CONTRIBUTION TO THE THEORY OF QUANTUM ELECTROMAGNETIC WAVES IN METALS IN A MAGNETIC FIELD

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Coupled longitudinal and transverse quantum electromagnetic waves in metals located in a strong magnetic field are investigated. It is shown that waves with velocity close to the helicon phase velocity are elliptically polarized in a plane perpendicular to the constant magnetic field. Waves whose velocity differs significantly from that of the helicon are longitudinal and their electric vector is polarized along the wave vector. Explicit expressions for the velocity dispersion of the waves are derived for slow waves.

1. INTRODUCTION

A number of recent theoretical papers predict the existence of specific quantum electromagnetic waves in metal in quantizing magnetic fields.[1-6] These waves are low-frequency oscillations of the electron-sound type in the degenerate-hole plasma of the metal. The frequency of these waves is much lower than the characteristic cyclotron frequency of the conduction electron. The quantum-wave dispersion law is linear, just as for ordinary acoustic oscillations, and the velocity of the quantum waves is determined by the quantum values of the longitudinal component (relative to the magnetic field) of the carrier velocity on the Fermi surface. The number of these quantum waves coincides with the number of the Landau levels on the Fermi surface (or differs from them by unity). McWorter and May,[1] and also Ginzburg, Konstantinov, and Perel'[2] analyzed longitudinal quantum waves propagating along a constant magnetic field. These longitudinal waves are analogous to longitudinal plasma oscillations of the acoustic type, which can exist in a degenerate plasma with strongly differing electron and hole masses (Pines and Schrieffer[7]). In [3] we considered transverse quantum waves in an isotropic metal with one group of carriers. Such transverse quantum waves should be observed also in compensated metals (with identical electron and hole concentrations) in the case of wave propagation along a magnetic field.[4,5] Transverse quantum waves, in the case of an isotropic metal, take place in the case of propagation at an angle to the magnetic field.[3] In compensated metals, the transverse quantum waves can propagate also along the magnetic field, if the Fermi surface is anisotropic and has many sheets. Zyryanov, Okulov, and Silin[6] found quantum spin waves.

The physical cause of the quantum electromagnetic waves in metals is the presence of collisionless absorption of the waves as a result of the Landau damping. If the conduction electrons have a large mean free path, then their interaction with the electromagnetic field can be regarded as direct absorption of quanta of an electromagnetic wave. In a strong magnetic field H, the states of the electrons are described by the magnetic quantum number n, the projection of the momentum p_z (the z axis is parallel to the external magnetic field H), and by the coordinate of the center of rotation of the electrons. We shall not take into account the spin splitting of the energy levels and the Fermi-liquid interaction of the conduction electrons. In the simplest case of a spherical Fermi surface (alkali metals), the conduction-electron energy levels are determined by the well known formula

$$\varepsilon_n(p_z) = \hbar\Omega n + p_z^2/2m, \tag{1}$$

where $\Omega = eH/mc$ is the cyclotron frequency, e is the absolute value of the electron charge, m is the effective mass, and c is the velocity of light. For simplicity, we shall not take into account the displacements of the energy levels by $\hbar\Omega/2$. On the Fermi surface, the longitudinal velocity of the electrons v_z can assume only discrete values

$$|v_z| = v_n \equiv [2m^{-1}(\varepsilon_F - n\hbar\Omega)]^{1/2}, \quad n = 0, 1, 2, \ldots, \tag{2}$$

where ϵ_F is the Fermi energy. When a quantum is absorbed, the waves should satisfy the energy and momentum conservation laws

$$\varepsilon_{n'}(p_z + \hbar k_z) - \varepsilon_n(p_z) = \hbar\omega, \tag{3}$$

where ω and **k** are the frequency and wave vector of the electromagnetic wave. In the limiting case of long waves and low frequencies, when

$$kR \ll 1, \quad \omega \ll \Omega, \quad R = v_F/\Omega, \tag{4}$$

the conservation laws (3) are satisfied only for transitions without a change of the magnetic quantum number: $n' = n$. It follows from (3) that absorption takes place if the following condition is satisfied

$$\omega/k_z = v_n + \hbar k_z/2m. \tag{5}$$

The quantity ω/k_z will be denoted by u and will henceforth be called the phase velocity of the wave. At absolute zero temperature, the wave can be absorbed only by those electrons, whose longitudinal velocities lie in the interval from $v_n - \hbar k_z/2m$ to v_n. If the phase velocity falls in this interval, then strong (giant) absorption takes place. Outside this interval, the absorption is small. These are indeed the quantum oscillations of the Landau damping, first predicted for ultrasound by

