduces to replacing the equilibrium distribution function F_0 by the function $\tilde{F}$ which is the solution of the equation

$$e\mathbf{E}\frac{\partial \tilde{F}}{\partial \mathbf{p}}+\hat{I}\tilde{F}=0. \qquad (30)$$

Expressing the antisymmetric part of the function $\tilde{F}$ in terms of its symmetric part $\tilde{F}_+$ and discarding the terms of order $mw^2/\bar{\epsilon}$, we have

$$\mathbf{q}\left(\frac{\partial \tilde{F}_+}{\partial \mathbf{p}}-\frac{\partial}{\partial \mathbf{p}}\hat{I}^{-1}e\mathbf{E}\frac{\partial \tilde{F}_+}{\partial \mathbf{p}}\right)=\mathbf{q}\left(\mathbf{v}-\frac{\tau_p}{\bar{\tau}_p}\mathbf{v}_{dr}\right)\frac{\partial \tilde{F}_+}{\partial \varepsilon}, \qquad (31)$$

where

$$\bar{\tau}_p=\frac{(2m)^{3/2}}{3\pi^2\hbar^3 n_0}\int \tau_p(\varepsilon)\left(-\frac{\partial \tilde{F}_+}{\partial \varepsilon}\right)\varepsilon^{3/2}\,d\varepsilon. \qquad (32)$$

From Eq. (31) it is clear that one can take the effect of an external electric field into account by changing the velocity of sound w to $w - A_i v_{dr}$ in the expressions, leading to corrections to Γ and Δw. The coefficients $A_i \sim 1$ depend on the scattering mechanisms. One can represent them in the form

$$A_i=\left(\bar{\tau}_p\int d\varepsilon g_i(\varepsilon)\right)^{-1}\int d\varepsilon \tau_p(\varepsilon)g_i(\varepsilon), \qquad (33)$$

where $g_0 = \partial F_+/\partial\epsilon$ appears in the expression for Γ_0, for a nonlinearity of the type (17) $g_1 = \epsilon^{-2}(ql)^3\,\partial F_+/\partial\epsilon$, but for the heating type of nonlinearity $g_2 = \partial\Delta F_+/\partial\epsilon$, where ΔF_+ is the correction to the symmetric part of the distribution function due to heating. It is easy to follow the way in which for $A_0 v_{dr} > w$ the absorption of sound changes into its amplification, and here the nonlinear effects are determined by the same parameters as for absorption.

3. WAVE OF FINITE AMPLITUDE

Let us consider the case when the generation of higher harmonics and the heating of the electrons by sound is unimportant; however the nonlinearity due to distortion of the distribution function is arbitrary.[2)] We have

$$\left(\frac{e\varphi_0}{\varepsilon}\right)ql\ll 1, \quad \left(\frac{e\varphi_0}{\varepsilon}\right)\sqrt{\frac{\tau_\varepsilon}{\tau_p}}\ll 1, \quad \left(\frac{e\varphi_0}{\varepsilon}\right)(ql)^2\gtrsim 1.$$

The parameter $e\varphi_0(ql)^2/\bar{\epsilon}$ has a clear physical meaning. In fact it is equal to $(\omega_0\tau_p)^2$ where $\omega_0 = \sqrt{2e\varphi_0/m}\,q$ is the frequency of the electron oscillations in the potential well created by the field of the sound wave. If this parameter is large, there is a group of electrons, "captured" by the wave, and these electrons are able to undergo many oscillations in the wave during the time between collisions. The distortion of the distribution function near $v_x = w$ is a reflection of the creation of a group of captured electrons.

It is easy to verify that in the case we are interested in the problem can be reduced to a one-dimensional problem. In fact, in the absorption of sound the electrons moving in phase with the wave, for which $v_x \sim w \ll \bar{v}$, give a contribution. Therefore one can regard the frequency of collisions ν_p, which depends on the total energy of an electron to within terms of order $mw^2/\bar{\epsilon}$, as being independent of v_x. Therefore Eq. (13) can be reduced to the form

$$-\frac{d}{dx}\frac{1}{x^2+a^2}\frac{dF}{dx}+F=F_0, \qquad (34)$$

where $x = (v_x - w)/\tilde{v}$, $\tilde{v} = \sqrt{2e\varphi_0/m}$, $a = \nu_p/qv = 1/\omega_0\tau_p$, considering that F and a depend on p_y and p_z as on parameters.

2) A strong nonlinearity of the heating type associated with the presence of electron temperature is considered in article [12]. In the absence of an electron temperature, the interaction of the wave with the electrons in the resonance region is treated incorrectly in this article.

Let us introduce the function $\Phi = (x^2+a^2)^{-1} dF/dx$; then

$$K_q(\omega) \sim \int dp_y dp_z \int_{-\infty}^{\infty} (x+ia)\Phi dx. \qquad (35)$$

We obtain the following equation for Φ

$$\frac{d^2\Phi}{dx^2} - x^2\Phi = a^2\Phi - \frac{dF_0}{dx} \qquad (36)$$

and the boundary conditions $\Phi \to 0$ as $x \to \pm\infty$. We seek the solution of Eq. (36) satisfying the boundary conditions in the form

$$\Phi = \sum_n \Phi_n(p_y, p_z)\psi_n(x), \qquad (37)$$

where

$$\psi_n(x) = \frac{\pi^{-1/4}}{\sqrt{2^n n!}} e^{-x^2/2} H_n(x),$$

and $H_n(x)$ are the Hermite polynomials.

