PARAMETRIC EXCITATION OF ELECTROSTATIC WAVES NEAR THE ELECTRON-CYCLOTRON FREQUENCY IN A FULLY IONIZED PLASMA

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Kinetic theory is used to analyze parametric excitation of electrostatic waves in a fully ionized plasma which is subject to a weak microwave electric field and a fixed magnetic field. The dispersion equation is derived for the electrostatic oscillations using the kinetic equation with the Landau collision term. Parametric excitation of waves is considered for the case in which the waves propagate essentially along the magnetic field when the frequency of the external microwave signal ω_0 is approximately equal to the electron cyclotron frequency Ω_e. Threshold values are found for the microwave field above which the plasma becomes unstable against the excitation of electrostatic waves. Depending upon the plasma parameters, the fixed magnetic field can act as a stabilizing factor or as a destabilizing factor. A qualitative comparison is made with the experimental data that are presently available and this comparison shows agreement with the theory.

1. INTRODUCTION

In recent years, in connection with problems of radiation acceleration and heating of fully ionized plasmas by means of electromagnetic fields a theory has been developed for the stability of a plasma that is subject to strong high frequency electric fields.[1–4] It has been shown in[5–9] that even in relatively weak fields ($v_E \ll v_{Te}$) there are a number of anomalous effects that arise by virtue of the interaction of the external high frequency field with the plasma; these effects are due to the development of parametric resonances and turbulence in the plasma. Threshold values have been determined for the high frequency field; above these values electrostatic waves are excited, that is to say, the plasma becomes unstable.

The introduction of a fixed magnetic field leads to the expansion of the region of parametric resonance since the number of characteristic plasma modes is increased significantly.[10–12] When the frequency of the external microwave field is approximately equal to one of the hybrid frequencies characteristic of a cold unmagnetized plasma it is found that low-frequency waves are excited; the growth rate for these waves is a maximum for wavelengths of the order of the excursion of the particle in the microwave field, that is to say[10], when the condition $|\mathbf{k}\cdot\mathbf{r}_B| \sim 1$ obtains. High-frequency electron oscillations are excited when the overtones of the frequency of the microwave field $p\omega_0$ are approximately equal to the sum of the hybrid frequencies. The threshold field required for the excitation of these modes is found to be a minimum[11] when $p = 1$ and $|\mathbf{k}\cdot\mathbf{r}_B| \ll 1$.

There have been a large number of experiments indicating anomalous absorption and heating of a magnetoactive plasma which interacts with external microwave fields (cf. for example[13–15]) under conditions such that the field frequency ω_0 is approximately equal to the electron cyclotron frequency $\Omega_e = eB_0/mc$. In addition to the strong absorption of the external wave in the vicinity of the cyclotron resonance, effects have been observed which cannot possibly be explained by binary collisions or by the theory of stochastic particle acceleration. For example, electrons have been observed with high longitudinal velocities, indicating the existence of a mechanism for the conversion of transverse electron energy into longitudinal electron energy. Evidently these effects can be attributed to collective processes which arise in the parametric interaction of the microwave field with the plasma; this picture supported by the fact that the electric field is characterized by a threshold value below which the indicated anomalous effects are not observed.

In the present work we investigate the stability of a fully ionized plasma subject to a weak external microwave electric field and a fixed magnetic field. It is assumed that the wavelength of the low-frequency electrostatic waves is smaller than the system dimensions L_0, the field inhomogeneities, and the electron mean free path $l = v_{Te}/\nu_{ei}$ (i.e., $k \gg L_0^{-1}, k_0, l^{-1}$). The frequency of the external electric field $\mathbf{E}(t) = \mathbf{E}_0 \sin\omega_0 t$ is assumed to be approximately equal to the electron cyclotron frequency Ω_e and much greater than the electron collision frequency ($\omega_0 \gg \nu_{ei}$). The electric field is assumed to be weak, so that thermal velocities of the particles are larger than the velocities associated with their oscillations in the external fields $v_B^2 \ll v_{Te}^2$; the orientation of the fields is chosen in the following manner: $\mathbf{B}_0(0, 0, B_0)$ and $\mathbf{E}_0(E_{\perp 0}\sin\chi, E_{\perp 0}\cos\chi, E_z)$. The problem lies in determining the threshold values of the microwave field that are required for the excitation of aperiodic ($\omega \ll kv_{Ti}$) as well as periodic ($\Omega_i, kv_{Ti} \ll \omega \ll k_z v_{Te}$) long wave ($k_\perp^2\rho_\alpha^2 \ll 1$) electrostatic waves that propagate at small angles with respect to the magnetic field.