Gurevich, Skobov, and Firsov.[8] The plot of the damping against the phase velocity u reveals an aggregate of narrow and high maxima, separated by intervals in which the absorption is small. If the inequalities (4) are satisfied, the distances between the maxima are much larger than their widths, and their height increases with decreasing temperature and with increasing mean free path. In the limiting case of absolute zero temperature and infinite mean free path, these maxima can be regarded as δ-like. From the Kramers-Kronig relations it follows that the nondissipative part of the corresponding dielectric constant has infinite discontinuities where the absorption has δ-like singularities. In the intervals between them, the nondissipative part of the conductivity (i.e., the dielectric constant) changes from $-\infty$ to $+\infty$. Such sharp changes of the dielectric constant lead to the existence of quantum electromagnetic waves.

The present paper is devoted to a theoretical investigation of coupled longitudinal and transverse quantum electromagnetic waves. We show that if the angle Φ between the wave vector and the direction of the magnetic field differs from zero, then there exists a connection between the longitudinal[2] and the transverse[3] quantum waves. It turns out that the quantum waves, whose velocities are close to the phase velocity of the helicon w, are transverse and their electric vector is polarized in a plane perpendicular to the magnetic field **H**. Those quantum waves whose velocities greatly differ from w are in the main longitudinal and their electric vector is polarized along the wave vector **k**.

2. FORMULATION OF PROBLEM

The spectrum and polarization of the electromagnetic waves are determined from Maxwell's equations. Eliminating the alternating magnetic field and neglecting the displacement current, we write these equations in the form

$$k^2E_\alpha - k_\alpha k_\beta E_\beta = 4\pi i\omega c^{-2}\sigma_{\alpha\beta}E_\beta, \quad \alpha, \beta = x, y, z, \tag{6}$$

where $\sigma_{\alpha\beta}(\mathbf{k}, \omega, \mathbf{H})$ are the Fourier components of the conductivity tensor with allowance for the quantization of the electron states in the magnetic field. We choose a coordinate system such that the z axis is parallel to the vector **H** and the x axis is perpendicular to the vectors **k** and **H**. Then $k_z = 0$, $k_y = k \sin \Phi$, and $k_z = k \cos \Phi$. In the absence of scattering, the Fourier components of the conductivity-tensor elements are determined by the expression

$$\sigma_{\alpha\beta}(\mathbf{k}, \omega, \mathbf{H}) = \frac{2}{V}\sum_{a'a}\frac{f_a - f_{a'}}{\hbar\omega_{a'a}}\,\frac{\langle a|j_\alpha(\mathbf{k})|a'\rangle\langle a'|j_\beta(-\mathbf{k})|a\rangle}{\delta + i(\omega_{a'a} - \omega)}. \tag{7}$$

Here a denotes the set of quantum numbers n, p_z, and X, characterizing the state of the electron in the magnetic field; $\hbar\omega_{a'a} = \epsilon_{a'} - \epsilon_a$ is the energy difference between these states. The eigenvalues of the energy $\epsilon_a = \epsilon_n(p_z)$ do not depend on the coordinate X of the center of rotation; f_a is the Fermi function of argument $(\epsilon_a - \epsilon_F)/kT$; in the case when $T \to 0$ the function $f_a = 1$ for $\epsilon_a < \epsilon_F$ and $f_a = 0$ for $\epsilon_a > \epsilon_F$; V is the volume of the crystal; the coefficient 2 takes into account the spin degeneracy; $\delta \to 0$. The quantity

