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MECHANISMS OF ABSOLUTE NEGATIVE CONDUCTIVITY OF THIN FILMS IN A TRANSVERSE QUANTIZED MAGNETIC FIELD

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The nonequilibrium electron electrical conductivity of thin semiconducting films in strong transverse magnetic fields is investigated. Both quantization in the magnetic field and size quantization are assumed. It is shown that absolute negative conductivity should be possible in electric fields $\hbar/e\tau L \ll E \ll \hbar\omega_c/eL$ (ω_c—cyclotron frequency, τ—relaxation time, L—magnetic radius) when optical phonon interaction is predominant (the electric current being directed opposite to the static electric field).

1. THE possibility of realizing absolute negative conductivity (ANC) in semiconductors has been recently discussed in a number of papers.[1–3] In a state with such conductivity, the electric current is directed against the static electric field. ANC was apparently first investigated in semiconductors by Kromer.[4] However, as shown by Zakharov[5] and by Elesin and Manykin,[6] in semiconductors with ANC, as a rule, the homogeneous distribution of the electric field is unstable and the inhomogeneous distribution is stable. Thus, a semiconductor with ANC can in principle serve as a current source. On the other hand, the ANC phenomenon can apparently be used to investigate the electron spectrum, relaxation mechanisms, etc.

In this paper we consider certain possible ANC mechanisms in a semiconducting film in a transverse magnetic field. They are due to scattering of nonequilibrium electrons by optical phonons and are connected with the structure of the energy spectrum under conditions when both quantization in the magnetic field and size quantization take place.

To observe the effects considered above, it is necessary to have

$$\omega_c\tau \gg 1, \qquad \Delta\tau \gg \hbar \tag{1}$$

(ω_c—cyclotron frequency, τ—relaxation time, Δ—characteristic distance between film levels), and also to have the electrons populate the lower film level and not go over to higher levels upon scattering. The most stringent of these conditions can be satisfied in sufficiently thin and perfect semiconducting films with low effective masses in realistic magnetic fields. For example, for an n-InSb film at a mobility $\mu = e\tau/m \sim 2 \times 10^4$ cm/V-sec, the magnetic field is $H \gtrsim 2 \times 10^4$ Oe, and the film thickness is $d \lesssim 5 \times 10^{-6}$ cm.

2. The energy spectrum and the wave functions of the stationary state of the electron in a film placed in crossed fields are given by

$$\varepsilon_{N,k} = \hbar\omega_c(N + 1/2) - eEX_k, \tag{2}$$

and

$$\Psi_{N,k}(\mathbf{r}) = \frac{1}{\sqrt{d}}\psi(z)\frac{1}{\sqrt{L_y}}e^{iky}\frac{1}{\sqrt{L}}\varphi_N\left(\frac{x - X_k}{L}\right), \tag{3}$$

where N is the magnetic quantum number, we have left out the index y of k_y, $X_k = -L^2k + eE/m\omega_c^2$ is the coordinate of the center of the electron orbit, $L = (c\hbar/eH)^{1/2}$ is the magnetic radius, the function $\psi(z)$ differs from 0 in the interval (0, d) and is normalized to unity, and $\varphi_N(x)$ is the N-th Hermite function. The z axis is directed along the magnetic field (perpendicular to the plane of the film), and the x axis is along the electric field (in the plane of the film). It is assumed that the linear dimensions L_x and L_y greatly exceed d. The energy is measured from the first film level in the absence of a magnetic field.

The transverse electric conductivity in quantizing magnetic fields is due, as is well known[8] to migrations of the centers of the electron orbits. We can therefore write in our case for the conduction current density

$$j_x = eL^2 \sum_{N,k;\,N',k'} f_N(k - k')W_{N,k;\,N',k'}, \tag{4}$$

where f_N is the number of electrons at the N-th Landau level per unit volume ($\sum_N f_N = n$, where n is the concentration); $W_{N,k;\,N',k'}$ is the probability that the scattered electron, will go from the state (N, k) to the state (N′, k′).

In the case of interaction with optical phonons, which we shall henceforth consider, the transition probability is

$$W_{N,k;\,N',k'} = \frac{2\pi}{\hbar}\sum_{q_x,q_y} A(\mathbf{q})\,|I_{N,N'}(\mathbf{q})|^2 \times\{(N_0 + 1)\delta[\varepsilon_{N',k'} - \varepsilon_{N,k} + \hbar\omega_0]\delta_{k,k'+q_y} + N_0\delta[\varepsilon_{N',k'} - \varepsilon_{N,k} - \hbar\omega_0]\delta_{k-q_y,k'}\}. \tag{5}$$

Here N_0 and ω_0 are the number and limiting frequency of the optical phonon;

$$|I_{N,N'}(\mathbf{q})|^2 \equiv |\langle N,0|e^{iq_xx}|N',\pm q_y\rangle|^2 = \frac{N!}{N'!}\left(\frac{1}{2}L^2q^2\right)^{(N'-N)}\exp\left(-\frac{1}{2}L^2q^2\right)\left|L_N^{(N'-N)}\left(\frac{1}{2}L^2q^2\right)\right|^2.$$

