TELESCOPIC-RESONATOR LASER

Yu. A. ANAN'EV, G. N. VINOKUROV, L. V. KOVAL'CHUK, N. A. SVENTSITSKAYA, and V. E. SHERSTOBITOV

Submitted July 29, 1969

Zh. Eksp. Teor. Fiz. 58, 786—793 (March, 1970)

The characteristics of a laser with an unstable resonator at large Fresnel numbers and radiative losses are considered. The possibility of describing certain properties of such a laser in the geometrical optics approximation without the use of the diffraction theory of open resonators is discussed. The results of an experimental investigation of a laser with an unstable resonator formed by a telescopic system of mirrors are presented.

We have previously shown[1,2] the promise of application of "unstable" resonators in lasers with a high spatial coherence of radiation. We shall now consider in more detail the properties of this little studied class of open resonators, using as an example a telescopic system of convex and concave mirrors with coincident foci (Fig. 1a). Such an arrangement of mirrors provides the best use of the volume of a cylindrical active body[2]; in addition, the analysis of the processes of formation of the wave front becomes quite lucid.

1. STRUCTURE OF THE WAVE FRONT IN THE GEOMETRIC APPROXIMATION

Let us compare the processes of formation of oscillations in a telescopic resonator and one with plane mirrors. In doing this we use the geometric approximation, assuming that radiation through a weakly inhomogeneous medium is propagated along straight lines, and the running bend of the wave front is equal to the difference in the optical paths along corresponding trajectories.[1)]

In a resonator with plane mirrors (its equivalent diagram is given in Fig. 1b), as we begin to have multiple passages of the radiation through an inhomogeneous active sample, an unlimited accumulation of wave front distortions takes place, and the geometrical approximation ceases to be applicable. Finally, the wave aberrations of the established field distribution are determined by diffraction effects and, as a rule, are many times greater than the magnitude of the aberrations in one passage δ. The corresponding estimates in the diffraction approximation for small δ can be made by the methods of perturbation theory.[3,4]

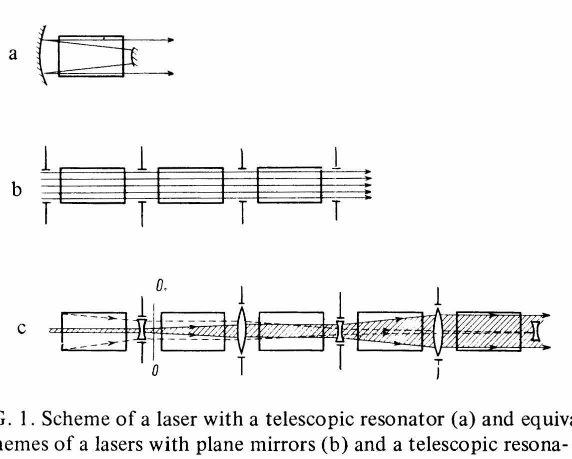

FIG. 1. Scheme of a laser with a telescopic resonator (a) and equivalent schemes of a lasers with plane mirrors (b) and a telescopic resonator (c).

[1)]Note that by introducing a complex index of refraction it is possible to account uniquely for not only phase but amplitude distortions that are due to a nonuniform distribution of the gain coefficient.

Since in actuality we accumulate not only distortions due to inhomogeneity of the medium but also perturbations due to diffraction at the open edge, it is obvious that in resonators with plane mirrors diffraction effects always will play the deciding role.

Now let us consider the passage of a coherent light beam along an optical path equivalent to a telescopic resonator (Fig. 1c). As is seen from the figure, the radiation which fills the entire cross section of the active element has several passages before this "spread out" from a small central portion of the cross section. The size of this portion diminishes with distance from the exit section as a geometric progression and quickly becomes sufficiently small that one can neglect wave aberrations within it.

It follows that the accumulation of aberrations in a telescopic resonator occurs only during the course of a few passes. The number of passes that give a significant contribution to the deformation of the front of the established wave diminishes with an increase in the magnification of the telescopic system M and an increase of the characteristic dimension of the optical inhomogeneities h (at distances of the order h the change in δ is small). If the inhomogeneity of the medium amounts to the presence of a slowly changing gradient of refractive index (due, for example, to thermal effects, the final magnitude of the aberrations of the established front can be found easily from a simple summation. Estimates show that this magnitude does not greatly exceed the aberrations that are inevitable in any laser scheme when M is as low as 2 and rapidly approximates it as M increases.