$$\langle a|\mathbf{j}(\mathbf{k})|a'\rangle = \frac{e}{2m}\int d^3\mathbf{r}e^{-i\mathbf{kr}}\Big\{\Psi_a^*(\mathbf{r})\left(\frac{\hbar}{i}\nabla + \frac{e}{c}\mathbf{A}_0\right)\Psi_{a'}(\mathbf{r}) - \Psi_{a'}(\mathbf{r})\left(\frac{\hbar}{i}\nabla - \frac{e}{c}\mathbf{A}_0\right)\Psi_a^*(\mathbf{r})\Big\} \tag{8}$$

is the Fourier component of the matrix element of the current-density operator, and $\Psi_a(\mathbf{r})$ is the electron wave function in the state a:

$$\Psi_a(\mathbf{r}) = (L_yL_z)^{-1/2}\exp[i(zp_z/\hbar - \gamma Xy)]\Phi_{n,X}(x),$$
$$\Phi_{n,X}(x) = \gamma^{1/4}\exp[-{}^1/{}_2\gamma(x - X)^2]H_n[\gamma^{1/2}(x - X)],$$
$$\gamma = eH/\hbar c. \tag{9}$$

Here $H_n(x)$ is a Hermit polynomial normalized to unity; L_y and L_z are the dimensions of the crystal along the axes y and z. The gauge of the vector potential $\mathbf{A}_0$ is chosen in the form

$$A_{0x} = A_{0z} = 0, \quad A_{0y} = xH. \tag{10}$$

3. CALCULATION OF THE CONDUCTIVITY TENSOR

Let us calculate the elements of the conductivity tensor $\sigma_{\alpha\beta}$ in the limiting case of long waves and low frequencies (4). The effect of giant quantum oscillations is described by those elements of the conductivity tensor, in which there are non-vanishing matrix elements of the current-density operator which are diagonal in n. These terms do not contain the large cyclotron frequency Ω in the energy denominators. It is easy to verify that only the components j_x and j_z have non-zero matrix elements that are diagonal in n, and that the diagonal matrix element j_y vanishes identically. The non-zero matrix elements j_x and j_z are determined by the formulas

$$\langle n, p_z, X|j_x(\mathbf{k})|n, p_z + \hbar k_z, X - \gamma^{-1}k_y\rangle = \frac{e\hbar}{im}\int_{-\infty}^{\infty}dx\Phi_{n,X}(x)\frac{\partial}{\partial x}\Phi_{n,X-\gamma^{-1}k_y}(x), \tag{11}$$

$$\langle n, p_z, X|j_z(\mathbf{k})|n, p_z + \hbar k_z, X - \gamma^{-1}k_y\rangle = \frac{e}{m}\left(p_z + \frac{\hbar k_z}{2}\right)\int_{-\infty}^{\infty}dx\Phi_{n,X}(x)\Phi_{n,X-\gamma^{-1}k_y}(x). \tag{12}$$

We substitute here the expression for $\Phi_{n,X}(x)$ from (9) and use the formula

$$\int_{-\infty}^{\infty}dx\,H_n(x + y)H_m(x + z)\exp\left\{-\frac{1}{2}[(x + y)^2 + (x + z)^2]\right\}$$
$$= 2^{(m-n)/2}(y - z)^{n-m}L_m^{n-m}[(y - z)^2/2]\exp[-(y - z)^2/4], \quad n \geqslant m, \tag{13}$$

where $L_m^\alpha(x)$ is a generalized Laguerre polynomial, normalized to unity. We then obtain for the matrix element (12) the expression

$$\langle n, p_z, X|j_z(\mathbf{k})|n, p_z + \hbar k_z, X - \gamma^{-1}k_y\rangle = (e/m)(p_z + \hbar k_z/2)L_n^0(\gamma^{-1}k_y^2/2)\exp(-\gamma^{-1}k_y^2/4) \tag{14}$$

