INSTABILITY OF SPATIALLY SEPARATED PLASMA BEAMS

Yu. A. ROMANOV and V. F. DRYAKHLUSHIN

Physico-technical Institute of the Gor'kiĭ State University

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The interaction and instability of two spatially separated homogeneous semi-infinite plasma beams are investigated in the kinetic and quasihydrodynamic approximations. The interface between the beams is assumed to be sharp and their temperatures identical. The boundaries of beam instability and the wave increments and also the nature of behavior of the excited wave field are determined. The dependence of the minimal drift velocity, at which oscillations arise, on the collision frequency in the plasma is investigated. It is shown that the wave increments may be quite large and comparable with the corresponding quantities for mutually penetrating beams.

IT is known that two-stream instability can develop in a plasma containing streams of charged particles. In the case of mutually penetrating unbounded beams, this instability has been investigated in sufficient detail[1,2]. In particular, the necessary conditions for its existence have been clarified[3,4]. As to two-stream instabilities in a bounded plasma, say a layered plasma, they have been rather little investigated. In particular, there are practically no studies of the instability limit, of the influence of thermal effects (spatial dispersion), etc.

The features of a layered plasma are of interest because of the possibility of amplification of surface waves and of the development of different instabilities due to the interaction between the spatially-separated beams. This question is of particular importance for a plasma, in connection with the possible use of the plasma for the amplification and generation of electromagnetic waves. The instability of mutually-penetrating beams in a solid-state plasma was first considered in[5]. In order for it to be realized in a two-component single-temperature plasma, it is necessary to satisfy the condition

$$v > 0.926(v_{Te} + v_{Th}), \tag{1}$$

where v is the relative drift velocity of the electrons and holes, and $v_{Te,h}$ are their thermal velocities. For a solid-state plasma, the condition (1) is quite stringent. More realistic is two-stream instability in a two-temperature plasma. In this case the instability can arise when the relative velocity of the electrons and holes is much smaller than the thermal velocity. To this end, however, it is necessary that the temperature of the electrons exceed the temperature of the holes by not less than one order of magnitude. It is quite difficult to ensure this condition for spatially unseparated electrons and holes. In particular, when a strong electric field is superimposed, it is apparently not realized[6]. So far, two-stream instability has not been observed in solids, this being possibly due to the indicated difficulties.

Instabilities in spatially-separated beams offer great possibilities in this respect. First, both beams can consist of particles having high mobility (for example, two layers of a semiconductor of n-type, separated by a thin dielectric layer). Second, by placing the semiconductor structure in liquid helium or nitrogen and passing electric current through the individual layers, it is possible to obtain large values of the ratios of the temperatures of the individual beams.

An investigation of the instability of spatially separated beams has been initiated relatively recently. In the papers devoted to this question[6-11], the analysis is limited to the elementary theory, so that the question of the boundaries of the instability, of the influence of thermal effects, etc., remain open.

In the present paper we investigate the interaction and the instability of two spatially separated homogeneous semi-infinite beams in the kinetic and in the quasi-hydrodynamic approximations. The separation boundary of the beams is assumed to be sharp, and the temperatures are assumed to be identical. The limits of the two-stream instability and the increments of the growing waves are determined.

1. DISPERSION EQUATION OF WAVES IN SPATIALLY SEPARATED BEAMS

Let us consider waves propagating along two separate-infinite spatially separated homogeneous beams. The corresponding dispersion equations were obtained from equality of the surface impedances of the beams. Surface impedances of a semibounded immobile plasma for different boundary conditions for the carriers (specular and diffuse reflections) have been found in[12]. We shall use the Lorentz-transformation method to find the corresponding expressions for a moving plasma.

Let the surface impedance of a semi-bounded plasma for E-waves, in a reference frame in which the plasma is at rest, be

$$\xi'(\omega', k_1') = \frac{E_x'(\omega', k_1')}{H_y'(\omega', k_1')}, \tag{2}$$

where ω' and k_1' are the frequency of the wave and the wave vector parallel to the boundary of the plasma; the XY plane coincides with the plasma boundary. The surface impedance in the reference frame in which the plasma moves with velocity v is determined by using the Lorentz transformation for the fields,

$$\xi(\omega, \kappa_1) = \frac{E_x(\omega, k_1)}{H_y(\omega, k_1)} = \frac{E_x'(\omega', k_1')\sqrt{1 - v^2/c^2}}{H_y'(\omega', k_1') - (v/c)E_z'(\omega', k_1')}. \tag{3}$$

