INTERACTION BETWEEN OPTICAL PHONONS AND ELECTRONS IN A QUANTIZING MAGNETIC FIELD

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The dispersion law for longitudinal optical phonons interacting with electrons in a quantizing magnetic field is studied. An analysis of the conservation laws shows that in the presence of magnetic quantization there appear additional sections in the (ω, q) plane in which Landau attenuation is absent. Dispersion curves in the regions $q \ll \omega/v_0$ and $q \gg 2k_0$ are found. It is demonstrated that in the set under consideration there exist solutions of the acoustic type, analogous to acoustic plasmons.

It is well known that in a quantizing magnetic field the interaction of electrons with other elementary excitations has a specific character. This is connected with the fact that the magnetic field substantially changes the spectrum of the electrons and breaks them up into groups, corresponding to different Landau levels. The change in the classification of the electron states and in the form of the conservation laws leads to a rearrangement of the attenuation regions in the (ω, q) plane, to an important transformation of the dispersion laws at the boundaries of these regions, and also to the appearance of new types of elementary excitations.

The influence of a quantizing magnetic field on elementary excitations in solids has been studied in a number of papers. The influence of Landau quantization on the attenuation of longitudinal ultrasound was investigated for the first time by Gurevich, Skobov and Firsov[1] (see also[2]). The spectrum of plasma oscillations in a quantizing magnetic field has been studied by Zyryanov[3]. In a paper by Glick and Callen[4] the problem of the propagation of helicons in the presence of Landau quantization was treated. Ginzburg, Konstantinov and Perel'[5] (see also[6]) showed that in a degenerate electron gas magnetic quantization leads to the appearance of acoustic plasma oscillations.

The dispersion law for longitudinal optical phonons interacting with electrons in polar semiconductors in the absence of a magnetic field was studied in the papers of Gurevich, Larkin and Firsov[7] and Lang and Pashibekova[8].

The present paper is devoted to the study of the spectrum of longitudinal optical vibrations interacting with an electron gas placed in a quantizing magnetic field.

We first perform an analysis of the conservation laws. This enables us to find the damping regions and to draw a number of qualitative conclusions about the excitation spectrum in a quantizing magnetic field. Generally speaking, the presence of a magnetic field leads to anisotropy in the properties of the electron gas, with the result that elementary excitations cannot be divided into purely longitudinal and transverse ones. However, if the wave is propagated along the field, it is possible to treat the longitudinal and transverse solutions independently.

First of all we shall find the sections of the (ω, q) plane corresponding to excitation of an electron and hole with energy ω and total momentum q, in the presence of Landau quantization ($\hbar = 1$). The edges of these sections define the regions of attenuation of the quasiparticles interacting with electrons.

We shall assume that the longitudinal wave is propagated along the field ($\mathbf{q} \parallel \mathbf{H}$) and that, consequently, transitions without change of n (n labels the Landau level) are allowed.

In this case it follows from the energy-momentum conservation laws and the Pauli principle that the sections of the (ω, q) plane corresponding to attenuation can be found from the relations (see Figs. 1 and 2)

$$\omega \leqslant \frac{q^2}{2m} + \frac{k_n}{m} q, \qquad \omega \geqslant -\frac{q^2}{2m} + \frac{k_n}{m} q,$$
$$\omega \geqslant \frac{q^2}{2m} - \frac{k_n}{m} q, \tag{1}$$

where $k_n = \sqrt{2m[\epsilon_F - (n + {}^1/_2)\Omega]}$ is the height of the n-th tube, $\Omega = eH/mc$ is the cyclotron frequency, ϵ_F is the Fermi energy and m is the effective mass. The vector $\mathbf{q}$ has a single component in the direction of the field, $\mathbf{q} = (0, 0, q)$. Here and below we assume that the electron spectrum is isotropic and quadratic in the absence of a magnetic field.

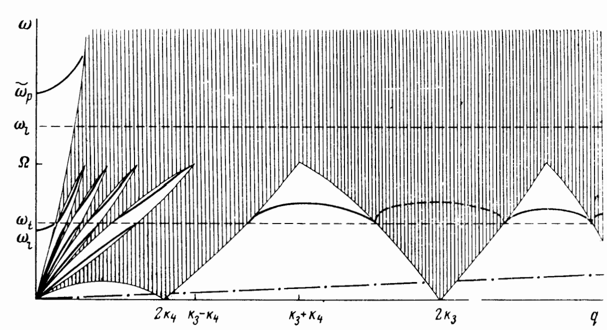

FIG. 1. Schematic form of the dispersion curves of longitudinal optical phonons and plasmons. Five magnetic levels are occupied. The Landau attenuation region is shaded. The dot-dash line indicates the occurrence of giant oscillations of the absorption coefficient of the acoustic phonons. The dashed line describes the dispersion law for slowly decaying optical phonons.