$L_N^{(M)}(x)$ is a generalized Laguerre polynomial; $\mathbf{q} = (q_x, q_y)$;

$$A(\mathbf{q}) = \sum_{q_z}|C_{\mathbf{q},q_z}|^2\chi(q_z),$$

where $C_{\mathbf{q},q_z}$ is the matrix element of the electron-phonon interaction. The factor

$$\chi(q_z) = \left| \int \psi^2(z) e^{iq_z z} dz \right|^2$$

expresses the possible nonconservation of the transverse phonon-momentum component during scattering (see, for example, [7]). The results do not depend qualitatively on the explicit form of the dependence, so that to simplify the final formulas we assume that $A(\mathbf{q}) \sim q^{-2}$. We have neglected in (5) the attenuation of the electron spectrum, something possible $E \gg \hbar/e\tau L$ (see [9]), and also the quantization (see, for example, [7] on this subject) and the dispersion of the optical phonon.

Let us substitute (5) in (4). Summing over k and k′, we obtain

$$\begin{aligned} j_x = \frac{2\pi e L^2}{\hbar} \sum_{N, N', q_x, q_y} f_N q_y A(\mathbf{q}) \, |I_{N,N'}(\mathbf{q})|^2 \\ \times \{(N_0+1)\delta[\hbar\omega_c(N'-N) + \hbar\omega_0 - eEL^2 q_y] \\ - N_0 \delta[\hbar\omega_c(N'-N) - \hbar\omega_0 + eEL^2 q_y]\}. \end{aligned} \quad (6)$$

We confine ourselves to the region of the electric fields $E \ll \hbar\omega_c/eL$. In such electric fields, as shown by Tavger and Erukhimov,[10] owing to the exponential smallness of the matrix elements $I_{N,N'}(\mathbf{q})$ with $q > L^{-1}$, the probability of a transition with a change of N and the conduction current, due to the elastic scattering mechanisms, are negligibly small. For the same reason, an insignificant contribution to the current is made by processes with emission of an optical phonon and a transition to a higher Landau level, and also by processes with absorption of a phonon and a transition to a lower level.[1)]

Accordingly, summing over q_i in (6) and replacing the summation with respect to q_x by integration, using the explicit expressions for $I_{N,N'}(\mathbf{q})$ and the interaction constant (see, for example, [11]), we arrive at the following expression for the current density

$$\begin{aligned} j_x = \frac{2\pi\omega_0 \hbar^2}{|eE| EL\tau_{op}} \sum_{N,\Lambda} \omega_\Lambda \exp\left[-\frac{1}{2}\left(\frac{\hbar\omega_\Lambda}{eEL}\right)^2\right] \\ \times \left[(N_0+1) f_{N+\Lambda} P_{N+\Lambda}^{(-\Lambda)}\left(\frac{\hbar\omega_\Lambda}{eEL}\right) - N_0 f_N P_N^{(\Lambda)}\left(\frac{\hbar\omega_\Lambda}{eEL}\right)\right]. \end{aligned} \quad (7)$$

Here

$$\tau_{op}^{-1} \sim 2\pi^2 Z^2 e^4 \gamma^2 m^{1/2} / M a_0^3 (\hbar\omega_0)^{1/2}$$

(Z—ion charge, a_0—distance between ions, γ—dimensionless polarizability coefficient, M—mass of unit cell) cell); $\omega_\Lambda = \omega_0 - \Lambda\omega_c$;

$$P_N^{(\Lambda)}(\xi) = \frac{N!}{2^\Lambda (N+\Lambda)!} \int_{-\infty}^{\infty} dx \, (x^2+\xi^2)^{(\Lambda-1)} e^{-x^2/2} \left| L_N^{(\Lambda)}\left(\frac{x^2+\xi^2}{2}\right)\right|^2.$$

3. Assume that electrons are produced at the lower Landau level under the influence of an external source, and that their concentration is much higher than the equilibrium value, i.e., $f_0 \gg f_0^{(\mathrm{therm})}$ (different mechanisms of such pumping are possible, for example optical pumping of the electrons from the valence band or from impurity levels, injection, etc.). Let us consider for concreteness the first resonance $\omega_c \sim \omega_0$. We assume that

$$\tau^{-1} \ll |\omega_0 - \omega_c| \ll \omega_c. \quad (8)$$

Then

$$f_1 \approx \alpha f_0, \quad f_2 \sim \alpha^2 f_0, \ldots, \quad \alpha = \frac{N_0 \tau_e}{\tau_{op} + (2N_0+1)\tau_e}, \quad (9)$$

where τ_e is a quantity of the order of the electron lifetime in the conduction band. When $\tau_e \ll \tau_{op}$, we have by virtue of (9)[2)] $f_0 N_0 \gg f_N(N_0+1)$, $(N > 0)$, and processes with phonon emission can be neglected (the first term in the square brackets of formula (7)). In this case we get from (7)

$$j_x = N_0 f_0 \frac{(\omega_c - \omega_0)}{\omega_0} I(E, \omega_c - \omega_0), \quad (10)$$

$$I(E, \omega_c - \omega_0) = \frac{\pi^{3/2}}{\sqrt{2}} \frac{\omega_0^2 \hbar^2}{|eE| E\tau_{op} L} \exp\left[-\frac{\hbar^2(\omega_c - \omega_0)^2}{2e^2E^2L^2}\right]. \quad (10')$$