Using Maxwell's equation

$$\operatorname{rot} \mathbf{H} = \frac{4\pi}{c}\mathbf{j} - \frac{i\omega}{c}\mathbf{E}, \tag{4}$$

and the Lorentz transformation for ω and k, and assuming the normal component of the current density on the surface of the plasma to be equal to zero (which is true for reflections of the specular type and of the diffuse type of the first kind[12]), we obtain finally

$$\xi(\omega,k_1)=\frac{\omega-k_1v}{\omega\sqrt{1-v^2/c^2}}\xi'(\omega',k_1'), \tag{5}$$

where

$$\omega'=\frac{\omega-k_1v}{\sqrt{1-v^2/c^2}},\quad k_1'=\frac{k_1-v\omega/c^2}{\sqrt{1-v^2/c^2}}. \tag{6}$$

The expression for ξ' in the case of specular reflection is given in[12]. Equating the surface impedances determined by formula (5) for different beams, we obtain the sought dispersion equation. We shall henceforth confine ourselves to potential oscillations and assume that $v \ll c$ and $\mathbf{k}_1 \parallel \mathbf{v}$. In this case the dispersion equation for E-waves propagating along the beams takes the form

$$\int_{-\infty}^{+\infty}\frac{dk_3}{k^2\varepsilon_1(k,\omega)}=-\int_{-\infty}^{+\infty}\frac{dk_3}{k^2\varepsilon_2(k,\omega-k_1v)}, \tag{7}$$

where ϵ_1 and ϵ_2 are longitudinal electric constants of the first (immobile) and second (moving velocity v) beams, and $k^2 = k_1^2 + k_3^2$.

It is interesting to note that if the thermal effects are neglected Eq. (7) becomes $\epsilon_1(\omega) = -\epsilon_2(\omega - k_1v)$, and the investigation of the instability under consideration reduces completely to the well investigated problem of the two-stream instability of two mutually interpenetrating beams[6].

2. INSTABILITY LIMIT

Let us find the limit of instability of the oscillations determined by (7). We consider first the quasihydrodynamic approximation, in which the expression for the longitudinal dielectric constant is given by

$$\varepsilon_{1,2}(k,\omega)=1-\frac{\omega_{0\,1,2}^2}{\omega(\omega+i\nu_{1,2})-k^2v_{T1,2}^2/2}, \tag{8}$$

where $v_T = \sqrt{2\kappa T/m}$ is the thermal velocity of the plasma particles and ω_0 is the plasma frequency. Substituting (8) in (7), we obtain

$$\frac{1}{\varepsilon_1(0,\omega)}\left\{1+\frac{ik_1}{k_3'}[\varepsilon_1(0,\omega)-1]\right\}=-\frac{1}{\varepsilon_2(0,\omega-k_1v)}\left\{1+\frac{ik_1}{k_3''}[\varepsilon_2(0,\omega-k_1v)-1]\right\}, \tag{9}$$

where

$$k_3'=\frac{\omega}{v_{T1}}\left\{2\left(1-\frac{\omega_{01}^2}{\omega^2}\right)-k_1^2v_{T1}^2+2i\frac{\nu_1}{\omega}\right\}^{1/2},$$
$$k_3''=\frac{\omega-k_1v}{v_{T2}}\left\{2\left[1-\frac{\omega_{02}^2}{(\omega-k_1v)^2}\right]-k_1^2v_{T2}^2+2i\frac{\nu_2}{\omega-k_1v}\right\}^{1/2}. \tag{10}$$

In the general case the solution of (9) is rather complicated. For two identical beams it simplifies somewhat. We confine ourselves to this case.

To find the limiting curve we write Eq. (9) for the imaginary and real parts, assuming ω and k_1 to be real. It is easy to see that one of the solutions of the obtained equation for the imaginary parts will be

$$\omega = k_1v/2. \tag{11}$$

The equation for the real parts then takes the form

$$Cx^2(x^2y^2+\alpha^2-1)-\alpha A+\frac{x^2y^2-1}{xy}B=0, \tag{12}$$

where

$$x=\frac{\omega}{k_1v_T}=\frac{v}{2v_T},\quad y=\frac{k_1v_T}{\omega_0},\quad \alpha=\frac{\nu}{\omega_0},$$
$$C=\{[2(x^2y^2-1)-y^2]^2+4\alpha^2x^2y^2\}^{1/2}$$
$$A=\sqrt{1/2[C+2(x^2y^2-1)-y^2]},\quad B=\sqrt{1/2[C-2(x^2y^2-1)+y^2]}.$$

Equation (12) determines the "principal" boundary curve. Furthermore, in the case of a small number of collisions ($\nu/\omega_0 \lesssim 0.03$) Eq. (9) contains one more "additional" boundary curve. Both curves, calculated with a computer for several values of ν, are shown in Fig. 1.

The boundary curve corresponding to the elementary theory ($v_T \to 0$) constitutes two straight lines, $\omega = k_1v/2 = 0$ and $\omega = k_1v/2 = \sqrt{\omega_0^2 - \nu^2}$. The "additional" boundary curve is doubly degenerate and yields two curves in the (ωv) plane. In the c.m.s., one curve corresponds to a wave traveling in the direction of motion of one beam, and the other to a wave traveling in the direction of motion of the other beam. (In a system connected with one of the beams, one wave moves with velocity larger than v/2 and the other with velocity smaller than v/2). Figure 2 shows the dependence of the minimal drift velocity, at which instability sets in, on the collision frequency. The presented curves show that the two-stream instability can occur at relatively small relative velocities $v \approx 1.5\, v_T$.

