SOVIET PHYSICS JETP VOLUME 30, NUMBER 1 JANUARY, 1970

COULOMB INTERACTION IN THE FINAL STATE

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Submitted January 17, 1969

Zh. Eksp. Teor. Fiz. 57, 225–231 (July, 1969)

The matrix element for an arbitrary process with account of the Coulomb interaction between the charged particles with small relative velocities in the final (or initial) state is found by direct summation of the contributions from the main graphs of perturbation theory.

1. If in a reaction, slow charged particles (with small relative velocities) occur in the final (or initial) state, the Coulomb interaction between these is of great importance. The problem of the Coulomb interaction between the members of an electron-positron pair was first considered within the framework of non-covariant perturbation theory in the known work of Sakharov.[1] Here account was taken of the fact that the interaction leading to the production (annihilation) of the pair acts over distances[1)] $\lesssim 1/m$, while the Coulomb interaction acts over distances $\sim 1/m\alpha \gg 1/m$. This circumstance allows one to factorize the matrix element, and the problem of the final state interaction can be solved with the help of the nonrelativistic Schrödinger equation. In solving the problem of the Coulomb interaction with the help of the graph technique one must keep in mind that the characteristic expansion parameter in the perturbation series is α/v (v is the relative velocity of the particles), and for small v (where $\alpha/v \gtrsim 1$) it is impossible to carry out the summation over the main graphs (with respect to α/v) of perturbation theory. Solov'ev and Yushin[2] have attempted to carry out such a summation for the process of two-quantum annihilation; however, there are some inaccuracies in the corresponding part of their paper. Below we shall give the solution of the problem of the Coulomb interaction in the final state for any reaction involving slow charged particles.

2. Let us consider the general graph giving a contribution to the Coulomb interaction in the final state. We begin our discussion with the vertex graph (Fig. 1), since the result is trivially generalized to graphs with an arbitrary number of external photon lines ($q_1, \ldots, q_s$). The matrix element of n-th order is

$$M_n = \left[\frac{-ie^2}{(2\pi)^4}\right]^n (-ie) \int \frac{d^4k_1' \ldots d^4k_n'}{(k_1'^2 - \lambda^{02} + i\varepsilon)\ldots(k_n'^2 - \lambda^{02} + i\varepsilon)}$$
$$\times \frac{\bar{u}(p_1)\gamma^{\mu_1}(\hat{p}_1 + \hat{k}_1' + m)\gamma^{\mu_2}(\hat{p}_1 + \hat{k}_1' + \hat{k}_2' + m)\gamma^{\mu_3}\ldots}{(2(p_1k_1') + k_1'^2 + i\varepsilon)[2(p_1(k_1' + k_2')) + (k_1' + k_2')^2 + i\varepsilon]\ldots}$$
$$\times \frac{\ldots\gamma^{\mu_{m3}}(-\hat{p}_2 + \hat{k}_{m2}' + \hat{k}_{m1}' + m)\gamma^{\mu_{m2}}(-\hat{p}_2 + \hat{k}_{m1}' + m)\gamma^{\mu_{m1}}v(p_2)}{\ldots[-2(p_2(k_{m1}' + k_{m2}')) + (k_{m1}' + k_{m2}')^2 + i\varepsilon](-2(p_2k_{m1}') + k_{m1}'^2 + i\varepsilon)} \quad (1)$$

In our further considerations we shall use the system of the center of inertia of the final particles, $\mathbf{p}_1 + \mathbf{p}_2 = 0$,

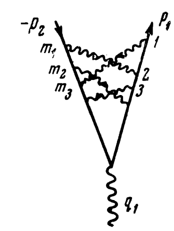

FIG. 1.

$|\mathbf{p}_1| = |\mathbf{p}_2| = p$, $E = \sqrt{p^2 + m^2}$. For $|\mathbf{k}_m'| \gtrsim m$ and the case of small p of interest to us, the terms $\mathbf{k}_m' \cdot \mathbf{p}$ in the denominator of the integral (1) can be omitted, so that the integral ceases to depend on p at all. Therefore, we consider below only the important region $|\mathbf{k}_m'| < m$. It follows from these considerations that we can restrict ourselves to the region $|k_m'^0| < p$.

