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THEORY OF LOW-FREQUENCY ELECTROMAGNETIC PROPERTIES OF A TURBULENT PLASMA

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A theory is developed for linear low-frequency electromagnetic waves and instabilities in a turbulent plasma and it is shown that the electromagnetic properties of such a plasma are modified substantially in the low-frequency region i.e., at frequencies much lower than the effective frequencies that characterize the turbulence collisions. An approach is developed that is based on the expansion of the particle collision integrals and the turbulence fluctuation collision integrals in terms of the turbulence energy. Within the framework of this analysis it is shown that in an isothermal plasma in which the turbulence energy W exceeds a critical value W_{cr}, ($W_{cr} \ll nT$), it is possible to have propagation of waves that are similar to acoustic waves, the velocity of these waves depending on the turbulence energy. A criterion for the applicability of this approach is developed. This criterion is based on the expansion of the turbulence collision integrals in terms of the turbulence energy. These criteria impose stringent conditions on possibility of appearance of electrostatic instabilities in the turbulent plasma. A new approach is also developed for computing the particle collision integrals and the turbulence fluctuation integrals, this approach being based on the weak correlation of the turbulent fluctuations between themselves and with perturbations of the turbulence fluctuations. An integral equation is derived and solved which makes it possible to sum the series in terms of turbulence energy in the collision integrals indicated above. It is shown that the result is the renormalization of the plasmon Green's function and the particle charge; this renormalization is related to the nonlinear modification of the dispersion properties of the plasma by the turbulence fluctuations. The dielectric constant obtained in this way is used to investigate new electrostatic instabilities in the turbulent plasma.

1. FORMULATION OF THE PROBLEM

1. The present work is devoted to the analysis of the properties of weak linear electromagnetic perturbations and waves in a turbulent plasma. In recent years many of the conclusions that follow from the theory of electromagnetic properties of a plasma in the absence of turbulence[1-3] have received experimental verification (cf. for example the review in[4]). The development of some of the general ideas as to the nature of plasma turbulence[5,6] makes it possible, at the present time, to formulate various linear electromagnetic properties of the turbulent plasma and to indicate how these properties can be distinguished from the properties of a non-turbulent plasma (attention has been directed to this problem in[7]). Physical arguments leading to this conclusion can be understood easily if one considers the propagation of a low-frequency linear perturbation in a plasma in which strong high-frequency fluctuations are excited.

Let us assume that the high-frequency turbulence is stationary. As in the case of fluids, the stationarity of the turbulence arises as a result of a balance between the generation of fluctuations in one region of wave numbers and the spectral transfer to another region, with subsequent dissipation of the fluctuations in this latter region.[8,9] The field associated with the linear low-frequency perturbation not only modifies (weakly) the particle distribution in the plasma (electrons and ions), but also modifies the flow of turbulence fluctuations. For example, if the frequency of the perturbation is much lower than a frequency given by the reciprocal time for the transfer of energy from the generation region to the absorption region, the perturbation can have a marked effect on such flows.

It is convenient to introduce the idea of an effective turbulence collision frequency ν_{turb} which is defined as the effective frequency of collisions of fluctuations between themselves and with plasma particles. These frequencies depend on the turbulence energy. In the case of weak turbulence, which is the only case that will be considered in the following analysis, the effective turbulence collision frequencies are proportional to higher powers of the turbulence energy and are characterized by low-frequencies.[1)] The highest frequency is characteristic of turbulence collisions that are proportional to the first power of the turbulence energy. The corresponding terms in the collision integral describe quasi-linear relaxation processess, the decay interaction, and induced scattering.[6] In the region of perturbation frequencies that are smaller than the effective turbulence frequencies the dielectric tensor cannot be expanded in the effective turbulence frequencies i.e., the turbulence energy. Perturbations of low frequency in the turbulent plasma cannot be regarded as nonlinear interactions between various modes and one must then speak of the actual modification of the mode, in particular, the dissappearance of higher modes and the appearance of new modes.

[1)] Specific expressions for the effective frequency of turbulence collisions (nonlinear growth rates) are given, for example, in [6].