When M is much greater than 1, the only criterion for the applicability of the geometric approximation is, in essence, the relation $h^2/\lambda L \gg 1$ (L is the resonator length), which is fulfilled for lasers based on such highly homogeneous media as neodymium glass.

From the figure it is clear that the external portion of the beam, in which there could be significant diffraction perturbations, leaves the resonator after a single passage through the telescopic system (cf.[5]). As a consequence, edge diffraction effects cannot significantly change the pattern given by the geometrical approximation.[2)]

Thus, the introduction of "unstable" resonators leads to a qualitatively new situation. For a large cross section of the active body and large radiative losses at the same time, diffraction effects cease to be determinative, at least for the oscillations with the highest Q. It becomes possible to describe certain properties of the laser, in particular its limiting characteristics that apply to the directivity of the radiation, by the methods of geometrical optics.

From the above it is also possible to conclude that the utilization of "unstable" resonators is particularly advantageous in cases of active media with a small gradient of refractive index. At large M in such resonators, it should be possible to observe low angular divergence of the radiation without the use of angular selectors with their concomitant energy losses. All these advantages of unstable resonators become obvious when one realizes that a laser with a telescopic resonator corresponds to the system of a master oscillator and amplifier with a matching telescope between them. The role of the oscillator is played by the central zone of the sample, and that of the amplifier by its periphery.

2. REMARKS ON THE PROPERTIES OF REAL LASERS

Let us consider a number of effects which can be important in real lasers and which do not fall within the framework of the discussion presented above. One of these is the effect of light scattering at large angles; microinhomogeneities of the active medium and the various interfaces (e.g., the ends of the active element) are the most frequent sources of this.

In resonators with plane mirrors the effect of light scattering (as well as the low-order aberrations considered in Sec. 1) can be treated successfully by perturbation theory.[6] For a resonator with large diffraction losses the system of eigenfunctions ceases to be complete,[7] and perturbation theory is inapplicable. Since light scattering at large angles leads to "fine" structure in the light wave front, the simplest geometrical approximation of Sec. 1 is also inapplicable.

We make use of the following procedure. We assume that the wave arising from light scattering can be represented in the form of a system of spherical waves, and we follow each of these separately as it passes through the resonator. For simplicity we consider only waves the centers of which are on the resonator axis. The reference plane is placed directly outside the convex mirror, as shown in Fig. 1c (OO_1). Then in the geometric approximation the curvature of the wave c after n passages through the resonator is formed according to the following rule:

$$y_{n+1} = y_1 M^{2n} / [y_1(M^{2n} - 1) + 1], \tag{1}$$

where $y \equiv cf$ is the curvature in dimensionless units (f is the focal length of the convex mirror). At the same time the size of the transverse section of the region occupied by the wave varies as

$$K_n(y_1) = [y_1(M^{2n} - 1) + 1] / M^n. \tag{2}$$

It follows from Eq. (1) that in the geometrical approximation there exist two waves the curvature of which is reproduced on passage through the resonator (see also[8]). The value y = 1 corresponds to the fundamental stable wave in which generation is possible. Its section per passage increases, of course, M times. The waves with $y \neq 0$ that form as a result of light scattering quickly converge to the fundamental. The number of passages required for this is determined from the condition $y_1 M^{2n} \gg 1$.

The second, unstable configuration with repeating curvature is the converging wave y = 0 (shown dashed in Fig. 1c). Diffraction effects prevent its cross section from decreasing without limit.

