SCATTERING OF LIGHT BY CARRIERS IN A SEMICONDUCTOR WITHOUT AN INVERSION CENTER

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Scattering of light in semiconductors without inversion centers is considered. It is shown that the cross section for light scattering due to transitions between the spectrum branches may be much greater than the cross section for scattering of light by density fluctuations in degenerate semiconductors. Scattering of light in semiconductors of the p-Ge type is discussed.

It is well known that scattering of light by free carriers decreases strongly when screening effects become appreciable. Recent experiments[1] in the semiconductors GaAs, InP, and CdTe have shown that the cross section for the scattering of light by single-particle excitations greatly exceed the screen Thomson scattering cross section. Wolff[2] has shown that when account is taken of the non-parabolicity of the conduction band, a new mechanism of single-particle scattering of light becomes possible, which he called scattering by "energy fluctuations." Unlike the ordinary single-particle scattering by conduction electrons, scattering by "energy fluctuations" is not connected with charge fluctuations, and is therefore not screened by the free carriers. This mechanism, however, can likewise not explain the anomalously large value of the scattering cross section observed in the experiments[1], which exceeds the theoretical value by two orders of magnitude.

As is well known, in semiconductors without an inversion center, owing to the spin-orbit interaction, the spin degeneracy at $k \neq 0$ in the conduction band is lifted. This leads to a combined resonance[3], to the appearance of the Pockels effect[4,5], and to other effects. The splitting of the band in cubic crystals is proportional in this case to k^3. The possibility of optical transitions between these two branches of the conduction band, as will be shown in the present article, leads to a strong quasielastic scattering of light, which explains quantitatively the observed large cross section for the scattering of light in semiconductors without an inversion center. Unlike the case of light scattering as the result of non-parabolicity effects, in our case the initial and final states lie on different branches of the energy spectrum. It is clear that such screening is not influenced by the screening which cuts off the ordinary scattering.

The Hamiltonian of the electron in such a band is of the form[3,5]

$$\hat{H} = \frac{\hat{k}^2}{2m_c} - \frac{\delta}{ms}(\sigma\,[\hat{\mathbf{k}}\hat{\pi}]), \quad \hbar = 1, \tag{1}$$

where

$$\frac{2}{m} = \frac{1}{m_c} + \frac{1}{m_v}, \quad 2ms^2 = \varepsilon_g, \quad \hat{\pi}_i = \hat{k}_{i+1}\hat{k}_{i+2}.$$

The quantity δ determines the intensity of the spin-orbit interaction, m_c is the effective mass of the electron at the bottom of the conduction band. The spectrum then takes the form

$$\begin{aligned}\varepsilon_{1,2} &= \frac{k^2}{2m_c} \pm \frac{\delta}{ms}[k^2(k_x^2k_y^2 + k_x^2k_z^2 + k_y^2k_z^2) - 9k_x^2k\ ^2k_z^2]^{1/2} \\ &= \frac{k^2}{2m_c} \pm \frac{\delta}{ms}k^3 f^{1/2}(\theta, \varphi), \\ &\quad 0 \leqslant f(\theta, \varphi) \leqslant 1/4.\end{aligned} \tag{2}$$

let us assume that the cross section $d\sigma/d\Omega$ for the scattering of light by electrons. The matrix element of the energy of interaction between the carriers and the light is of the order of $r_0A^2\delta_0k/ms$, where $r_0 = e^2/m_cc^2$, $\delta_0 = \delta m_c$, and A is the vector potential of the light wave. At low temperatures and in the case of strong degeneracy, the electrons taking part in the transitions can have momenta that differ from the Fermi momentum k_F by an amount on the order of $\delta_0k_F^2/ms$, and therefore

$$\frac{d\sigma}{d\Omega} \sim r_0^2\left(\frac{\delta_0k_F}{ms}\right)^2\frac{4\pi}{(2\pi)^3}k_F^2\frac{\delta_0k_F^2}{ms},$$

or, since the electron concentration is $N = (8/3)\pi k_F^3/(2\pi)^3$, we get

$$\left(\frac{d\sigma}{d\Omega}\right)_c \sim Nr_0^2\delta_0^3\left(\frac{\varepsilon_F}{\varepsilon_g}\right)^{3/2}. \tag{3}$$