To find the matrix element (11), we express the derivative of $\Phi_{n,X}(x)$ in terms of other Hermit functions

$$\frac{\partial}{\partial x}\Phi_{n,X}(x) = \gamma^{1/2}\left[\sqrt{\frac{n}{2}}\Phi_{n-1,X}(x) - \sqrt{\frac{n+1}{2}}\Phi_{n+1,X}(x)\right]. \tag{15}$$

Substituting (15) in (11) and using (13), we obtain

$$\langle n, p_z, X|j_x(\mathbf{k})|n, p_z + \hbar k_z, X - \gamma^{-1}k_y\rangle$$
$$= i\frac{e\hbar}{2m}k_y[\sqrt{n}L_{n-1}^1(\gamma^{-1}k_y^2/2) + \sqrt{n+1}L_n^1(\gamma^{-1}k_y^2/2)]\exp(-\gamma^{-1}k_y^2/4). \tag{16}$$

In the quasiclassical approximation, the quantum numbers n are large and therefore the Laguerre polynomials can be replaced by their asymptotic expression

$$L_m^{n-m}(x) = x^{(m-n)/2} e^{-x/2} J_{n-m}[\sqrt{2(m+n+1)x}], \quad n \geqslant m, \quad (17)$$

where $J_\alpha(x)$ is the Bessel function. Then the matrix elements (14) and (16) can be represented in the form

$$\langle n, p_z, X | j_x(\mathbf{k}) | n, p_z + \hbar k_z, X - \gamma^{-1} k_y \rangle = i \frac{e\hbar}{m} (2n\gamma)^{1/2} J_1(\sqrt{2n}\gamma^{-1/2} k_y) \approx ienk_y/m, \quad (18)$$

$$\langle n, p_z, X | j_z(\mathbf{k}) | n, p_z + \hbar k_z, X - \gamma^{-1} k_y \rangle = \frac{e}{m}(p_z + \hbar k_z/2) J_0(\sqrt{2n}\gamma^{-1/2} k_y) \approx \frac{e}{m}(p_z + \hbar k_z/2). \quad (19)$$

The last terms in formulas (18) and (9) constitute the asymptotic expression for the quasiclassical matrix elements at small values of $k_y R$.

We neglect the elements σ_{yy} and $\sigma_{yz} = \sigma_{zy}$ of the conductivity tensor $\sigma_{\alpha\beta}$. Since they do not contain terms with current-density matrix elements diagonal in n, the quantum oscillations in these elements are not giant, and can therefore be disregarded in the limiting case (4). We retain, however, the Hall conductivity $\sigma_{xy} = -\sigma_{yx}$, which is relatively large even though it does not contain spatial-dispersion terms. In the region of low frequencies and large wavelengths considered by us, the spatial inhomogeneity of the field has practically no influence on the Hall conductivity, which is determined by the well known classical expression

$$\sigma_{yx} = N_0 ec / H, \quad (20)$$

where N_0 is the electron concentration. The quantum corrections to the Hall mobility (20) are in the quasiclassical case of the order of $(\hbar\Omega/\epsilon_F)^{3/2}$. In the remaining elements of the conductivity tensor (σ_{xx}, σ_{zz}, $\sigma_{xz} = -\sigma_{zx}$) we retain only the matrix elements (18) and (19) which are diagonal in n and which describe the effect of the giant quantum oscillations. As already indicated above, the dissipative parts of these elements of the conductivity tensor differ from zero only in narrow intervals of the magnetic field or of the phase velocities. Obviously, the presence of strong absorption makes it impossible for electromagnetic waves to exist inside these intervals. In this connection, we present expressions only for nondissipative parts of the conductivity-tensor elements σ_{xx}, σ_{xz}, and σ_{zz}:

$$\operatorname{Im}\sigma_{xx} = -\frac{e^2\gamma k_y^2}{2\pi^2 m\omega k_z} \sum_{n=0}^{N} n^2 \left[\ln\left| \frac{(v_n + \hbar k_z/2m)^2 - u^2}{(v_n - \hbar k_z/2m)^2 - u^2} \right| - 2\ln\left| \frac{v_n + \hbar k_z/2m}{v_n - \hbar k_z/2m} \right| \right], \quad (21)$$