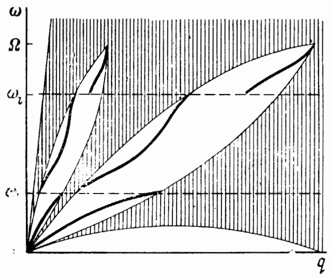

FIG. 2. Dispersion curves in a leaf-type "window."

At the intersection of attenuation regions corresponding to different Landau levels a number of "windows" appear in which there is no attenuation.

"Windows" of a first type have the form of leaves, spreading out from the coordinate origin. At small q they are bounded by the straight lines $\omega = k_n q/m$ and $\omega = k_{n+1} q/m$. Each "window" of this type is terminated on the line $\omega = \Omega$ at $q = k_n - k_{n+1}$. "Windows" of a second type border upon the axis $\omega = 0$. They begin at the points $q = 2k_n$ and end at $\omega = \Omega$ and $q = k_{n+1} + k_n$.

Attenuation of the quasi-particles is also absent at $\omega > q^2/2m + k_0 q/m$ and $\omega < q^2/2m - k_0 q/m$. "Windows" of the first and second type contract with decrease of the magnetic field and their height decreases. The total number of "windows" increases with increase in the number of occupied levels.

At the edges of the attenuation regions anomalies should arise in the spectra of the longitudinal excitations. For example, at the edges of "windows" of the second type, giant oscillations of the absorption coefficient and of the phase velocity of the longitudinal ultrasound arise[1,2]. Logarithmic singularities of the polarization operator at the points $q = 2k_n$ with $\omega = 0$ lead to oscillations of the static screening potential[9] with periods $\sim (2k_n)^{-1}$. The possibility of propagation of acoustic plasma oscillations is connected with the existence of the "leaf windows"[5].

Below it is shown that similar singularities arise also in the optical phonon spectrum. Furthermore, the spectrum of longitudinal plasmon oscillations is changed in a quantizing magnetic field.

We turn now to an analytic investigation of the problem. We shall assume the temperature to be zero and shall neglect collisions. We shall assume also that perturbation theory is valid, i.e., the electron gas satisfies the high density approximation and the coupling constant of longitudinal optical phonons with electrons is small.

We must, however, bear in mind that perturbation theory does not apply at the edges of the attenuation regions. The point is that the vertex part in the Dyson equation at these edges has a singularity corresponding to small momentum transfers. Moreover, we must note that, in the system under consideration, generally speaking, a superconducting transition is possible[10]. This corresponds to another singularity in the vertex part. We, however, shall assume that the Coulomb repulsion dominates the phonon attraction and that there is no transition to a superconducting state.

The spectrum of the elementary excitations can be found from the Dyson equation. In writing the Dyson equation we shall take into account both Coulomb interaction and the electron-electron interaction caused by exchange with the optical phonons[7]. Using the expression for the electron polarization operator in a quantizing field (see, for example,[2]) we obtain the dispersion equation in the form

$$\sum_{n=0}^{N} \ln\left|\frac{q^2(q-2k_n)^2-(2m\omega)^2}{q^2(q+2k_n)^2-(2m\omega)^2}\right| = \pi a_0 \alpha q^3 \frac{\omega^2-\omega_l^2}{\omega^2-\omega_t^2}, \qquad (2)$$

where $a_0 = \epsilon_\infty/me^2$ is the effective Bohr radius, $\omega_t = (\epsilon_\infty/\epsilon_0)^{1/2}\omega_l$ is the transverse optical phonon frequency, $\alpha = c/eH$ is the square of the magnetic length, N is the number of occupied Landau levels, ϵ_∞ is the dielectric constant due to the inner electrons, ϵ_c is the coupling constant for electrons and optical phonons, $1/\epsilon_0 = 1/\epsilon_\infty - 1/\epsilon_c$, ϵ_0 is the static dielectric constant and ω_l is the longitudinal optical phonon frequency.