We make a "scale" transformation of the integration variables, $\mathbf{k}_m' = p\mathbf{k}_m$, $k_m'^0 = p^2k_m^0/E$ and introduce the regularization mass of the photon $\lambda = \lambda_0/p$; we then obtain

$$M_n = \left(\frac{ie^2}{(2\pi)^4Ep}\right)^n (-ie) \int \left(\prod_{m=1}^{n} \frac{dk_m^0\, d\mathbf{k}_m}{\mathbf{k}_m^2 + \lambda^2}\right) \frac{\bar{u}(p_1)\gamma^{\mu_1}(\hat{p}_1 + m)\gamma^{\mu_2}}{(\mathbf{k}_1^2 - 2k_1^0 + 2\mathbf{kn} - i\varepsilon)}$$
$$\times \frac{(\hat{p}_1 + m)\gamma^{\mu_3}\ldots\gamma^{\mu_{m3}}(-\hat{p}_2 + m)\gamma^{\mu_{m2}}(-\hat{p}_2 + m)\gamma^{\mu_{m1}}v(p_2)}{[(\mathbf{k}_1 + \mathbf{k}_2)^2 - 2(k_1^0 + k_2^0) + 2\mathbf{n}(\mathbf{k}_1 + \mathbf{k}_2) - i\varepsilon]\ldots[(\mathbf{k}_{m1} + \mathbf{k}_{m2})^2 + 2(k_{m1}^0 + k_{m2}^0) + 2\mathbf{n}(\mathbf{k}_{m1} + \mathbf{k}_{m2}) - i\varepsilon]}$$
$$\times \frac{1}{(\mathbf{k}_{m1}^2 + 2k_{m1}^0 + 2\mathbf{n}\mathbf{k}_{m1} - i\varepsilon)}, \quad (2)$$

where $\mathbf{n} = \mathbf{p}_1/p$. In the denominators of (2) we neglect terms of the form $p^2(k_m^0)^2/E^2$, $p^2k_{m_i}^0k_{m_j}^0/E^2$ as compared to terms of the form k_m^0, $\mathbf{k}_m^2$ and terms containing $\mathbf{k}_m$ in the numerator. This approximation is justified, since the integral (2) converges for $p \to 0$. Thus the scale transformation allows us to separate the main terms with respect to α/v in the matrix element.

We begin the calculation of the integral (2) with the integration over the zero components k_m^0. Here it is convenient to make the following transformation of variables:

$$\begin{aligned} \tilde{k}_1^0 &= k_1^0, & k_1^0 &= \tilde{k}_1^0, \\ \tilde{k}_2^0 &= k_1^0 + k_2^0, & k_2^0 &= \tilde{k}_2^0 - \tilde{k}_1^0, \\ \tilde{k}_3^0 &= k_1^0 + k_2^0 + k_3^0, & k_3^0 &= \tilde{k}_3^0 - \tilde{k}_2^0, \\ &\cdots \\ \tilde{k}_n^0 &= k_1^0 + k_2^0 + k_3^0 + \ldots k_n^0, & k_n^0 &= \tilde{k}_n^0 - \tilde{k}_{n-1}^0. \end{aligned} \quad (3)$$

The integrals over the variables $\tilde{k}_m^0$ can be evaluated by

[1)]We use units such that $\hbar = c = 1$; $\alpha = e^2/4\pi = 1/137$; the metric is $(ab) = a_0b_0 - \mathbf{a}\cdot\mathbf{b}$.

closing the contour in the upper (or lower) half-plane and computing the residues in the poles. If m_1 does not coincide with 1, then all poles in the variable $k^0_{m_1-1}$ will lie in the lower half-plane [indeed, in the terms corresponding to the line p_1, this variable is encountered once in the combination $(\widetilde{k}^0_{m_1-1} + i\epsilon)$; in the terms corresponding to the line p_2, one has the combination $(\widetilde{k}^0_{m_1-1} + i\epsilon)$, or $\widetilde{k}^0_{m_1-1}$ does not enter at all]. But then the integral over the variable $\widetilde{k}^0_{m_1-1}$ vanishes everywhere except when $m_1 = 1$. Continuing this argument, one easily sees that only those graphs give a nonvanishing contribution in which $k^0_{m_j} = k^0_j$; these are the graphs which have the form of a straight ladder (Fig. 2).

We make a few remarks on the choice of graphs.