An analysis of the diffraction effects shows that they finally play a large role only when $|y_1| \leq 2f\lambda/a^2$, where 2a is the transverse dimension of the resonator cross section in which the scattered wave arises. Usually $2f\lambda/a^2 \ll 1$; in particular, for the case of the laser used in our work, this quantity was about 4×10^{-3}. Thus estimates obtained in the geometric approximation are valid down to extremely small values of y_1. At the same time, it follows from Eq. (2) that waves with small y_1 are propagated with smaller losses than the fundamental wave, and in the first passes generally without loss ($K_n < 1$). As a result, the intensity of these waves increases compared to the intensity of the fundamental wave in the ratio

$$\left[\frac{K_n(1)}{K_n(y_1)}\right]^2 = \left[\frac{M^{2n}}{y_1(M^{2n}-1)+1}\right]^2,$$

which tends to $1/y_1^2$ as n increases.

Thus, if converging waves arise in a laser with an unstable resonator because of light scattering, their intensity can increase by several orders as they pass through the resonator. The attendant increase in field close to the resonator axis in real lasers can markedly redistribute the number of excited atoms over the sample cross section and significantly change the generation pattern.

Let us also consider the role of diffraction at the resonator edge. We have already pointed out[2] that with increasing size and curvature of the mirrors there occurs in the diffraction approximation a successive shift of modes having the least losses (a similar situation is observed in the case of a plane resonator with misadjusted mirrors[9]). Figure 2 shows the results of numerical calculations that illustrate this shift.[3)] As is seen from the graphs, as the mirrors are deflected and as N_{eq}[4)] is increased from zero (plane resonator, curve I) to a value close to unity (curve II), the distribution of phase is practically unchanged. Simultaneously there is a growth in the intensity of the field at the resonator edge accompanied by an increase in loss. Finally, when $N_{eq} > 1$, another mode (curve III) with a phase lead greater by approximately π begins to have the least loss. The relative intensity of the field of this mode at the resonator edge is small for $N_{eq} \approx 1$; as a result the

[2)]We shall consider the role of diffraction effects in more detail in the next section.

[3)]In [11] the results of similar calculations are presented, but unfortunately without data on the phase distribution.

[4)]The definition of N_{eq} is given in [11,2].

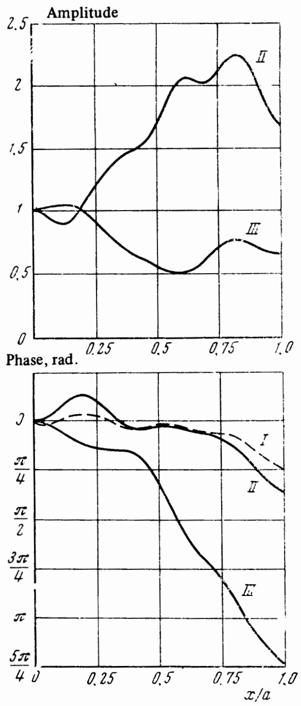

Fig. 2. Distribution of amplitude and phase of the field of the mode with least loss in the plane of one of the mirrors of an empty ideal resonator (x/a – transverse coordinate on the mirror surface); N = 2.5. I – plane mirrors, g = 1 – L/R = 1 [10]; II – symmetrical resonator with convex mirrors, g = 1.065 (M = 1.431, N_{eq} = 0.915); III – the same, g = 1.09 (M = 1.524, N_{eq} = 1.083).

loss for this configuration of field is smaller, in spite of the greater curvature of the wave front. With further increase of N_{eq} the field intensity of this mode at the edge of the resonator rises, and the process of mode shifting is repeated.

Numerical calculations show also that as N_{eq} increases, the differences between the lowest mode of the diffraction approximation and the spherical wave of the geometrical approximation diminish and practically disappear at $N_{eq} > 10$. We note also that these differences, like the differences between the exact solutions and the Vaĭnshteĭn functions for the case of the plane resonator, are determined in real lasers by so many factors that it would make no sense to take them into account. In particular, if the dimensions of the beam are determined in essence only by the size of one of the mirrors, then when the beam fills the same cross section of the active element, the values of N_{eq} corresponding to cases bound by convex and concave mirrors differ by a factor M.