We first consider the solutions for small q in the region $\omega > q^2/2m + k_0 q/m$, where there is no damping. After simple calculation, we obtain from Eq. (2)

$$\omega^2(q) = \bar{\omega}^2 + \frac{(\bar{\omega}^2 - \omega_l^2\varepsilon_\infty/\varepsilon_0)\beta\gamma_N}{\omega_l^2\bar{\omega}^4\varepsilon_\infty/\varepsilon_c + \frac{1}{2}\omega_p^2(\bar{\omega}^2-\omega_l^2\varepsilon_\infty/\varepsilon_0)^2} q^2, \qquad (3)$$

where

$$\bar{\omega}^2 = \frac{1}{2}\left\{\omega_l^2 + \omega_p^2 \pm \sqrt{(\omega_l^2+\omega_p^2)^2 - 4\frac{\varepsilon_\infty}{\varepsilon_0}\omega_l^2\omega_p^2}\right\},$$

$\omega_p^2 = 4\pi e^2 n_0/m\epsilon_\infty$ is the plasma frequency, n_0 is the electron density and $\gamma_N = \sum_{n=0}^{N} k_n^3$. For $(\omega_p/\omega_l)^2 \ll 1$, we obtain from (3)

$$\tilde{\omega}_l^2(q) = \omega_l^2 + \frac{\varepsilon_\infty}{\varepsilon_c}\omega_p^2 + \frac{\varepsilon_\infty\beta\gamma_N}{\varepsilon_c\omega_l^2} q^2, \qquad (4a)$$

$$\tilde{\omega}_p^2(q) = \frac{\varepsilon_\infty}{\varepsilon_0}\omega_p^2 + \frac{2\beta\gamma_N}{\omega_p^2} q^2, \qquad (4b)$$

where $\beta = \Omega e^2/\pi m^2\epsilon_\infty$. In another limiting case $(\omega_p/\omega_l)^2 \gg 1$ we obtain

$$\tilde{\omega}_l^2(q) = \frac{\varepsilon_\infty}{\varepsilon_0}\omega_l^2\left(1 - \frac{\omega_l^2}{\omega_p^2}\frac{\varepsilon_\infty}{\varepsilon_c}\right) + \frac{2\beta\gamma_N}{\omega_p^2} q^2, \qquad (5a)$$

$$\tilde{\omega}_p^2(q) = \omega_p^2 + \frac{\varepsilon_\infty}{\varepsilon_c}\omega_l^2 + \frac{\beta\gamma_N}{\omega_p^2} q^2. \qquad (5b)$$

In a strong magnetic field, when only one Landau level is occupied, formula (4a), for example, may be written in the form

$$\tilde{\omega}_l^2(q) = \omega_l^2 + \frac{\varepsilon_\infty}{\varepsilon_c}\omega_p^2 + \frac{\varepsilon_\infty\omega_p^2 v_0^2}{2\varepsilon_c\omega_l^2} q^2, \qquad (6)$$

where $v_0 = \sqrt{2(\epsilon_F - \Omega/2)/m}$. Consequently the coefficient of q^2, determining the spatial dispersion, decreases as H^{-2} with increase of the field. We must note that with increase of the field, the attenuation region in this case is displaced in the direction of greater q.

To calculate γ_N in (4a)–(5b) when a large number of levels are occupied, we use the Poisson summation formula. The result can be represented in the form

$$\gamma_N = \frac{(2m)^{1/2}}{\Omega}\left\{\frac{2}{5}\varepsilon_F^{5/2} - \frac{1}{16}\varepsilon_F^{1/2}\Omega^2 + \Omega^{5/2}\left[\sum_{l=1}^{\infty}(-1)^{l+1}l^{-3/2}\{\cos(2\pi l\varepsilon)S(\sqrt{2\pi l\varepsilon}) + \sin(2\pi l\varepsilon)C(\sqrt{2\pi l\varepsilon})\}\right]\right\}, \qquad (7)$$

where S(x) and C(x) are Fresnel integrals and $\epsilon = \epsilon_F/\Omega$.

The first term in the right-hand side of (7) defines the spatial dispersion in the absence of the field. The next two terms describe the monotonic field-dependence of the coefficient of q^2 and the oscillations of this coefficient of the de Haas—van Alphen type.

In another limiting case, when $q \gg k_0$, the dispersion law for the optical phonons has the form

$$\omega^2(q) = \omega_l^2\left\{1 - \frac{\varepsilon_\infty}{\varepsilon_c}\left(\frac{16\pi n_0}{a_0}q^{-4} + \frac{32\gamma_N}{3\pi a_0 \alpha}q^{-6}\right)\right\}, \quad (8)$$

where n_0 is the electron density. As in the case of small q, it is not difficult to find the form of formula (8) for $N = 0$ and $N \gg 1$.