2. DISPERSION EQUATION FOR ELECTROSTATIC WAVES

In describing the interaction of the weak external microwave field $\mathbf{E}(t) = \mathbf{E}_0 \sin\omega_0 t$ with the fully ionized plasma located in a fixed magnetic field under conditions such that $\omega_0 \sim \Omega_e$ and such that the external frequency does not exceed greatly the electron plasma frequency $\omega_{Le} = \sqrt{4\pi e^2 N_e/m}$ we can make use of the kinetic equation with the Landau collision integral:[16]

$$\frac{\partial f_\alpha}{\partial t}+\mathbf{v}\frac{\partial f_\alpha}{\partial \mathbf{r}}+\frac{e_\alpha}{m_\alpha}\left(\mathbf{E}(\mathbf{r},t)+\frac{1}{c}[\mathbf{v}\mathbf{B}_0]\right)\frac{\partial f_\alpha}{\partial \mathbf{v}}=\sum_\beta I_{\alpha\beta}(f_\alpha,f_\beta),$$

$$I_{\alpha\beta}(f_\alpha,f_\beta)=2\pi e_\alpha^2 e_\beta^2 L\frac{\partial}{\partial p_i}\int d\mathbf{p}'\frac{w^2\delta_{ij}-w_iw_j}{w^3}\left(f_\beta\frac{\partial f_\alpha}{\partial p_j}-f_\alpha\frac{\partial f_\beta}{\partial p'_j}\right), \tag{2.1*}$$

where the subscripts α and β refer to electrons and ions, $L = \ln(k_{max}/k_{min})$, and $\mathbf{w} = \mathbf{v}_\alpha - \mathbf{v}_\beta$. We shall limit ourselves to processes that occur in a time

$$t\ll\frac{1}{\nu_{ei}}\left(\frac{v_{Te}}{v_B}\right)^2,\ \frac{1}{\nu_{ei}}\frac{M}{m}, \tag{2.2}$$

that is to say, we neglect the change in temperature and the excitation of higher harmonics of the external field.[17] Then, we expand f_α in powers of ν_{ei}/ω_0, assuming that the distribution function for the ground state is a Maxwellian[10]

$$f_{\alpha 0}(\mathbf{v},t)=\frac{N_\alpha}{(2\pi m_\alpha T_\alpha)^{3/2}}\exp\left\{-\frac{m_\alpha(\mathbf{v}-\mathbf{v}_{\alpha B}(t))}{2T_\alpha}\right\}, \tag{2.3}$$

where

$$\mathbf{v}_{\alpha B}(t)=\frac{e_\alpha}{m_\alpha}\int_{-\infty}^{t}dt'\left(\frac{\mathbf{B}_0(\mathbf{B}_0\mathbf{E}_0(t'))}{B_0^2}\right.$$

$$\left.+\frac{[\mathbf{E}_0(t')\mathbf{B}_0]}{B_0}\sin\Omega_\alpha(t-t')+\frac{[\mathbf{B}_0[\mathbf{E}_0(t')\mathbf{B}_0]]}{B_0^2}\cos\Omega_\alpha(t-t')\right)$$

is the particle velocity associated with the oscillations in the external microwave field.