We note that the modification of the low-frequency properties of a plasma in the presence of turbulence is of special interest in view of the fact that the low-frequency regions represent the greatest danger for the confinement of plasma (in view of the hydrodynamic instabilities and the drift instabilities). By changing the conditions for excitation and dissipation of high-frequency turbulence, for instance the intensity and class of turbulence fluctuations, it should be possible to regulate the low-frequency properties and instabilities of the plasma.

2. The problems developed in the theory given below are the determination of the analytic form of the low-frequency dielectric tensor for the plasma $\epsilon_{ij}(\omega, \mathbf{k}, W_{\mathbf{k}_1})$ and the functional dependence on the spectral energy density of the turbulence $W_{\mathbf{k}_1}$

$$W = \int W_{\mathbf{k}_1} d\mathbf{k}_1.$$

Here, W is the energy associated with the turbulence (per cm^3), ω is the frequency, and $\mathbf{k}$ is the wave vector of the perturbation. In formulating the theory we shall make use of ideas similar to those which appear in the method of collective perturbations that describe weak turbulence.[9] The basis of this method lies in the truncation of the nonlinear equations and the expansion in terms of the number of plasmons in the plasmon-particle collision integral. Thus, the expansion is carried out in terms of the turbulence energy in the kernels of the collision integrals, and account is taken of all processess indicated by the increasing number of external plasmon lines.[6]

The general scheme for the calculations of the theory is as follows: the particle distribution function f^α for particles of species α and the electric field $\mathbf{E}$ are written in a sum of turbulence $(\varphi^\alpha, \mathbf{e})$ and regular $(\Phi^\alpha, \vec{\mathscr{E}})$ components:

$$f^\alpha = \Phi^\alpha + \varphi^\alpha, \quad \mathbf{E} = \vec{\mathscr{E}} + \mathbf{e},$$

where $\langle \varphi^\alpha \rangle = 0$, $\langle \mathbf{e} \rangle = 0$; the averaging is over the statistical ensemble. By averaging the equations of motion and Maxwell's equations over the statistical ensemble and subtracting the averaged equations from the original equations we can obtain a system of equations for the regular components and the turbulence components. Further, we separate quantities that characterize the original turbulence state, which are designated below by the subscript (0): $\varphi^{(0)\varphi}$, $\Phi^{(0)\varphi}$, $\mathbf{e}^{(0)}$.[2)] We then consider the perturbation of the turbulence field and the field related to it $\vec{\mathscr{E}}^{(1)}$ (the superscript (1) indicates the perturbation). All quantities $(\mathbf{e}, \Phi, \varphi\vec{\mathscr{E}})$ are expanded in terms of $\vec{\mathscr{E}}^{(1)}$ and only the terms linear in $\vec{\mathscr{E}}^{(1)}$ are retained. In this way we obtain two systems of equations for the basic turbulent state and deviations from this state. These equations are general and, in principle, can be used for strong turbulence.

In obtaining the abbreviated equations that describe the weakly turbulent state it is possible, as is usually done, to carry out an expansion in the turbulence field $\mathbf{e}^{(0)}$ in the kernels of the integral equation for the initial turbulent state. One of the methods used below is the expansion in terms of the field $\mathbf{e}^{(0)}$ in the kernels for equations that describe the deviation from the turbulent state. Taking into consideration only a finite number of terms of the expansion, it is possible to take account of only the first turbulence collision frequencies. This procedure allows us to describe the electromagnetic properties of the plasma for frequencies that are lower than these turbulence frequencies but higher than the neglected turbulence frequencies (of higher order in the turbulence energy). The necessity for this approach to the problem is dictated by the fact that in many practical cases the effective turbulence frequencies depend not only on the turbulence energy, but also on ω and $\mathbf{k}$, the frequency and wave vector of the perturbation.

In Sec. 3 we analyze the limits of applicability of this approach, which is based on the expansion of the particle collision integral and the turbulence collision integral in terms of the turbulence energy, and the required criteria are derived.[10] It is shown that these criteria are closely related to the effects of nonlinear modifications of the dispersion properties for the high-frequency fluctuations. Although the changes in the frequencies of the high-frequency fluctuations under conditions of weak turbulence are always small, as we have indicated, the low-frequency perturbations can have an effect on them at frequencies which are, roughly speaking, lower than the nonlinear shifts. It is shown that the problem reduces to the renormalization of the propagators of the high-frequency plasmons and the particle charge.