It is of very great interest to examine the solutions of equation (2) inside the "windows" of the first and second type. The polarization operator at the edges of the "windows" has a logarithmic singularity.

We consider the solutions in the "windows" of the leaf type. The polarization operator goes to $-\infty$ at the left-hand edge of the "leaf window" and to $+\infty$ at the right-hand edge. Therefore, solutions of Eq. (2) exist in these regions irrespective of the sign of the $D_{\bar{0}}$ function. In the case where $\Omega < \omega_t$, the solutions in the "windows" of the leaf type are acoustic plasmons, attenuating at $\omega = \Omega$. For small q these solutions were investigated in[5].

If $\Omega > \omega_t$, then for $\omega < \omega_t$, i.e., in the region where Coulomb repulsion is dominant ($D_0 > 0$) there exist solutions of the acoustic plasmon type, terminating at $\omega = \omega_t$. In the frequency interval $\omega_t < \omega < \Omega$, i.e., in the region where phonon attraction is dominant ($D_0 < 0$), solutions also exist which begin at the point of intersection of the straight line $\omega = \omega_t$ with the left-hand edges of the "leaf windows," and terminate at $\omega = \Omega$, if $\Omega < \omega_1$ (see Fig. 1). For $\omega_1 < \Omega$, when the $D_{\bar{0}}$ function changes its sign twice in the interval $(0, \Omega)$, the solutions in the "leaf windows" are decomposed into three parts (see Fig. 2).

In the "windows" of the second type solutions exist only when $\Omega > \omega_t$, in the interval $\omega_t < \omega < \Omega$. In the vicinity of the points of intersection of the straight line $\omega = \omega_t$ with the edges of the "windows," the solutions of Eq. (2) can be written in the form

$$q(\omega) = k_n + \sqrt{k_n^2 \pm 2m\omega_t} + B_n \exp\left\{\frac{\omega^2 - \omega_l^2}{\omega^2 - \omega_t^2}\right\}. \quad (9)$$

An explicit expression for the constant B_n can be found without difficulty from Eq. (2). The plus sign in (9) refers to the solution at the left-hand edge of the "window," the minus sign to that at the right-hand edge. To obtain the dispersion curves in the right-hand end of the "window" it is necessary to replace n by n − 1. A schematic form of the dispersion curves inside the "windows" of the second type is depicted in Fig. 1. Exact solutions of Eq. (2) in these regions, as in the "windows" of the leaf type, can be found only by means of a computer calculation.

The attenuation of the excitation experiences a jump at the edges of the "windows" we have considered. The attenuation of optical phonons at the edges of "windows" of the second type is analogous to the attenuation of acoustic phonons, considered in[2]. This does not imply, however, that optical phonons are not well-defined everywhere outside the "windows." With increase of q and ω the attenuation decreases, on the average, as $1/\omega q^3$. Estimates we have made show that for the typical semiconductor parameters $n \sim 10^{17}$ cm^{-3}, $m = 0.1m_0$, $\omega_t \sim 10^{12}$ sec^{-1}, $\epsilon \sim 10$ and $H \sim 10^4$–10^5 Oe with $q \sim 2k_0$ the attenuation is small. For $\Omega \ll \omega_t$, the solutions in the region where there is attenuation go over to the solutions found in[7] in the absence of the field.

Up to now we have assumed that there is no temperature-diffusion of the distribution function and we have not taken electron diffusion into account. From analysis of the imaginary part of the polarization operator at finite temperature

$$\operatorname{Im}\Pi(q,\omega) = -\frac{m}{8\pi\alpha q}\operatorname{sh}\left(\frac{\omega}{2kT}\right)\operatorname{ch}^{-1}\left[\frac{(m\omega/q - q/2)^2 - k_n^2}{4mkT}\right] \times \operatorname{ch}^{-1}\left[\frac{(m\omega/q + q/2)^2 - k_n^2}{4mkT}\right] \quad (10)$$

it follows that temperature-diffusion of the edges of the attenuation regions in the high frequency case under consideration will be exponentially small if the condition $kT \ll \Omega$, $kT \ll \omega_t$ is fulfilled. Electron collisions can be neglected if ω is much greater than the collision frequency.

The anomalies in the optical phonon and plasmon spectra described above can be detected in experiments on the scattering of cold neutrons and X-rays. For this it is necessary that the experimental accuracy guarantee a momentum resolution of not less than $2(k_n - k_{n+1})$. Modern experimental technique[11] enables us to obtain such accuracy in fields $H \sim 10^5$ Oe.

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