We now consider small perturbations of the ground state under the effect of the electrostatic field $\mathbf{E} = -\nabla\Phi$. The deviation from the equilibrium function δf_α and the perturbation of the field potential can be written

$$\delta f_\alpha(\mathbf{r},\mathbf{v},t)=\exp\{-i\omega t+i\mathbf{k}(\mathbf{r}-\mathbf{r}_{\alpha B}(t))\}\cdot$$

$$\times\sum_{n=-\infty}^{+\infty}\Psi_{\alpha n}(\mathbf{v}-\mathbf{v}_{\alpha B}(t))\exp\{-in\omega_0 t\},$$

$$\Phi(\mathbf{r},t)=\sum_{n=-\infty}^{+\infty}\Phi_n\exp\{-i(\omega+n\omega_0)t+i\mathbf{k}\mathbf{r}\}, \tag{2.4}$$

where $\mathbf{r}_{\alpha\beta}(t) = \int_{-\infty}^{t} dt'\mathbf{v}_{\alpha\beta}(t')$ is the amplitude of the particle oscillation in the microwave field. Substituting Eq. (2.4) in the linearized kinetic equation (2.1) and using the Poisson equation

$$\Delta\Phi=4\pi\sum_\alpha e_\alpha\int d\mathbf{v}\delta f_\alpha, \tag{2.5}$$

after some elementary but tedious calculations (cf., for example[9]) we find the dispersion equation¹⁾ for electrostatic oscillations to accuracy of order $a_B^4 = |\mathbf{k}\mathbf{r}_{eB}|^4$:

$$1+\frac{a_B^2}{4}\sum_{n=-\infty}^{+\infty}R_n+\frac{a_B^4}{16}\sum_{n=-\infty}^{+\infty}\left(\frac{P_n}{4}+\sum_{m>n+1}R_nR_m\right)=0, \tag{2.6}$$

where

$$(R_n,P_n)=\frac{(\delta\varepsilon_e^{(n+1)}-\delta\varepsilon_e^{(n,n-1)})(\delta\varepsilon_i^{(n+1)}-\delta\varepsilon_i^{(n,n-1)})}{\varepsilon^{(n,n-1)}\varepsilon^{(n+1)}},$$

while $\delta\epsilon_\alpha^{(n)} = \delta\epsilon_\alpha(\omega + n\omega_0, \mathbf{k})$ is the contribution of particles of species α in the longitudinal dielectric constant of the plasma $\epsilon^{(n)} = 1 + \delta\epsilon_e^{(n)} + \delta\epsilon_i^{(n)}$.

Below we shall limit ourselves to the case in which the frequency ω_0 is approximately equal to one of the hybrid frequencies $\omega_\pm$ and we shall thus eliminate a double resonance (i.e., $\omega_0 \sim \omega_-$ or $2\omega_0 \sim \omega_+$) which is only possible in a magnetoactive plasma. Assuming, as in[6-9] that the frequency of the excited waves ω is much smaller than ω_0 we can write the dispersion equation (2.6) in the form

$$\frac{1}{1+\delta\varepsilon_e(\omega,\mathbf{k})}+\frac{1}{\delta\varepsilon_i(\omega,\mathbf{k})}+K^2\frac{\omega_0\Delta k^2 r_{De}^2}{\Delta^2-(\omega+i\gamma)^2}=0, \tag{2.7}$$

where

$$K^2=\frac{1}{2}\left(\frac{\mathbf{k}\mathbf{r}_{eB}}{kr_{De}}\right)^2\Big/\omega_0\frac{\partial}{\partial\omega_0}\mathrm{Re}\,\varepsilon(\omega_0),$$

$$\Delta=\mathrm{Re}\,\varepsilon(\omega_0)\Big/\frac{\partial}{\partial\omega_0}\mathrm{Re}\,\varepsilon(\omega_0),$$

$$\gamma=\mathrm{Im}\,\varepsilon(\omega_0)\Big/\frac{\partial}{\partial\omega_0}\mathrm{Re}\,\varepsilon(\omega_0).$$