1) Only those graphs give contributions of interest to us (with the expansion parameter e^2/v_j) in which the virtual photon lines begin and end on external lines. Indeed, only in this case do terms with external momenta and masses occur in the propagators in the integrals (1) and (2), so that the factor $1/p$ separates out as a result of the scale transformation. Therefore the concrete form of the graph for which we consider the Coulomb interaction in the final (initial) state is not essential at all—it can be one electron line with an arbitrary number of external photon lines $(q_1, \ldots, q_s)$ [production (annihilation) of a pair through 1, 2, ..., s real (or virtual) photons] or several electron lines. The only important thing is that the momentum transferred in the graph of the basic process be much larger than the important region of momentum transfers along the photon lines of the Coulomb interaction. [In the integral (2), the main contribution comes from $|\mathbf{k}|, k^0 \sim 1$, which means that the important region for the integral (1) is

$$|\mathbf{k}'| \sim p, \quad k'^0 \sim p^2/E \ll |\mathbf{k}'|\text{.}^{[2]}]$$

2) The contributions from graphs where virtual photon lines begin and end on the same external line have poles which lie in the same half-plane, since the sign of the term k^0 in (2) is determined by the external momentum, and this is the same for all these lines. If there is more than one external photon line $(q_1, \ldots, q_s)$, there are virtual photon lines whose ends fall on an internal electron line between the external photon lines $(q_1, \ldots, q_s)$. Graphs with such lines do not give a contribution, as is seen, for example, from the fact that no factor $1/p$ can be pulled out from them after the scale transformation. Vacuum polarization graphs and graphs describing the scattering of light by light can be neglected in our approximation; their contributions are proportional to powers of p in the numerator for small momenta $\mathbf{k}'$, which are important in our problem. The inclusion of vacuum polarization graphs leads only to a renormalization of the charge.

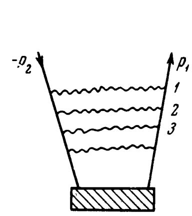

FIG. 2.

It follows from our consideration that the graph of the type of Fig. 2 is the most general form of graph which must be investigated in the problem of the Coulomb interaction in the final state.

3) Although it is the region of small momenta of the virtual photons which is important in our problem, the approximation of classical currents is nevertheless not applicable, since it is impossible to neglect terms of the form $k_i'^2$, $k_i'k_j$ compared to terms of the form (pk') in view of the smallness of the momentum of the external lines p. For this reason the procedure of[2] is incorrect, where the Coulomb interaction in the final state of the two-quantum annihilation process was calculated using the approximation of classical currents with a correction of the propagator of the internal electron line which is linear in $\mathbf{k}'$.

4) When we consider the "Coulomb interaction" of charged particles one of which is an initial, the other a final particle, then the poles in k^0_j in the integral (2) lie in one half-plane, and hence such graphs do not give contributions of order α/v.

3. Let us continue the calculation of the integral (2) for the ladder of Fig. 2. After commutation, we obtain for the numerator

$$[-4(p_1p_2)]^n \bar{u}(p_1)\gamma^\mu v(p_2).$$

We note that we consider the Coulomb interaction between an electron and a positron (attraction). Calculating the integral over $\widetilde{k}^0_j$ by the method of residues, we find

$$M_n = M_0 I_n; \quad I_0 = 1, \tag{4}$$

$$I_n = \left(\frac{\alpha}{\pi^2 v}\right)^n \int \left(\prod_{m=1}^{n} \frac{d^3k_m}{\mathbf{k}_m^2 - 1 - i\varepsilon}\right) \times \frac{1}{[(\mathbf{k}_1-\mathbf{n})^2+\lambda^2][(\mathbf{k}_2-\mathbf{k}_1)^2+\lambda^2]\ldots[(\mathbf{k}_n-\mathbf{k}_{n-1})^2+\lambda^2]}, \tag{5}$$

where $v = 2p/E$ is the relative velocity,[3] and M_0 is the matrix element for the basic process (without account of the final state interaction). In the integral (5) we have made a change of variables:

$$\mathbf{k}_1 + \mathbf{n} \to \mathbf{k}_1; \quad \mathbf{k}_1 + \mathbf{k}_2 + \mathbf{n} \to \mathbf{k}_2; \quad \ldots \sum_{i=1}^{n} \mathbf{k}_i + \mathbf{n} \to \mathbf{k}_n.$$

We note that (4) and (5) do not depend on the spin of the charged particles, since the same result will be obtained for particles with arbitrary spin (and minimal coupling) after $\mathbf{k}'$ is neglected in the numerator and the graph is reduced to the ladder graph.