It is more important to use diffraction theory to estimate the field distribution and to find the eigenvalues of the modes that differ from the "fundamental" but possess nearly the same losses. The theoretical papers[11-13] devoted to these questions are in error, since the discrete set of functions found in them without considering the diffraction effects at the resonator edge are far from being the set of lowest modes of the resonator in the diffraction approximation (with respect to[11] we pointed this out earlier[2]). Hence the conclusion made in these papers about the large magnitude of the difference of loss for the lowest modes of unstable resonators is also incorrect. It may be presumed that in actuality these differences are in order of magnitude equal to the amplitude of the oscillations on the graph representing the dependence of the losses of the "lowest" modes on N_{eq} ([2], Fig. 2) and are not too large.

When the loss differences are not so great, several transverse modes may be present. In lasers with resonators of the "stable" configuration or plane mirrors, the reason for this multimode character is usually the noncoincidence of the volumes of the lowest mode and the active medium (see, for example,[14]).

The distribution of the intensity of the lowest mode of an unstable resonator over the cross section of the active rod is close to uniform. In this case the presence of several transverse modes may arise because in the spiked regime there is an accumulation of inverted population between individual spikes up to a level in excess of threshold. This creates the condition for the appearance of one of the modes with the least losses depending into which of them the "seeding" radiation falls first. The source of this can be either spontaneous noise or residual radiation from the preceding spike remaining in the resonator.

The most important feature of the kinetics of lasers with unstable resonators is that the process of establishment of oscillations over the whole volume goes very fast.[15] This is guaranteed by the presence of a mechanism for the forced distribution of radiation over the whole cross section. At the same time, in the case of lasers with plane mirrors the connection between individual regions of the cross section is effected only by virtue of diffraction at high angles; hence it is very weak and can be destroyed easily.[16] It is just for these reasons that one frequently observes alternate generation of separate regions in lasers with plane mirrors. In unstable resonators this situation is evidently eliminated, and this again favors a greater spatial coherence of the radiation and can lead to more regular generation kinetics.

3. EXPERIMENTAL RESULTS

The above ideas were chekced with the laser used in our earlier work.[2] The diameter of the neodymium-glass cylinder was 45 mm and it was 600 mm long. The sample, placed in a cuvette with a liquid filter, was pumped with four IFP-20 000 lamps (the illuminating system of this laser is described in[17]). The resonator of plane and convex mirrors was replaced by a telescopic one with the same magnification (M = 2). The radius of curvature of the concave mirror was 400 cm, that of the convex mirror, 200 cm, and the diameter of the latter was about 22 mm ($N_{eq} \approx 50$). In order to eliminate the effect of radiation reflected from the ends of the sample, they were inclined at an angle of about 3° to the axis of the resonator.

Going from the system of plane and convex mirrors to the telescopic system led to an increase in the radiated energy (due to better utilization of the sample volume) up to the level characteristic of lasers with plane mirrors. The angular divergence of the radiation decreases simultaneously (Fig. 3, curve II). The reason for the greater angular divergence observed earlier (Curve I) was probably the perpendicularity of the ends of the sample to the resonator axis.

In the case of the telescopic resonator, one of the two waves reflected from the congruent surface (toward the concave mirror) is purely converging (y = 0). In the preceding section it was shown that the emergence of a

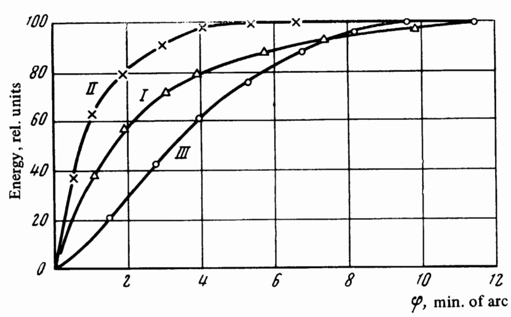

FIG. 3. Angular distribution of the energy of radiation: I – resonator with plane and convex mirrors, [2] II – telescopic resonator, II – the the same, transparent plate inside resonator. Along the ordinate axis is plotted the fraction of energy included in a cone with vertex angle φ.

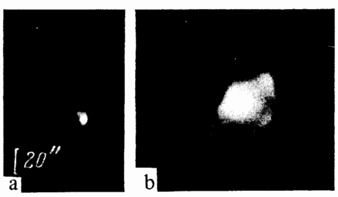

FIG. 4. Photograph of the distribution of radiation in the far zone: a – without plate, b – with plate.

converging wave even with a very small initial intensity can strongly affect the structure of the radiation field. In fact, if one places a glass plane-parallel plate perpendicular to the axis of the telescopic resonator, then even in the presence of bidirectional light transmission (coefficient of reflection not greater than 0.3%) the pattern of the generation changes strikingly (Fig. 4; Fig. 3, curve III).