3. ANALYSIS OF THE DISPERSION EQUATION

Since we are interested in the frequency region $\omega_0 \sim \Omega_e$ it is easy to show that waves are excited which propagate at a small angle θ with respect to the magnetic field ($\omega_0 \sim \Omega_e \sim \omega_+$). Hence it is reasonable to consider the following range of angles and wave vectors²⁾ $\mathbf{k}(k\sin\theta, \theta, k\cos\theta)$:

$$\max\left(\frac{\delta^2}{\omega_0^2},\frac{\delta^2}{\omega_{Le}^2}\right)\ll\sin^2\theta\ll\min\left(1,\frac{(\omega_{Le}^2-\Omega_e^2)^2}{4\Omega_e^2\omega_{Le}^2}\right), \tag{3.1}$$

$$\frac{\nu_{ei}}{v_{Te}}\ll k\ll\frac{\delta}{v_{Te}}\left(2\ln\frac{\delta}{\nu_{ei}}\right)^{-1/2},$$

which implies the condition $\omega_0 \sim \Omega_e$ and the resonance nature of the dielectric constant of the plasma $\epsilon(\omega_0)$; hence, we can neglect cyclotron wave damping in the term $\mathrm{Im}\,\epsilon(\omega_0)$. Here the quantity $\delta = \omega_0 - \Omega_e$, which characterizes the detuning, can be assumed to be greater than ν_{ei} or kv_{Te}. If this condition does not hold then the excitation of the waves requires a much stronger microwave field.

In what follows we shall consider two limiting orientations of the microwave field with respect to the magnetic field:

$$\frac{\alpha_\perp}{\alpha_\parallel}\gg\alpha_0=\frac{4\delta^2}{\omega_0^2}\frac{1}{\sin^2\theta}, \tag{3.2a}$$

$$\frac{\alpha_\perp}{\alpha_\parallel}\ll\alpha_0=\frac{4\delta^2}{\omega_0^2}\frac{1}{\sin^2\theta}, \tag{3.2b}$$

where

$$\alpha_\perp=\frac{E_{\perp 0}^2}{E_0^2}\left(\sin^2\chi+\frac{\Omega_e^2}{\omega_0^2}\cos^2\chi\right),\quad \alpha_\parallel=\frac{E_{z0}^2}{E_0^2}.$$

It is then evident from (3.1) that $\alpha \ll 1$ and this means that (3.2a) is almost always satisfied with the exception of strictly parallel $\mathbf{E}_0$ and $\mathbf{B}_0$. Taking account of these conditions the quantities in (2.7) are now written in the form

$$K^2=\left(\frac{\alpha_\perp/4}{\alpha_\parallel\delta^2/\omega_0^2\sin^2\theta}\right)\frac{\omega_{Le}^2}{\omega_0^2}Z^2, \tag{3.3a}$$

$$\Delta=\delta\left(\frac{2(\omega_0^2-\omega_{Le}^2)\omega_0\delta}{\omega_{Le}^2\Omega_e^2\sin^2\theta}-1\right), \tag{3.3b}$$

$$\gamma=\nu_{ei}, \tag{3.3c}$$

¹⁾This equation was obtained with Yu. M. Aliev.

*$[\mathbf{v}\mathbf{B}_0] \equiv \mathbf{v}\times\mathbf{B}_0$.

²⁾It is assumed below that $\cos\theta \sim 1$ but $k_z \sim k$.

where $Z^2 = E_0^2/4\pi n_e T_e$ is the ratio of the energy in the microwave field to the thermal energy of the electrons, while

$$\nu_{ei} = \frac{4}{3}\sqrt{\frac{2\pi}{m}}\frac{e^2 e_i^2 N_i L}{T_e^{3/2}}$$

is the effective frequency for electron-ion collisions; the upper and lower expressions in (3.3a) correspond to the two field orientations in (3.2a) and (3.2b).

We now examine the significance of the quantity Δ. In[8,9] this quantity characterizes the deviation of ω_0 from the plasma frequency. Here, however, Δ relates the angle of propagation of the wave θ to the detuning δ. Below we shall obtain an expression for the threshold field and the frequency of the excited waves as a function of Δ; the minimum value ($\Delta = \Delta_0$) actually represents a minimum value with respect to angle, that is to say, the determination of the direction of propagation for fixed δ:

$$\sin^2\theta = \frac{\delta^2}{(\delta+\Delta_0)\omega_0}\frac{2(\omega_0^2-\omega_{Le}^2)\omega_0^2}{\omega_{Le}^2\Omega_e^2}. \quad (3.4)$$

It will be evident from Eq. (3.3a) that the most favorable orientation for the field is given by (3.2a), in which the threshold field, i.e., Z^2, decreases with increasing angle between $\mathbf{E}_0$ and $\mathbf{B}_0$, reaching a minimum value for $\alpha_{\parallel} = 0$, that is to say, when these are mutually perpendicular. Only these two field orientations will be considered in the present work.