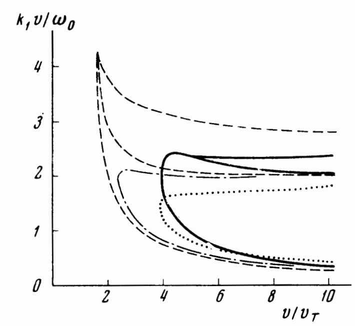

FIG. 1

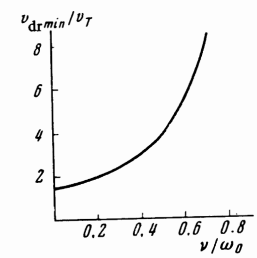

FIG. 2

FIG. 1. Limits of instability of oscillations: dashed curves – $\nu/\omega_0 = 0.01$; dash-dot – $\nu/\omega_0 = 0.3$, dotted – $\nu/\omega_0 = 0.5$, solid – kinetic theory.

FIG. 2. Dependence of minimal drift velocity, at which the instability sets in, on the collision frequency.

It is of interest to study the behavior of the fields of the excited wave at the boundary parameters. These fields in an immobile plasma, according to[12], are determined by the equations

$$E_x(z,k_1,\omega)=\text{const}\cdot\left[\omega(\omega+i\nu)e^{-k_1z}-\frac{ik_1}{k_3'}\omega_0^2e^{ik_3'z}\right],$$
$$E_z(z,k_1,\omega)=-\text{const}\cdot[\omega(\omega-i\nu)e^{-k_1z}-\omega_0^2e^{ik_3'z}]. \tag{12'}$$

The fields in a moving beam are derived from the obtained expressions by making the substitution $\omega \to \omega - k_1v$.

Let us examine, for example, the curve for $\nu/\omega_0 = 0.01$. An analysis shows that for the lower branch both terms in (12) attenuate exponentially; for the middle branch, the second term corresponds to a weakly-damped longitudinal plasma wave traveling from the boundary of the beams; for the upper, degenerate

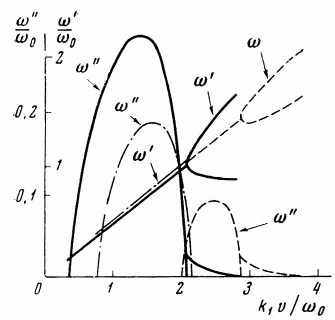

FIG. 3. Frequencies and increments of excited waves: dashed curves – $\nu/\omega_0 = 0.01$, $v/V_T = 2$; solid – $\nu/\omega_0 = 0.01$, $v/v_T = 8$; dash-dot – $\nu/\omega_0 = 0.1$, $v/v_T = 0.4$.

branch, in the case of the wave propagating with velocity larger than v/2, the second term in (12) describes a wave traveling from the boundary in the stationary beam and attenuating exponentially in the moving beam; for the wave propagating with velocity less than v/2, the picture is reversed.

As is well known, the quasihydrodynamic approximation takes into account the thermal effects rather crudely. We therefore consider the question of the instability limit in the kinetic approximation.

We shall assume that the electron beams are described by shifted Maxwellian distribution functions, and we shall neglect collisions. In this case the dielectric constants ϵ_1 and ϵ_2, which enter in (7), are determined by the expressions

$$\varepsilon(k,\omega) = 1 + 2\frac{\omega_0^2}{\omega^2}z^2\left(1 - 2ze^{-z^2}\int_0^z e^{\xi^2}d\xi\right) + 2i\frac{\omega_0^2}{\omega^2}z^3\sqrt{\pi}\,e^{-z^2}, \quad z = \omega/kv_T. \tag{13}$$

The solution of (7) with allowance for (13) is analogous to the solution of this equation in the quasihydrodynamic approximation, and leads to the boundary curve shown in Fig. 1 by the solid line. In this approximation, the normal drift velocity, at which the instability sets in, is $v_{dr} \approx 3.8 \times v_T$. This value is somewhat higher than that obtained in the quasihydrodynamic approximation, and exceeds by approximately a factor of 2 the corresponding quantity for mutually interpenetrating beams (see (1)).

3. INCREMENT OF EXCITED WAVES

We shall determine the increments of the excited waves in the quasihydrodynamic approximation. Substituting in (7) the expression (8) and putting $\omega = \omega' + i\omega''$, we obtain a system of two real equations determining ω' and ω''. The computer solution of these equations is shown in Fig. 3 for similar values of ν and v. The presented curves show that the increments of the investigated waves can be quite large and comparable with the corresponding values in the case of interpenetrating beams.

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