$$\operatorname{Re}\sigma_{xz} = \frac{e^2\gamma k_y}{\pi^2 \hbar k_z^2} \sum_{n=0}^{N} n \ln\left| \frac{(v_n + \hbar k_z/2m)^2 - u^2}{(v_n - \hbar k_z/2m)^2 - u^2} \right|, \quad (22)$$

$$\operatorname{Im}\sigma_{zz} = -\frac{e^2\gamma m\omega}{2\pi^2\hbar^2 k_z^3} \sum_{n=0}^{N} \ln\left| \frac{(v_n + \hbar k_z/2m)^2 - u^2}{(v_n - \hbar k_z/2m)^2 - u^2} \right|. \quad (23)$$

In these formulas N is the maximum Landau number on the Fermi surface.

Strong absorption of the wave takes place inside the intervals of the velocity u in which

$$|u - v_n| < \hbar k_z / 2m.$$

Therefore the spectrum of the quantum waves can exist only outside these intervals. To simplify further calculations, we assume that the following inequality holds

$$\hbar k_z / 2m \ll |u - v_n|. \quad (24)$$

In this case, the logarithms in (21)—(23) can be expanded in powers of $\hbar k_z/2m$. As a result we obtain

$$\operatorname{Im}\sigma_{xx} = -\frac{e^2\Omega k_y^2 N^2}{\pi^2 m\omega u} \sum_{n=0}^{N} \left(\frac{n}{N}\right)^2 \frac{u}{v_n} \left[\left(1 - \frac{u^2}{v_n^2}\right)^{-1} - 1 \right], \quad (25)$$

$$\operatorname{Re}\sigma_{xz} = \frac{e^2\Omega k_y N}{\pi^2\hbar\omega} \sum_{n=0}^{N} \frac{n}{N} \frac{u}{v_n} \left(1 - \frac{u^2}{v_n^2}\right)^{-1}, \quad (26)$$

$$\operatorname{Im}\sigma_{zz} = -\frac{e^2\Omega mu}{\pi^2\hbar^2\omega} \sum_{n=0}^{N} \frac{u}{v_n} \left(1 - \frac{u^2}{v_n^2}\right)^{-1}. \quad (27)$$

4. INVESTIGATION OF THE DISPERSION EQUATION

Let us write down the dispersion equation from which we should determine the spectrum of the quantum electromagnetic waves. This equation is the condition for the vanishing of the determinant of the system of homogeneous Maxwell equations (6). If we neglect the elements σ_{yy} and σ_{yz}, then the dispersion equation assumes the form

$$(kc)^4 - (4\pi\omega\sigma_{xy}\sec\Phi)^2 - 4\pi i\omega k^2 c^2 [\sigma_{xx} + (\operatorname{tg}\Phi\sigma_{xy} + \sigma_{xz})^2/\sigma_{zz}] = 0. \quad (28)$$

This dispersion equation differs from the corresponding equation in [3] in the presence of a term with $(\tan\Phi\sigma_{xy} + \sigma_{xz})^2/\sigma_{zz}$, which describes the coupling of the longitudinal and transverse quantum waves. It is convenient to rewrite Eq. (28) in a different form, in terms of the phase velocity u. To this end, we introduce the "phase" velocity of the helicon

$$w = c(\omega / 4\pi |\sigma_{xy}\cos\Phi|)^{1/2}. \quad (29)$$

The quantity w differs from the true helicon velocity $\omega/k = w_0$ by the factor $|\sec\Phi|^{1/2}$. Substituting (25)—(27) in (28), we represent the dispersion equation in the following form:

$$1 - u^4/w^4 = \mu[S_{xx} - (S_{xz} - {}^4/_3\sqrt{N})^2 / S_{zz}]. \quad (30)$$