1. Excitation of Aperiodic Low-frequency Waves $\omega \ll kv_{Ti}$

In this limit the particle contributions to the dielectric constant assume the form

$$\delta\varepsilon_\alpha(\omega,\mathbf{k}) = \frac{1}{k^2 r_{D\alpha}^2}\left(1 + i\sqrt{\frac{\pi}{2}}\frac{\omega}{kv_{T\alpha}}\right). \quad (3.5)$$

It is easy to show that when (3.5) is used Eq. (2.7) has a pure imaginary solution with respect to ω; this solution can change sign at some critical value of the external microwave field, that is to say, the Debye shielding is overcome and the plasma supports aperiodic growing waves

$$\omega = -i(\nu_{ei} - K^2\omega_0\xi_1 - [(\nu_{ei} - K^2\omega_0\xi_1)^2 - (\Delta^2 - \nu_{ei}^2 + K^2\Delta\omega_0\xi_2)]^{1/2}), \quad (3.6)$$

where

$$\xi_1 = \sqrt{\frac{\pi}{8}}\frac{\Delta}{kv_{Ti}}\frac{T_e T_i}{(T_e+T_i)^2}, \quad \xi_2 = \frac{T_e}{T_e+T_i}, \quad \frac{T_e}{T_i}\frac{m}{M} = \beta_0 \ll 1.$$

Taking $\omega = 0$ we can find the value of the threshold field:

$$Z^2_{\text{thr}} = \frac{4}{\alpha_\perp\xi_2}\frac{\omega_0}{\omega_{Le}}\frac{\Delta^2+\nu_{ei}^2}{|\Delta|\omega_{Le}}, \quad \Delta < 0. \quad (3.7)$$

It follows from the condition $\Delta < 0$ that aperiodic excitation is possible if the following inequalities are satisfied:

$$\omega_0 > \Omega_e > \omega_{Le}, \quad \omega_0 < \Omega_e < \omega_{Le} \quad (3.8a)$$

$$\omega_{Le} > \omega_0 > \Omega_e. \quad (3.8b)$$

The condition in (3.8a) actually denotes a resonance of ω_0 at the upper and lower hybrid frequencies $\omega_\pm$ close to Ω_e. In this case the threshold field, which is minimized with respect to Δ, is given by

$$(Z^2_{\text{thr}})_{min} = \frac{8}{\alpha_\perp\xi_2}\frac{\omega_0}{\omega_{Le}}\frac{\nu_{ei}}{\omega_{Le}}, \quad |\Delta_0| = \nu_{ei}. \quad (3.9)$$

In the case in (3.8b) the frequency ω_0 exceeds ω_- and the threshold field is much greater than (3.9) because of the inequality

$$|\Delta| > \delta \gg \nu_{ei}.$$

We now give the growth rate for microwave fields somewhat greater than the threshold value:

$$\omega = i\frac{\alpha_\perp\xi_2}{8}\omega_0\frac{\omega_{Le}^2}{\omega_0^2}\frac{Z^2 - Z^2_{\text{thr}\ min}}{1+2\nu_{ei}\xi_1/\omega_0\xi_2}. \quad (3.10)$$

It is evident from Eqs. (3.9) and (3.10) that the introduction of a fixed magnetic field can enhance the development of the instability and can also act as a stabilizing factor. If the condition $\omega_0 < \Omega_e < \omega_{Le}$ is satisifed the threshold field in Eq. (3.9) can be much smaller than the value obtained in[8,9] (specifically, for the condition $\omega_0 \sim \Omega_e < \alpha_\perp\omega_{Le}/2 \sim \omega_{Le}/2$), that is to say, the magnetic field enhances the development of the instability. The growth rate of the instability is also increased above threshold. In the opposite limiting case, that is to say when $\omega_0 > \Omega_e > \omega_{Le}$, the plasma is stabilized by the magnetic field.