As shown in Appendix A, the matrix element for the process can be written in the form

$$M = \sum_{n=0}^{\infty} M_n = M_0 \sum_{n=0}^{\infty} I_n = M_0 \psi_v^+(0) = M_0(\psi_v^-(0))^*, \tag{6}$$

where $\psi_v^+(\mathbf{r})$ is the solution of the Schrödinger equation for scattering in a Coulomb field (with regularization) in the form of a divergent wave. The distribution of the poles in the integrals (2) and (5) is determined by the

[2]It follows from this that one must neglect k′ in all internal lines of the basic process.

[3]In the case of particles with different masses, the factor $E_1E_2/p(E_1 + E_2) = 1/v$ is taken out.

"causal" term $i\epsilon$ in the denominators of the propagators; thus (5) and (6) show (cf. also Appendix A) that in solving scattering problems, causality arguments require that the initial states must be chosen in the form of divergent waves, and final states in the form of convergent waves.

4. It is desirable to calculate the integral (5) directly, taking into account that the solution (6) was obtained with a regularized Coulomb potential and that the solution in a Coulomb field has a number of features which are connected with the slow fall-off of the potential at infinity. This calculation is carried out in Appendix B. It yields for the attractive case

$$M = M_0 \exp\left\{-i\frac{\alpha}{v}\left(C + \ln\frac{\lambda_0}{2p}\right)\right\} e^{\alpha\pi/2v}\Gamma\left(1 - \frac{i\alpha}{v}\right) \qquad (7)$$

and for the repulsive case

$$M = M_0 \exp\left\{i\frac{\alpha}{v}\left(C + \ln\frac{\lambda_0}{2p}\right)\right\} e^{-\alpha\pi/2v}\Gamma\left(1 + \frac{i\alpha}{v}\right), \qquad (8)$$

where C is the Euler constant. These formulas contain the photon mass λ_0 only in the phase of the matrix element, so that no infrared regularization of the cross section is necessary; this is natural since the emission of real photons by slow particles is strongly suppressed.

The advantage of our approach to the Coulomb interaction problem is that it is universal, since, as shown above, the problem reduces to the summation of straight ladder graphs (of the type of Fig. 2) for an arbitrary process; the only requirement is that the characteristic momentum transfers in the basic process be much larger than the relative momenta of the final (or initial) particles, which corresponds to the requirement that the basic process occurs over distances which are small compared to atomic dimensions.

As a result the cross section is multiplied by the factor $|\psi_v^{\pm}(0)|^2$, which leads to an essential change in the cross section in a narrow region of small relative velocities (this corresponds to a narrow region of energies near the reaction threshold, or a narrow region of angles of emission). In special cases this result has been obtained earlier by Sommerfeld[3] (for bremsstrahlung of nonrelativistic particles), and Sakharov[1] (for the production of electron-positron pairs).

APPENDIX A

The Schrödinger equation in momentum space for scattering from a regularized Coulomb potential ($e^{-\lambda_0 r}/r$, $\lambda_0 \to 0$) has the form

$$\psi_v^{\pm}(\mathbf{k}) = \delta(\mathbf{p}-\mathbf{k}) - \frac{\alpha m}{\pi^2}\frac{1}{\mathbf{p}^2 - \mathbf{k}^2 \pm i\varepsilon}\int d^3k_1 \frac{1}{(\mathbf{k}-\mathbf{k}_1)^2 + \lambda_0^2}\psi_v^{\pm}(\mathbf{k}_1), \qquad (A.1)$$

where $v = p/E$. Using $\psi_v^{\pm}(\mathbf{r}=0) = \int d^3k\,\psi_v^{\pm}(\mathbf{k})$ and integrating (A.1) we obtain

$$\psi_v^{\pm}(\mathbf{r}=0) = 1 + \sum_{n=1}^{\infty}\left(-\frac{\alpha m}{\pi^2}\right)^n \int d^3k_1 \frac{1}{[(\mathbf{k}_1-\mathbf{p})^2 + \lambda_0^2](\mathbf{p}^2 - \mathbf{k}_1^2 \pm i\varepsilon)}$$

$$\times \int d^3k_2 \frac{1}{[(\mathbf{k}_2-\mathbf{k}_1)^2+\lambda_0^2]}\frac{1}{(\mathbf{p}^2-\mathbf{k}_2^2 \pm i\varepsilon)}$$

$$\ldots\int d^3k_n \frac{1}{[(\mathbf{k}_n - \mathbf{k}_{n-1})^2 + \lambda_0^2](\mathbf{p}^2 - \mathbf{k}_n^2 \pm i\varepsilon)}. \qquad (A.2)$$