Here the parameter

$$\mu = 3\pi N_0 \varepsilon_F \sin^2\Phi / H^2\sqrt{N} \quad (31)$$

plays the role of the "coupling constant" of a classical helical wave having a velocity $u = w(\omega)$, with the quantum waves. The quantities S_{ik} are connected in simple fashion with (25)—(27) and are given by

$$S_{xx} = \sum_{n=0}^{N} \left(\frac{n}{N}\right)^2 (N + \Delta - n)^{-1/2} \left[\left(1 - \frac{u^2}{v_n^2}\right)^{-1} - 1 \right], \quad (32)$$

$$S_{xz} = \sum_{n=0}^{N} \frac{n}{N} (N + \Delta - n)^{-1/2} \left(1 - \frac{u^2}{v_n^2}\right)^{-1}, \quad (33)$$

$$S_{zz} = \sum_{n=0}^{N} (N + \Delta - n)^{-1/2} \left(1 - \frac{u^2}{v_n^2}\right)^{-1}, \quad (34)$$

where $\epsilon_F = (N + \Delta)\hbar\Omega$, N is the integer part of the ratio $\epsilon_F/\hbar\Omega$, and Δ is its fractional part. The roots $u = u_n(\omega)$ of Eq. (30) determine the spectrum of the quantum waves. At large values of N this equation is very complicated and cannot be solved in general form. It is possible, however, to determine the number of real roots and to analyze the character by qualitatively investigating equations (30). Let us examine the behavior of the right side of (30) as a function of u. Each of the terms of the sum S_{ik} has a singularity as $u \to v_n$. In the immediate vicinity of the given singular point $u \approx v_n$ it is possible to retain one singular term in each of the sums (32)—(34). The structure of the right side of Eq. (30) is such that the singularity at $u = v_n$ vanishes. Consequently, the velocities $u = v_n$ do not lead to a singularity in the right side of Eq. (30). The singularities in the right side of Eq. (30) will occur at those values of u, where

$$S_{zz}(u) = 0. \tag{35}$$

Equation (35) is the dispersion equation of the longitudinal quantum waves, which differs from the analogous equation in [2] only that it contains $k_z = k \cos \Phi$ instead of k. The roots of this equation. $u = s_n$ were investigated in detail in [2]. It was shown there that Eq. (35) has exactly N positive roots located between neighboring values of the quantized velocities v_n. For slow waves, in which the phase velocity $u \approx v_n \ll v_F$ (i.e., for magnetic quantum numbers n close to N), the distances between the roots of Eq. (35) and v_n are much shorter than the intervals between the neighboring velocities v_n. For velocities $u \lesssim v_F$, $(N-n \sim N)$, the roots of Eq. (35) lie approximately halfway between the neighboring velocities v_n.

The solid lines in the figure are a plot of the right side of (30). The singularity points where there are infinite discontinuities are the solutions $u = s_n$ of Eq. (35). The dash-dot line shows the parabola $1 - u^4/w^4$. The points of intersection of the parabola with the solid lines give the solutions of Eq. (30). Consequently, there are altogether N + 1 quantum waves. It is seen from the figure that the quantum-wave velocities that differ greatly from w are close to the solutions $u = s_n$ of Eq. (35). Near w, the velocities of the quantum waves differ significantly from s_n. As we shall show later, the results obtained in [3] for transverse quantum waves actually pertain to those solutions of Eq. (30), for which

$$|1 - u^4/w^4| \ll 1. \tag{36}$$

In a magnetic field $H \sim 10^5$ Oe at a frequency $\omega \sim 10^{10}$ sec^{-1} the velocity is $w \sim 10^6$ cm/sec. If the inclination angle Φ is small, then $\mu < 1$ and decreases with increasing magnetic field like $H^{-3/2}$. We present explicit expressions for the velocities of the slow quantum waves ($u \ll v_F$) at a small coupling constant $\mu \ll 1$. In this case we can retain one singular term in each of the formulas (32)—(34), and the sum of the remaining terms can be estimated by replacing the summation over n by integration in the sense of the principal value. As a result we obtain

$$\begin{aligned} S_{xx} &\approx (N+\Delta-n)^{-1/2}(1-u^2/v_n^2)^{-1}, \\ S_{xz} &\approx {}^4/_3 N^{1/2} + (N+\Delta-n)^{-1/2}(1-u^2/v_n^2)^{-1}, \\ S_{zz} &\approx 2N^{1/2} + (N+\Delta-n)^{-1/2}(1-u^2/v_n^2)^{-1}. \end{aligned} \tag{37}$$