In Sec. 4 we develop a new method that allows us to take account of this renormalization by formulation of an integral equation for the kernel of the integrals for the particle collisions and the collisions between turbulent fluctuations. These effects are found to be most important for the plasma oscillation turbulence. The instability of a turbulent plasma corresponds to the so-called instability of a gas of cold Langmuir plasmons, which was first investigated by Vedenov and Rudakov.[11] This feature arises in our analysis within the framework of the approach used for expansion of the collision integrals for particle collisions and turbulence fluctuations in terms of the turbulence energy.[3)] According to the criterion that is derived, this instability is found to be possible in a narrow range of plasma parameters and turbulence parameters, in particular, only for very low plasmon phase velocities. The use of a dielectric constant found by summing the series in the perturbation theory in turbulence energy in the kernels of the collision integrals for the particles and turbulence fluctuations (Sec. 4) indicates the existence of new instabilities in a turbulent plasma.

The methods that are developed also have application in problems of stabilization of drift instabilities by high-frequency turbulence and stochastic radio-frequency fields[13], spontaneous excitation of magnetic fields, and the skin effect and anti-skin effect in a turbulent plasma.[7] Solely for reasons of simplification, the presentation is restricted to the example of magnetized electrons and ions. We also limit the analysis of the radio-frequency turbulence to the case of rather high phase velocities so that the resonance

[2)] For simplicity, in what follows we take $\vec{\mathscr{E}}^{(0)} = 0$.

[3)] The instability criterion [11] has also been obtained by Galītis [12] by means of an energy principle.

particles in the plasma can be neglected as can binary collisions between particles.

2. EXPANSION OF THE COLLISION INTEGRALS FOR THE PARTICLES AND OF THE TURBULENCE FLUCTUATIONS IN TERMS OF THE TURBULENCE ENERGY

1. The motion of particles in a magnetized plasma can be described in terms of the motion of the guiding centers. The drift kinetic equation, which describes the distribution of guiding centers in the limit $H \to \infty$ that is to say, neglecting drift effects due to inhomogeneities in the plasma) is given by[14]

$$\frac{\partial f^\alpha}{\partial t}+v_z\frac{\partial f^\alpha}{\partial z}+\frac{e_\alpha}{m_\alpha}E_z\frac{\partial f^\alpha}{\partial v_z}=0, \tag{2.1}$$

where v_z is the particle velocity while E_z is the component of electric field along H.[4)] Using the method described above, from Eq. (2.1) we obtain a system of equations for the basic turbulent state [superscript (0), $\mathscr{E}^{(0)}=0$] and perturbations about this state [superscript (1)]

$$-i(\omega-k_zv_z)\varphi_k^{(0)\alpha}+\frac{e_\alpha}{m_\alpha}e_k^{(0)}\frac{\partial\Phi^{(0)\alpha}}{\partial v_z}+\frac{e_\alpha}{m_\alpha}\frac{\partial}{\partial v_z}\int dk_1\,dk_2\delta(k-k_1-k_2)\,(e_{k_1}^{(0)}\varphi_{k_2}^{(0)\alpha}-\langle e_{k_1}^{(0)}\varphi_{k_2}^{(0)\alpha}\rangle)=0, \tag{2.2}$$

$$\left(1+\frac{k_\perp^2}{k_z^2-\omega^2}\right)e_k^{(0)}=-\frac{4\pi i}{k_z}\sum_\alpha e_\alpha\int\varphi_k^{(0)\alpha}\,dv_z, \tag{2.3}$$

$$-i(\omega-k_zv_z)\Phi_k^{(1)\alpha}+\frac{e_\alpha}{m_\alpha}\mathscr{E}_k\frac{\partial\Phi^{(0)\alpha}}{\partial v_z}=-\frac{\partial}{\partial v_z}\int dk_1\,dk_2\times\frac{e_\alpha}{m_\alpha}\delta(k-k_1-k_2)\,[\langle e_{k_1}^{(1)}\varphi_{k_2}^{(0)\alpha}\rangle+\langle e_{k_1}^{(0)}\varphi_{k_2}^{(1)\alpha}\rangle], \tag{2.4}$$