Making the scale transformation $\mathbf{k}_i \to p\mathbf{k}_i$ in (A.2), we find that the n-th term of the series agrees with I_n in (5).

APPENDIX B

Calculating the integral (5) over the angles, we obtain

$$I_n = \left(\frac{\alpha}{\pi v}\right)^n \int_0^{\infty}\left(\prod_{m=1}^{n}\frac{dk_m}{k_m^2 - 1 - i\varepsilon}\right) k_n \ln\frac{(k_1+1)^2+\lambda^2}{(k_1-1)^2+\lambda^2}$$

$$\times \ln\frac{(k_2+k_1)^2+\lambda^2}{(k_2-k_1)^2+\lambda^2}\ldots\ln\frac{(k_n+k_{n-1})^2+\lambda^2}{(k_n-k_{n-1})^2+\lambda^2}. \qquad (B.1)$$

For the following it is convenient to parametrize the logarithms entering in this expression:

$$\ln\frac{(k_m+k_{m-1})^2+\lambda^2}{(k_m-k_{m-1})^2+\lambda^2} = \int_{\lambda^2}^{\infty} d\beta_m^2$$

$$\times\left[\frac{1}{(k_m-k_{m-1})^2+\beta_m^2} - \frac{1}{(k_m+k_{m-1})^2+\beta_m^2}\right]. \qquad (B.2)$$

Substituting this representation in (B.1) and taking account of the symmetry of the integrand, we have

$$I_n = \left(\frac{\alpha}{\pi v}\right)^n \int_{\lambda^2}^{\infty}\left(\prod_{j=1}^{n} d\beta_j^2\right)\int_{-\infty}^{+\infty}\left(\prod_{m=1}^{n}\frac{dk_m}{k_m^2-1-i\varepsilon}\right)k_n$$

$$\times\frac{1}{[(k_1-1)^2+\beta_1^2][(k_2-k_1)^2+\beta_2^2]\ldots[(k_n-k_{n-1})^2+\beta_n^2]}. \qquad (B.3)$$

It is convenient to begin the integration with the integral over k_n:

$$\int_{\lambda^2}^{\infty} d\beta_n^2 \int_{-\infty}^{+\infty}\frac{k_n dk_n}{[(k_n-k_{n-1})^2+\beta_n^2]}\frac{1}{(k_n^2-1-i\varepsilon)}$$

$$= \int_{\lambda}^{\infty} d\beta_n (2\pi i)\frac{ik_{n-1}}{(1+i\beta_n)^2 - k_{n-1}^2}. \qquad (B.4)$$

Calculating the next integral, we have

$$(-2\pi)\int_{\lambda}^{\infty} d\beta_n \int_{\lambda^2}^{\infty} d\beta_{n-1}^2 \int_{-\infty}^{+\infty}\frac{k_{n-1}dk_{n-1}}{[(k_{n-1}-k_{n-2})^2+\beta_{n-1}^2]}$$

$$\times\frac{1}{(k_{n-1}^2-1-i\varepsilon)[(1+i\beta_n)^2-k_{n-1}^2]} = (-2\pi)^2\int_{\lambda}^{\infty} d\beta_n \int_{\lambda}^{\infty} d\beta_{n-1}k_{n-2}$$

$$\times\left[\frac{1}{(1+i\beta_{n-1})^2-k_{n-2}^2} - \frac{1}{[1+i(\beta_n+\beta_{n-1})]^2-k_{n-2}^2}\right]\frac{1}{(1+i\beta_n)^2-1}$$

$$= (-2\pi)^2\int_{\lambda}^{\infty} d\beta_n \int_{\lambda}^{\lambda+\beta_n} d\beta_{n-1}\frac{1}{(1+i\beta_n)^2-1}\frac{k_{n-2}}{(1+i\beta_{n-1})^2-k_{n-2}^2}. \qquad (B.5)$$