From (10) and (10′) we see directly that when $\omega_0 > \omega_c$ the direction of the conduction current is opposite to that of the electric field, i.e., ANC takes place. As already noted, the usual conduction current due to scattering by impurities and acoustic phonons is small in the electric-field region under consideration.[7, 10]

In the general case, when $\omega_0 \sim M\omega_c$ (M—positive integer), the electrons will populate mainly the levels with N = 0, M, 2M, . . . ,

$$\begin{aligned} f_0, f_M, f_{2M}, \ldots \gg f_N \quad (N \neq 0, M, 2M, \ldots), \\ \text{но} \; f_0 N_0 \gg f_M(N_0+1), \quad f_{2M}(N_0+1), \ldots \end{aligned} \quad (11)$$

However, the results obtained in this case do not differ qualitatively from (10) or (10′). In this case the ANC will take place when $\omega_0 > M\omega_c$.

A similar ANC mechanism is possible when the electrons with $\tau_e \ll \tau_{op}$ are almost in equilibrium ($f_0 \approx f_0^{(\mathrm{therm})}$, $f_N \approx 0$, $N > 0$) and the number of phonons is $N_0 \gg N_0^{(\mathrm{therm})}$.

The foregoing mechanisms have a simple physical interpretation. The average electron energy is smaller in this case than the average phonon energy, energy is transferred from the phonons to the electrons, and in each interaction events a fraction of the energy $\hbar(\omega_0 - M\omega_c)$ is consumed in changing the potential energy of the electron. If $\omega_0 > M\omega_c$, then the potential energy of the electrons increases, i.e., they are displaced along the electric field when $e < 0$ (when $e < 0$ the displacement is in the opposite direction), and this leads to the ANC.

4. We consider now the case when an external source causes electrons to be produced at some higher level. Assume for concreteness that the pumping is to the level with N = 1. Then, at $T \ll \hbar\omega_0$ (T is the lattice temperature) two lower levels (N = 0, N = 1) become populated. The main contribution to the current at not too large lifetimes ($\tau_c \lesssim \tau_{op}/N_0$) will then be made by transitions from the first Landau level to the zeroth level, with emission of a phonon. In this case we obtain from (7)

[1)] Allowance for the attenuation of the spectrum adds a correction of the order of $(\omega_c\tau)^{-1}$, or even smaller, to the principal expression.

[2)] For equilibrium electrons and phonons $f_0 N_0 \sim f_1(N_0+1)$, and $\omega_c \sim \omega_0$.

$$j_x = -f_1 \frac{(\omega_c - \omega_0)}{\omega_0} I(E, \omega_c - \omega_0). \qquad (12)$$

We note that in this mechanism the ANC takes place when $\omega_c > \omega_0$, and the possibility of its realization is not limited to short electron lifetimes (unlike the mechanisms considered in Sec. 3 of the present paper and in [1]).

At high temperatures, $T \sim \hbar\omega_0$, the absorption of phonons and the repopulation of the electrons among the levels with $N > 1$ become significant. However, if $\tau_e \lesssim \tau_{op}$, then ANC is possible also in this case.

In this mechanism, unlike in the preceding ones, the energy is pumped over from the electrons to the lattice, for in this case the average electron energy exceeds the lattice temperature. In each energy-transfer act, however, part of the energy is used up to change the potential energy of the electron, which increases if $\omega < \omega_c$ ($\omega_0 < M\omega_c$).

5. Let us present estimates for the electric fields. When $m \sim 0.01 m_0$, $\mu \sim 3 \times 10^4$ cm²/V-sec, and $H \sim 3 \times 10^4$ Oe we have $E_{min} = \hbar/e\tau L \sim 2.1 \times 10^3$ V/cm and $E_{max} = \hbar\omega_c/eL \sim 21 \times 10^3$ V/cm.

We call attention to the fact that the aforementioned mechanisms may not predominate if the required conditions are satisfied with not too large a margin. Then no ANC will appear. Nonetheless, these mechanisms can greatly influence, for example, the current-voltage characteristics (the negative differential conductivity), the film photoconductivity spectra, etc. In particular, even weak pumping will lead to a change of the relative height of the current maxima in magnetophonon resonance in thin films,[9] something can be used to identify the maxima (for example, in pumping to the lower Landau level, each first maximum of the pair, corresponding to a given N, increases, and in pumping to a higher level every other maximum increases).

In conclusion, we note that since the possibility of the considered mechanisms is due to the limitation on the motion of the electrons in a direction perpendicular to the plane of the film (two-dimensional electron motion), similar effects can take place also in layered structures and in a thin surface layer of bulky samples in the case of strong bending of the bands.[12]

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