Substituting (37) in (30) and gathering like terms, we obtain

$$1 - u^4/w^4 = \mu_n(1-u^2/s_n^2)^{-1}, \tag{38}$$

where

$$s_n = v_n[1 + {}^1/_4(N/(N+\Delta-n))^{1/2}], \tag{39}$$

$$\mu_n \approx \mu(N+\Delta-n)^{-1/2}. \tag{40}$$

If the relative difference between the velocities w and s_n exceeds $(\mu_n/2)^{1/2}$, then the phase velocity of the quantum wave is

$$u = s_n[1 - {}^1/_2\mu_n(1 - s_n^4/w^4)^{-1}]. \tag{41}$$

If the velocity w is close to one of the velocities s_n, the phase velocities of two neighboring quantum waves are described by the following approximate formula:

$$u = {}^1/_2(w+s_n) \pm {}^1/_2[(w-s_n)^2 + {}^1/_2\mu_n s_n w]^{1/2}. \tag{42}$$

We shall show below that waves with velocities (41) are polarized mainly along the wave vector **k**. The "strongly coupled" quantum waves with velocities (42) have the polarization of a helical wave

$$E_z \approx 0, \quad E_y = i \sec \Phi E_x. \tag{43}$$

5. POLARIZATION OF QUANTUM WAVES

In order to determine the polarization of the quantum waves, it is necessary to substitute the spectrum obtained above in Maxwell's equation (6). It is easy to obtain from these equations the following relations between the components of the electric field:

$$\frac{E_z}{E_y} = \operatorname{ctg}\Phi \frac{A}{1+A}, \quad \frac{E_x}{E_y} = -i\cos\Phi \frac{w^2/u^2}{1+A}, \tag{44}$$

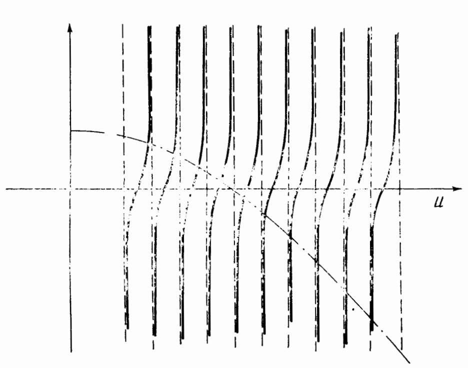
where

$$A = \frac{4}{3}\mu\sqrt{N}\frac{w^4}{u^4}\frac{S_{xz} - {}^4/_3\sqrt{N}}{S_{zz}}. \tag{45}$$

It is easy to verify that the spectrum (41) corresponds to large values of |A|, and the spectrum (42) corresponds to $|A| \ll 1$. Indeed, for quantum waves whose velocities differ significantly from w, condition (35) is satisfied with good accuracy, as a result of which we have $|A| \gg 1$. It then follows from (44) that

$$|E_x| \ll |E_y|, \quad E_z = \operatorname{ctg}\Phi E_y,$$

i.e., the vector **E** is parallel to **k**. For quantum waves whose velocities are close to w, the quantity A can be transformed with the aid of Eq. (38). Substituting in (45) the expressions for S_{xz} and S_{zz} from (37), we find after simple transformations that



Graphic investigation of the dispersion equation. Solid lines – right side of Eq. (30). Dash-dot line – left side of (30). Vertical dashed lines – positions of the roots $u = s_n$ of Eq. (35).

$$|A| = {}^2/_3 |w^4/u^4 - 1|.$$

For those waves whose velocities are determined by formula (42), we have $|A| \ll 1$ and consequently $|E_z| \ll |E_y|$, and the ratio between E_x and E_y is determined by (43).

Thus, quantum waves whose velocities are close to w have the same polarization as the helicon.[3] Quantum waves whose velocities differ significantly from the velocity of the classical wave, constitute longitudinal oscillations whose electric vector is directed parallel to the wave propagation direction.

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