2. Excitation of Periodic Waves in the Frequency Region $kv_{Ti} \ll \omega \ll kv_{Te}$

Here we are interested in the excitation of waves at frequencies $\omega \gg \Omega_i = e_i B_0/Mc$. In this limit, the quantity $\delta\epsilon_e(\omega, \mathbf{k})$ in Eq. (2.7) is taken to be the expression in (3.5) while the quantity $\delta\epsilon_i(\omega, \mathbf{k})$ is given by

$$\delta\varepsilon_i(\omega,\mathbf{k}) = -\frac{\omega_{Li}^2}{\omega^2}\left(1 - i\frac{8}{5}\frac{\nu_{ii}k^2v_{Ti}^2}{\omega^3} - i\sqrt{\frac{\pi}{2}}\frac{\omega^3}{k^3v_{Ti}^3}\exp\left\{-\frac{\omega^2}{2k^2v_{Ti}^2}\right\}\right). \quad (3.11)$$

Assuming that ω is real, from the dispersion equation in (2.7) we obtain a system of equations whose solution gives the value of the threshold field and the oscillation frequency:

$$(\omega^2-\omega_s^2)(\Delta^2+\nu_{ei}^2-\omega^2) + 4\omega^2\nu_{ei}\gamma_s(\omega) = K^2\omega_s^2\omega_0\Delta,$$
$$\gamma_s(\omega)(\Delta^2+\nu_{ei}^2-\omega^2) - \nu_{ei}(\omega^2-\omega_s^2) = 0. \quad (3.12)$$

Here, $\omega_s = kv_s$, where $v_s = \sqrt{T_e/M}$ is the ion-acoustic velocity,

$$\gamma_s(\omega) = \gamma_i(\omega) + \gamma_L = {}^4/_5\nu_{ii}\frac{k^2v_{Ti}^2}{\omega^2} + \sqrt{\pi m/8M}\,\omega,$$

is the linear growth rate and

$$\nu_{ii} = \frac{4}{3}\sqrt{\frac{\pi}{M}}\frac{e_i^4 N_i L}{T_i^{3/2}}$$

is the effective frequency for ion-ion collisions. In order to facilitate the subsequent analysis of (3.12) we investigate the two limiting cases in which the damping of the wave is due to Cerenkov absorption on electrons (we neglect Cerenkov absorption on the ions, as is valid when $\omega^2 \gg \omega_s^2(T_i/T_e)\ln\beta_0^{-1}$ where $\beta_0 = T_e m/T_i M$) or ion-ion collisions, and take $|e| = e_i$.

In the first case $\gamma_L \gg \gamma_i$ and the damping rate is independent of ω. In this case the solution of Eq. (3.12) coincides formally with the solution given in[7]. The only difference lies in the fact that in a fully ionized plasma in this limit only the shortwave ion-acoustic oscillations can be supported:

$$\omega_{\rm thr} = \omega_s \gg \frac{8}{5\sqrt{\pi}}\left(\frac{T_e}{T_i}\right)^{1/2} \nu_{ei}, \Omega_i. \qquad (3.13)$$

In this case the plasma is not isothermal $T_e/T_i \gg \max(1, \ln \beta_0^{-1})$ while the threshold field necessary for exciting these waves is

$$(Z^2_{\rm thr})_{min} = \frac{4}{\alpha_\perp}\left(\frac{\pi}{2}\frac{m}{M}\right)^{1/2}\frac{\omega_0}{\omega_{Le}}\frac{\nu_e}{\omega_{Le}}, \quad \Delta_0 = \omega_s. \qquad (3.14)$$

We note that as a consequence of Eq. (3.1) and the inequalities $\omega_0 \sim \Omega_e \gg kv_{Te}$ and $kr_{De} \ll 1$ the region in which ion-acoustic waves can be supported is bounded by the conditions

$$\left(\frac{m}{M}\right)^{1/2}\nu_{ei} \ll \omega_s < \left(\frac{m}{M}\right)^{1/2}\delta\left(2\ln\frac{\delta}{\nu_{ei}}\right)^{-1/2}; \quad \omega_{Li}\left(\frac{M}{m}\right)^{1/2}\Omega_i. \qquad (3.15)$$

Using Eqs. (3.13) and (3.15) for specified plasma parameters N_i and $T_{e,i}$ we can then find the allowable values of the magnetic field or the detuning δ.