The cross section for the scattering of light by the "energy fluctuations," according to[2], is

$$\left(\frac{d\sigma}{d\Omega}\right)_w = r_0^2N\left(\frac{\varepsilon_F}{\varepsilon_g}\right)^2\frac{qv_F}{\varepsilon_F}, \tag{4}$$

where q is the wave vector of light and

$$\left(\frac{d\sigma}{d\Omega}\right)_c \Big/ \left(\frac{d\sigma}{d\Omega}\right)_w \sim \delta_0^3\frac{ms}{q}. \tag{4'}$$

According to estimates[5], $\delta_0 \approx 0.6$ for ZnTe. For GaAs this constant, obtained from Pockels-effect data, is of the same order. Therefore the ratio (4) can reach in GaAs two orders of magnitude, thus agreeing with the results of[1].

If account is taken of the contribution made to the scattering of the light by the non-parabolicity of the band due to terms proportional to k^3, unlike Wolff[2] who took into account only the k^4 terms, then we obtain

$$\left(\frac{d\sigma}{d\Omega}\right)_{cc} \approx r_0^2N\frac{qv_F}{\varepsilon_F}\frac{\varepsilon_F}{\varepsilon_g}\delta_0^2. \tag{5}$$

At not very small δ_0, the scattering cross section, given by (5), will also be larger than the cross section determined from (4) for the scattering of light. A rigorous analysis confirms these qualitative considerations.

The differential cross section for the scattering of the light is

$$\frac{d\sigma}{d\omega\, d\Omega} = \frac{3}{\pi\sqrt{2}} r_0^2 \delta_0^3 \left(\frac{\varepsilon_F}{\varepsilon_g}\right)^{3/2} \frac{N}{\omega_{max}} \int |D|^2 w \delta(w - f^{1/2}(\theta, \varphi))\, d\Omega, \quad (6)$$

where $\omega = \omega_1 - \omega_2$ is the difference between the frequencies of the incident and scattered light,

$$w = \frac{\omega}{\omega_{max}}, \quad \omega_{max} = \frac{\delta_0 k_F^3}{m_- m s}.$$

It should be noted that since $q \ll \delta_0 k_F^2/ms$ and q does not enter into the expression for the scattering cross section, the cross section for scattering of the light depends little on the scattering angle. The angular dependence is determined here by the anisotropy of the probabilities of transitions between the two branches of the spectrum, i.e., by the function $|D|^2$, which depends on the angles θ and φ and on the mutual orientation of the polarization vectors of the incident and scattered light $\mathbf{e}_1$ and $\mathbf{e}_2$. Owing to its complexity, we shall not present here the general expression, and confine ourselves to the case when the light propagates along one of the cubic axis of the crystal and is scattered through 90°, corresponding to the experimental setup in[1]. Under these conditions, if both photons are polarized perpendicular to the scattering plane (($\perp$, $\perp$) in Mooradian's notation[1]), then

$$|D|^2 = \frac{4}{w^2} \{w^2(1 - xy) - x^2[(1 - x)(2 - 3y) - xy(1 - y)]^2\}, \quad (7)$$

where $x = \sin^2\theta$ and $y = \cos^2\varphi$.

On the other hand, if the polarization of the scattered photon is perpendicular to the polarization of the incident photon (the case ($\parallel$, $\perp$) or ($\perp$, $\parallel$)), then

$$|D|^2 = \frac{4}{w^2} \{w^2[1 - x(1 - y)] - x(1 - x)y[1 - 3x(1 - y)]^2\}. \quad (8)$$

Expressions (7) and (8) shows that a polarization dependence of the scattering cross section should be observed in the case of scattering, as was indeed observed in[1]).