We have

$$\Phi = \sum_n \frac{C_n}{2n+1+a^2}\psi_n(x), \qquad (38)$$

where the $C_n(p_y, p_z)$ are the coefficients of the expansion of the function dF_0/dx in terms of the function ψ_n. In the case of Boltzmann statistics we obtain

$$K_q(\omega) = -\frac{2^{5/2}n}{T}\sum_k \frac{(2k)!}{2^{2k}(k!)^2} e^{-4ke\varphi_0/T}\Big[2^{3/2}(2k+1) \times \left(\frac{e\varphi_0}{T}\right)^{1/2}\left\langle\frac{1}{4k+3+a^2}\right\rangle + i\frac{w}{\bar{v}}\left\langle\frac{a}{4k+1+a^2}\right\rangle\Big], \qquad (39)$$

$$\langle f\rangle \equiv \left(\int dp_y dp_z F_0|_{x=0}\right)^{-1} \int dp_y dp_z f(p_y, p_z) F_0|_{x=0}.$$

In the expression for Im $K_q(\omega)$ values of $k \sim a^2 \ll T/e\varphi_0$ are essential. Therefore one cannot neglect the factor $\exp(-4ke\varphi_0/T)$. For large values of a, by using the asymptotic behavior of $\psi_k(x)$, one can follow the transition to the linear theory. For small values of a one can neglect a^2 in the denominator of Eq. (39), and then

$$\frac{\Gamma}{\Gamma_0} = \frac{1}{2\sqrt{\pi}}\langle a\rangle \sum_{k=0}^{\infty} \frac{(2k)!}{2^{2k}(k!)^2(4k+1)}. \qquad (40)$$

In the expression for Re $K_q(\omega)$ we need to take account of the factors $\exp(-4ke\varphi_0/T)$ notwithstanding the smallness of $e\varphi_0/T$. In fact, for large values of k the factor $(2k)!/2^{2k}(k!)^2 \sim 1/\sqrt{k}$, and without taking these factors into consideration the series diverges. This means that values of $k \sim T/e\varphi_0 \gg a^2$ are essential; therefore one can neglect the quantity a^2 in the denominator and replace the summation by an integral. In this connection, in order of magnitude we obtain the same result as in the linear theory. This result is quite natural since, in contrast to absorption, all of the electrons give a contribution to the screening.

In the case of Fermi statistics all the results differ by the replacement of the temperature T by the Fermi energy ϵ_F.

It is easiest of all to take the influence of an external electric field into account for $\omega\tau_p \ll 1$. In this case it is only necessary to replace the equilibrium distribution function F_0 by the solution of Eq. (30). Thus, the electronic part of the sound absorption in an external electric field with $e\varphi_0(ql)^2/\bar{\epsilon} \gg 1$ is determined by the expression

$$\frac{\Gamma}{\Gamma_0} = \left[\langle a\rangle - \frac{v_{dr}}{w}\frac{\langle a\tau(\varepsilon_\perp)\rangle}{\bar{\tau}}\right]\frac{1}{2\sqrt{\pi}}\sum_k \frac{(2k)!}{2^{2k}(k!)^2(4k+1)}. \qquad (41)$$

We note that in the case being considered of a strong nonlinearity, without taking the lattice absorption into account the electronic processes do not lead to the appearance of a stationary wave. However, the nonlinear effects reduce the amplification coefficient by a factor of $\sqrt{e\varphi_0/\bar{\epsilon}}\, ql \gg 1$ times; therefore taking account of the small lattice absorption $\Gamma_l \ll \Gamma_0$ might lead to a stationary wave.

In conclusion we take this opportunity to express our gratitude to V. L. Gurevich and B. D. Laĭkhtman for helpful discussions.

[1] V. L. Gurevich and V. D. Kagan, Fiz. Tverd. Tela 4, 2441 (1962) [Sov. Phys.-Solid State 4, 1788 (1963)].
[2] E. M. Ganapol'skiĭ and A. N. Chernets, Dokl. Akad. Nauk SSSR 149, 72 (1963) [Sov. Phys.-Doklady 8, 265 (1963)].
[3] K. W. Nill and A. L. McWhorter, J. Phys. Soc. Japan, Supplement to Vol. 21, 000 (1966).
[4] S. P. Ivanov and G. D. Mansfel'd, Radiotekhnika i élektronika 14, 2 (1969).
[5] C. A. Nanney, Phys. Letters 26A, 66 (1967).
[6] P. E. Zil'berman, Fiz. Tverd. Tela 12, 1014 (1970) [Sov. Phys.-Solid State 12, 796 (1970)].
[7] A. R. Hutson, Phys. Rev. Letters 9, 296 (1962).
[8] Yu. M. Gal'perin, I. L. Drichko, and B. D. Laĭkhtman, Fiz. Tverd. Tela 12, 1437 (1970) [Sov. Phys.-Solid State 12, 1128 (1970)].
[9] V. L. Gurevich and B. D. Laĭkhtamn, Zh. Eksp. Teor. Fiz. 46, 598 (1964) [Sov. Phys.-JETP 19, 407 (1964)].
[10] L. D. Landau, Zh. Eksp. Teor. Fiz. 16, 574 (1946).
[11] Yu. M. Gal'perin, Fiz. Tverd. Tela 10, 2338 (1968) [Sov. Phys.-Solid State 10, 1840 (1969)].
[12] V. V. Belyĭ and Yu. L. Klimontovich, Fiz. Tekh. Poluprov. 3, 339 (1969) [Sov. Phys.-Semicond. 3, 291 (1969)].

Translated by H. H. Nickle
190