Attention is directed to the fact that even in the absence of light scattering the angle which includes the major portion of the radiated energy exceeds by several times the width of the central kernel. A series of experiments to elucidate this was undertaken.

One of these was to photograph the angular distribution integrated over time of the radiation filling the central portion of the sample cross section. For this we used the beam reflected from the end of the rod on the side of the concave mirror. A portion of the beam was cut out by a circular diaphragm standing outside the resonator. The diameter of the isolated beam varied from 10 to 45 mm.

Photometry of these photographs showed that as the beam diameter was increased the width of the kernel of the angular distribution diminished in inverse proportion to the diameter and was extremely close to the diffraction limit. At the same time the relative intensity of the wings of the distribution increased markedly. This behavior is evidently associated with the presence of thermal deformations, which, as special measurements showed, are greatest and most irregular just in the outer layers of the sample (their greatest magnitude reaches about 2λ).

Multimode generation could also have affected the radiant distribution integrated over time of a flash. Photographs of the angular distribution in individual "spikes" were taken to elucidate this. It was found that the pattern of generation from spike to spike did not remain unchanged.

Toward the end of the pump pulse the angular divergence increases on the average, and the pattern of distribution of radiation in the individual spikes becomes less clear. This is obviously associated with the thermal deformations of the resonator increasing with time.

More noteworthy is the fact that along with this general tendency there is observed an alternation of spikes with markedly differing character of distribution (Fig. 5). Some of them possess a distinct diffraction structure (Fig. 5a, b); it may be presumed that they correspond to the lowest mode of the resonator, extremely close to a spherical wave. The remaining spikes evidently pertain to modes with other transverse indices. Any attempt to identify these modes must await the establishment of a theory of unstable resonators in the diffraction approximation.

It is necessary to note also that during the time of a pump pulse we observed irregular displacements of the kernel of the angular distribution which we were unable to explain.

The distribution in the far zone was accomplished by photographing the interference pattern from two apertures placed at the output of the laser. The distance between the apertures was many times greater than in other experiments of this kind[18] and amounted to 30 mm. An example of a time scan of the interference pattern, obtained with a slit and camera with rotating drum, is shown in Fig. 6.

In the majority of the individual spikes the radiation was completely coherent. The phase difference in corresponding portions of the wave front from spike to spike varies, which corresponds to the data obtained in the photography of the angular distribution.

For some of the spikes the interference pattern was diffuse, the radiation incoherent. The nature of these spikes requires explanation (see also[2]).

Besides investigations of the angular distribution, we also took photographs of the distribution of radiation over the cross section of the resonator. For this we placed a small (~2 mm diameter) opaque screen inside the resonator, not far from its axis. The presence of the screen led to the absence of radiation not only within the corresponding portion of the resonator cross section, but also within portions which were separated from the axis at distances M and M^2 times greater and had correspondingly greater dimensions (see Fig. 1c). Also visible was a sharp structure caused by diffraction both at the edge of the convex mirror and at the screen, a phenomenon characteristic of systems with high spatial coherence.

This observation confirmed the statements made in the preceding section that independent generation in separate regions of the sample cross section is impossi-

FIG. 5. Photographs of the angular distribution of radiation in separate spikes during one pump pulse: a – one of the first spikes, b – after ~ 300 μsec, and c – after ~ 500 μsec.

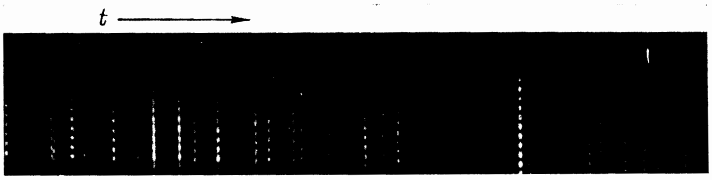

FIG. 6. Time scan of the interference pattern from two slits placed at the output of the laser (beyond the convex mirror). Distance between slits 30 mm, photograph taken in the focal plane of a lens with f = 50 m.

ble in the case of unstable resonators. Obvious also is the particularly important role of the central portion, from which the radiation "spreads" over the entire cross section. By introducing into this portion radiation from an external source, it is possible to provide effective control of the radiation of the whole laser and thus obtain desirable time and spectral characteristics.