It follows from (6) that the spectrum of the scattered light has the form shown in the figure. At small ω, the cross section $d\sigma/d\omega d\Omega$ increases in proportion to ω^2. The limiting value is $\omega_{max} \ll \omega_1$.

A characteristic feature of the spectrum is the appearance of a logarithmic divergence in the spectrum of the scattered light and $\omega_0 = 4\sqrt{2}\,\omega_{max}/9$, which is connected with the singularity in the number of the electron states in the interval $d\epsilon$ and with the modulus of the momentum dk, which determines, as can be seen from the foregoing, the number of electrons that take part in the scattering of light with a given change of frequency. From (6) we have

$$\frac{d\sigma}{d\Omega} = \frac{3}{\pi\sqrt{2}} \delta_0^3 N r_0^2 \left(\frac{\varepsilon_F}{\varepsilon_g}\right)^{3/2} \times \int f^{1/2}(\theta', \varphi') |D|^2 d\Omega'. \quad (9)$$

Since the integral entering in (9) is a number of the order of unity, it follows that

$$\frac{d\sigma}{d\Omega} \approx N r_0^2 \delta_0^3 \left(\frac{\varepsilon_F}{\varepsilon_g}\right)^{3/2},$$

which coincides with (3).

It must be noted that the obtained expression for the spectral dependence of the scattered light is valid if $\omega\tau \gg 1$, where τ is the characteristic relaxation time. When account is taken of the scattering of the electrons in each sub-band, the logarithmic singularity becomes "washed out" in the same manner as the spectrum becomes sharply cut off. At the same time, the integral

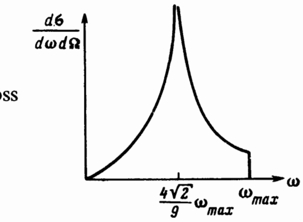

FIG. 1. Differential scattering cross section vs. light frequency.

light-scattering cross section does not depend on τ if $\hbar/\tau \ll \epsilon_F$, since it is determined by the correlation function at equal times, and therefore expression (9) is valid also under the inverse condition $\omega\tau \ll 1$[1)].

The considered light-scattering mechanism can always occur naturally if there are several branches of the energy spectrum that lie close to each other or are degenerate at some point of k-space. In this sense, it is of interest to consider the scattering of light in p-Ge by the light—heavy hole transitions. Since the divergence of the bands is proportional to k^2, the matrix element of the interaction between the holes and the two photons does not depend on k, and therefore we can expect the light-scattering coefficient to be a quantity of the same order as the unscreened Thomson cross section. Indeed, in the approximation of spherical energy bands

$$\frac{d\sigma}{d\omega\, d\Omega} = \frac{\omega^{1/2}}{1 - e^{-\omega/k_B T}} \left(\frac{3}{4\pi} r_-\right)^2 \frac{I}{\sqrt{2}} m_-^{3/2} \times [n_l(\sqrt{2\omega m_-}) - n_h(\sqrt{2\omega m_-})], \quad (10)$$

where $m_-^{-1} = m_h^{-1} - m_l^{-1}$, and m_l and m_h are the masses of the light and heavy holes; $r_- = e^2/m_- c^2$; $n_l(|k|)$ and $n_h(|k|)$ are respectively the distribution functions of the light and heavy holes, which we assume to be functions of only the modulus of the momentum. I is a quantity on the order of unity, which depends on the mutual orientation of the polarizations of the incident and scattered photons. It is seen from (10) that by measuring the frequency dependence of the spectrum of the scattered light it is possible, in principle, to determine the relative form of the distribution functions of the light and heavy holes in the region of energies much larger than $\hbar/\tau$.

For example, in the case of Fermi statistics, the cross section is

$$\frac{d\sigma}{d\Omega} = \frac{I}{64\sqrt{2}} r_-^2 N \left[\frac{(m_h - m_l) m_0}{(m_0 - m_l)(m_0 - m_h)}\right]^{3/2}$$

we see that it is of the same order as the unscreened Thomson light-scattering cross section.

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1) This was pointed out to us by V. L. Gurevich.