In the second case $\gamma_L \ll \gamma_i$ it is possible to excite electrostatic waves at a frequency $\omega_{\rm thresh} = \omega_s$ and at frequencies which are very different from the ion-acoustic frequency. We shall first consider the excitation of ion-acoustic waves. In this limit the wavelengths that are excited are longer than those given by (3.13):

$$\Omega_i, \nu_{ei} \ll \omega_s \ll \frac{8}{5\sqrt{\pi}}\left(\frac{T_e}{T_i}\right)^{1/2}\nu_{ei}, \qquad (3.16)$$

or

$$\Omega_i, \left(\frac{16}{15\sqrt{2}}\right)^{1/3}\beta_0^{1/6}\nu_{ei} \ll \omega_s \ll \nu_{ei}. \qquad (3.17)$$

The corresponding minimum threshold microwave fields are

$$(Z^2_{\rm thr})_{min} = \frac{64}{5\sqrt{2}}\frac{\beta_0^{1/2}}{\alpha_\perp}\frac{\omega_0}{\omega_{Le}}\frac{\nu_{ei}}{\omega_{Le}}\frac{\nu_{ei}}{\omega_s}, \quad \Delta_0 = \omega_s, \qquad (3.18)$$

$$(Z^2_{\rm thr})_{min} = \frac{256}{15\sqrt{6}}\frac{\beta_0^{1/2}}{\alpha_\perp}\frac{\omega_0}{\omega_{Le}}\frac{\nu_{ei}}{\omega_{Le}}\frac{\nu_{ei}^2}{\omega_s^2}, \quad \Delta_0 = \frac{\nu_{ei}}{\sqrt{3}}. \qquad (3.19)$$

As in the preceding case, the plasma cannot be isothermal. In addition there is a wave branch characterized by a threshold frequency

$$\omega_{\rm thr} = (\omega_s\nu_{ei})^{1/2}\left(\frac{6}{5\sqrt{2}}\right)^{1/4}\beta_0^{1/8}, \qquad (3.20)$$

the corresponding threshold field being

$$(Z^2_{\rm thr})_{min} = \frac{12}{\alpha_\perp}\left(\frac{3\beta_0^{1/2}}{5\sqrt{2}}\right)^{1/2}\frac{\omega_0}{\omega_{Le}}\frac{\nu_{ei}}{\omega_{Le}}\frac{\nu_{ei}}{\omega_s}, \quad \Delta_0 = \frac{\nu_{ei}}{\sqrt{2}}. \qquad (3.21)$$

Here, the values of ω_s are specified by the relation

$$\Omega_i\frac{\Omega_i}{\nu_{ei}}\left(\frac{5\sqrt{2}}{6\beta_0^{1/2}}\right)^{1/2} \ll \omega_s \ll \nu_{ei}\beta_0^{1/4}\left(\frac{6}{5\sqrt{2}}\right)^{1/2}\min\left\{1; \frac{T_e}{T_i\ln(1/\beta_0)}\right\}. \qquad (3.22)$$

It has been assumed in the derivation of Eqs. (3.18)–(3.22) that $\beta_0 \ll 1$. In the opposite limit only longwave oscillations can be excited

$$\Omega_i\frac{\Omega_i}{\nu_{ei}}\left(\frac{5\sqrt{2}}{6\beta_0^{1/2}}\right)^{1/2} \ll \omega_s \ll \frac{3}{16}\left(\frac{15}{\sqrt{2}}\right)^{1/2}\beta_0^{-1/4}, \qquad (3.23)$$

for which the threshold frequencies and fields are given by Eqs. (3.20) and (3.21) respectively.