If for this purpose one makes an aperture in the center of one of the mirrors, the system in essence becomes an amplifier. It differs from other amplifying systems in that, as follows from Fig. 1c, it provides, simultaneously with amplification, a broadening of the beam cross section and a corresponding decrease in its angular divergence.

The information presented in this paper, together with the results of our previous work,[2] is obviously sufficient to serve as a basis for making conclusions about the promise and possible fields of application of systems with unstable resonators.

[1] Yu. A. Anan'ev, N. A. Sventsitskaya, and V. E. Sherstobitov, Dokl. Akad. Nauk SSSR **179**, 1304 (1968) [Sov. Phys.-Dokl. **13**, 351 (1968)].

[2] Yu. A. Anan'ev, N. A. Sventsitskaya, and V. E. Sherstobitov, Zh. Eksp. Teor. Fiz. **55**, 130 (1968) [Sov. Phys.-JETP **28**, 69 (1969)].

[3] V. V. Lyubimov, Opt. Spektrosk. **21**, 224 (1966); Opt. Spektrosk. **24**, 815 (1968).

[4] Yu. A. Anan'ev, V. V. Lyubimov, and I. B. Orlova, Zh. Tekh. Fiz. **39**, 1872 (1969) [Sov. Phys.-Tech. Phys. **14**, 1408 (1970)].

[5] A. E. Siegman, Proc. IEEE **53**, 177 (1965).

[6] Yu. A. Anan'ev, V. V. Lyubimov, and B. M. Sedov, Zh. Prikl. Spektrosk. **8**, 955 (1968).

[7] L. A. Vaĭnshteĭn, Otkrytye rezonatory i otkrytye volnovody (Open Resonators and Open Waveguides), Sov. Radio, 1966 (Engl. Transl., Golem Press, Boulder, Colo., 1969).

[8] W. Kahn, Appl. Optics **5**, 407 (1966).

[9] H. Ogura, Y. Yoshida, and J. Ikenoue, J. Phys. Soc. Japan **20**, 598 (1965).

[10] A. G. Fox and T. Li, Bell Syst. Tech. J. **40**, 453 (1961).

[11] A. E. Siegman and R. Arrathoon, IEEE J. Quant. Electronics **QE-3**, 156 (1967).

[12] S. R. Barone, Appl. Optics **6**, 861 (1967).

[13] L. Bergstein, Appl. Optics **7**, 495 (1968).

[14] Yu. A. Anan'ev, Zh. Tekh. Fiz. **37**, 139 (1967) [Sov. Phys.-Tech. Phys. **12**, 97 (1967)].

[15] Yu. A. Anan'ev, N. A. Sventsitskaya, and V. E. Sherstobitov, Zh. Tekh. Fiz. **39**, 1325 (1969) [Sov. Phys.-Tech. Phys. **14**, 997 (1970)].

[16] N. G. Basov, É. M. Belenov, and V. S. Letokhov, Zh. Tekh. Fiz. **35**, 1098 (1965) [Sov. Phys.-Tech. Fiz. **10**, 845 (1965)]; Dokl. Akad. Nauk SSSR **161**, 799 (1965) [Sov. Phys.-Dokl. **10**, 311 (1965)].

[17] Yu. A. Anan'ev, I. M. Buzhinskiĭ, M. P. Vanyukov, E. F. Dauengauer, and O. A. Shorokhov, Opt.-Mekhan. Promysh. **9**, 26, 1968.

[18] V. V. Korobkin, A. M. Leontovich, M. N. Popova, and M. Ya. Shchelev, Zh. Eksp. Teor. Fiz. **53**, 16 (1967) [Sov. Phys.-JETP **26**, 10 (1968)].

Translated by L. M. Matarrese
95