The integral over k_{n-2} has the same form as the integral over k_{n-1}, so that the successive integrations can be carried out in elementary fashion. Making the replacement $\beta_m \to \lambda\beta_m$, we obtain

$$I_n = \left(\frac{i\alpha}{v}\right)^n \int_1^{\infty} d\beta_n \int_1^{1+\beta_n} d\beta_{n-1}\ldots\int_1^{1+\beta_2} d\beta_1 \prod_{m=1}^{n}\frac{1}{\beta_m(1+i\lambda\beta_m/2)}. \qquad (B.6)$$

This integral is equal to the following integral up to terms of order $O(\lambda)$:

$$I_n = \left(\frac{i\alpha}{v}\right)^n \int_1^{2/i\lambda} d\beta_n' \int_1^{1+\beta_n'} d\beta'_{n-1}\ldots\int_1^{1+\beta_2'} d\beta'_1\frac{1}{\beta_1'\beta_2'\beta_3'\ldots\beta_n'}, \qquad (B.7)$$

which can be seen by making the replacement

$$\beta_m \to \frac{\beta_m'}{1 - i\lambda\beta_m'/2}. \qquad (B.8)$$

The integral (B.7) can be calculated with the help of the exponential parametrization

$$\frac{1}{\beta_m} = \int_0^\infty e^{-t_m\beta_m} dt_m. \tag{B.9}$$

Let us consider

$$\int_0^\infty dt_1 \int_0^\infty dt_2 \int_1^{1+\beta_3} d\beta_2 \int_1^{1+\beta_2} d\beta_1 e^{-\beta_1 t_1} e^{-\beta_2 t_2} \tag{B.10}$$

$$= \int_0^\infty dt_1 \frac{e^{-t_1}}{t_1} \int_0^\infty dt_2 \int_1^{1+\beta_3} d\beta_2 (e^{-\beta_2 t_2} - e^{-\beta_2(t_1+t_2)}) = \int_0^\infty dt_1 \frac{e^{-t_1}}{t_1} \int_0^{t_1} dt_2 \int_1^{1+\beta_3} d\beta_2 e^{-\beta_2 t_2}.$$

It is seen that the integral over the next variables β_m can be evaluated in the same way. As a result we have

$$I_n = \left(\frac{i\alpha}{v}\right)^n \int_0^\infty dt_1 \frac{e^{-t_1}}{t_1} \int_0^{t_1} dt_2 \frac{e^{-t_2}}{t_2} \cdots \int_0^{t_{n-2}} dt_{n-1} \frac{e^{-t_{n-1}}}{t_{n-1}} \int_0^{t_{n-1}} dt_n \int_1^{2/i\lambda} d\beta_n e^{-t_n\beta_n}. \tag{B.11}$$

Changing the order of integration and taking account of the symmetry of the integrand, we find

$$I_n = \left(\frac{i\alpha}{v}\right)^n \int_0^\infty dt_n \int_1^{2/i\lambda} e^{-t_n\beta_n} d\beta_n \frac{1}{(n-1)!} \left[\int_{t_n}^\infty dt \frac{e^{-t}}{t}\right]^{n-1}$$

$$= \left(\frac{i\alpha}{v}\right)^n \int_0^\infty d\beta e^{-\beta} \frac{1}{n!} \left[\int_{i\lambda\beta/2}^\infty dt \frac{e^{-t}}{t}\right]^n. \tag{B.12}$$

Summing over n, we obtain [up to terms of order $0(\lambda)$]

$$M = M_0 \sum_{n=0}^\infty I_n = M_0 \int_0^\infty d\beta e^{-\beta} \exp\left[\frac{i\alpha}{v} \int_{i\lambda\beta/2}^\infty dt \frac{e^{-t}}{t}\right]$$

$$= M_0 \exp\left\{-\frac{i\alpha}{v}\left(C + \ln\frac{\lambda}{2} + \frac{i\pi}{2}\right)\right\} \Gamma\left(1 - \frac{i\alpha}{v}\right)$$

$$= M_0 \exp\left\{-\frac{i\alpha}{v}\left(C + \ln\frac{\lambda_0}{2p}\right)\right\} e^{\alpha\pi/2v} \Gamma\left(1 - \frac{i\alpha}{v}\right). \tag{B.13}$$

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Translated by R. Lipperheide
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