It follows from a comparison of the expressions given above for the determination of the threshold microwave fields that the smallest values are those associated with Eq. (3.14). As in Eqs. (3.18), (3.19) and (3.21), the necessary condition for the existence of an instability is that the quantity Δ_0 be positive. Starting from this condition we can determine the range of values of ω_0 in which parametric excitation of periodic waves occurs. It turns out that the inequality in (3.8a) must be realized together with the inverse relation in (3.8b). Hence, all that has been indicated above with respect to the excitation of aperiodic waves holds here and in the region $\omega_{Le} < \omega_0 < \Omega_e$ the oscillations are excited with greater difficulty than in the frequency range given by (3.8a). This feature is explained as follows: if both (3.8b) and the inverse relation hold then, as is clear from (3.3b), we have $\Delta > \delta \gg \omega_s$, ν_{ei} and minimization with respect to Δ ($\Delta = \Delta_0$) is impossible. We note further that the allowable values of the magnetic field, as follows from (3.13), (3.15), (3.16), (3.17), (3.22) and (3.23), are always bounded; in turn this limits the effects caused by the application of the fixed magnetic field both with regard to stabilization of the instability as well as the reduction in the threshold value of the microwave field.

It is still too early to speak about a detailed comparison of theory with experiment. Even in the work containing the most significant experiments[14,15] a detailed analysis of the instability threshold has not yet been carried out. Nonetheless, on the basis of the results contained in[14] one can make a qualitative comparison. Since the experiment was carried out under conditions satisfying the relation

$$\omega_{Le} \lesssim \omega_0 \lesssim \Omega_e, \qquad (3.24)$$

then, as follows from Eqs. (3.8a) and (3.8b), only the periodic instability can develop. As we have indicated above, in the frequency region denoted by (3.24) the threshold fields are considerably greater than those considered in the present paper. To make a practical verification of this assertion we can estimate the calculated threshold values of the microwave field (3.14), (3.18), (3.19) and (3.21), which are written in the form

$$(Z^2_{\rm thr})_{min} = \frac{8\omega_0|\delta|}{\omega_{Le}^2}\varkappa. \qquad (3.25)$$

The representation of the threshold field given in Eq. (3.25) is convenient from the point of view of the comparison with the experimental data[14] and gives the value $\kappa \approx 10^{-2}$. The calculation of κ for the experimental conditions[14] ($\nu_{ei} \sim 10^5$ sec^{-1}, $\omega_0 \sim 10^{10}$ sec^{-1}) carried out starting from the expression for the threshold field (3.14) yields the value $\kappa \sim 10^{-6}$, whereas estimates made on the basis of (3.18), (3.19) and (3.21) show that $\kappa \ll 10^{-4}$. This result does not contradict the derivation of the theory developed in the present paper as indicated in (3.24), which shows that the value of threshold field is considerably greater than the value determined here for the periodic instability.

To make a comparison of the theoretical results with the experimental data of[15] we note the changes that are introduced when the microwave field is not linearly polarized. In the simplest case of circular polarization, for waves propagating along the magnetic field it is easy to show[10] that all of the results obtained above for the threshold fields will differ by the factor

$$\chi=\alpha_\perp \begin{pmatrix} 1/2 \\ 2\omega_0^2/\delta^2 \end{pmatrix}. \quad (3.26)$$

Here, the upper row in the brackets refers to right-hand polarization while the lower row refers to left-hand polarization. It will be evident that for the right-hand polarization the threshold value of the microwave field is much lower than for the left-hand polarization. This means that the anomalous effects must be rather strong in the first case. This qualitative conclusion is in complete agreement with the results[15] in which the cyclotron resonance case have been investigated ($\omega_0 \sim \Omega_e$). Hence, it is reasonable to assume that the rapid plasma heating observed in[15] (a time $t \sim 10-20$ μsec) is associated with the parametric instability that has been investigated above.

In conclusion we note that under cyclotron resonance conditions ($\omega_0 \sim \Omega_e$) it is easiest to excite short waves which propagate almost parallel to the magnetic field. In this case it is found that the threshold values of the microwave field, beyond which the magnetoactive plasma becomes unstable against electrostatic waves, have minima when the external microwave electric field and the fixed magnetic field are perpendicular and satisfy the relation $\omega_0 < \Omega_e < \omega_{Le}$. In the inverse case the magnetic field has a stabilizing effect on the instability caused by the microwave field. The authors wish to thank A. A. Rukhadze and Yu. M. Aliev for help and valuable comments.

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