$$-i(\omega-k_zv_z)\varphi_k^{(1)\alpha}+\frac{e_\alpha}{m_\alpha}e_k^{(1)}\frac{\partial\Phi^{(0)\alpha}}{\partial v_z}=-\frac{\partial}{\partial v_z}\int dk_1\,dk_2\times\delta(k-k_1-k_2)\frac{e_\alpha}{m_\alpha}\Big[e_{k_1}^{(0)}\Phi_{k_2}^{(1)\alpha}+\mathscr{E}_{k_1}\varphi_{k_2}^{(0)\alpha}+e_{k_1}^{(0)}\varphi_{k_2}^{(1)\alpha}+e_{k_1}^{(1)}\varphi_{k_2}^{(0)\alpha}-\langle e_{k_1}^{(1)}\varphi_{k_2}^{(0)\alpha}+e_{k_1}^{(0)}\varphi_{k_2}^{(1)\alpha}\rangle\Big]; \tag{2.5}$$

$$\left(1+\frac{k_\perp^2}{k_z^2-\omega^2}\right)\begin{Bmatrix}e_k^{(1)}\\ \mathscr{E}_k\end{Bmatrix}=-\frac{4\pi i}{k_z}\sum_\alpha e_\alpha\int dv_z\begin{Bmatrix}\varphi_k^{(1)\alpha}\\ \Phi_k^{(1)\alpha}\end{Bmatrix}.$$

Here, the velocity of light c is taken equal to unity, A_k is the four-dimensional Fourier component for the quantity A, $k=\{\mathbf{k},\omega\}$, $k_\perp^2=k^2-k_z^2$, $dk=d\mathbf{k}\,d\omega$ and the basic turbulent state is stationary, i.e., $\Phi_k^{(0)}=\Phi^{(0)}\delta(k)$ while the spectrum of stationary turbulence U_k is determined from the relation

$$\langle e_{k_1}^{(0)}e_{k_2}^{(0)}\rangle=U_{k_1}\delta(k_1+k_2). \tag{2.6}$$

The frequency of linear turbulence fluctuations is determined by the dispersion equation

$$\Pi(k)\equiv\Pi(\omega,\mathbf{k})=\varepsilon_0^{(e)}(\omega,\mathbf{k})+\varepsilon_0^{(i)}(\omega,\mathbf{k})-1+\frac{k_\perp^2}{k_z^2-\omega^2}=0, \tag{2.7}$$

$$\varepsilon_0^{(\alpha)}(\omega,\mathbf{k})=1+\frac{4\pi e^{\alpha2}}{m_\alpha k_z}\int dv_z\frac{\partial\Phi^{(0)\alpha}}{\partial v_z}\frac{1}{(\omega-k_zv_z+i\delta)},\quad\delta\to+0.$$

For the high-frequency fluctuations, whose phase velocities are much higher than the mean electron velocities, using Eq. (2.7) and neglecting spatial dispersion we have ($\omega_{pe}^2=4\pi n_0e^2/m_e$)

$$\omega^2=\omega^2_{\mathbf{k}_1,\pm1}=\frac{1}{2}[k_1^2+\omega_{pe}^2\pm\sqrt{(k_1^2+\omega_{pe}^2)^2-4\omega_{pe}^2k_{1z}^2}], \tag{2.8}$$

while the quantity U_k is related to the spectral density of the turbulence $W_\mathbf{k}$ by the relation

$$U_{k_1}=2\pi\sum_{s=\pm1}[W^s_{\mathbf{k}_1}\delta(\omega-\omega_{\mathbf{k}_1,s})+W^s_{-\mathbf{k}_1}\delta(\omega+\omega_{\mathbf{k}_1,s})]\times\left[1+\frac{k_{1\perp}^2k_{1z}^2}{(k_{1z}^2-\omega^2_{\mathbf{k}_1,s})^2}\right]^{-1}. \tag{2.9}$$

For the case of plasma fluctuations along H we have $\omega_{\mathbf{k}_1}=\omega_{pe}+3k_1^2v_{Te}^2/2\omega_{pe}$.

By limiting ourselves to high-frequency fluctuations we can treat the ions in linear fashion. We now consider the integral for collisions between electrons and turbulent fluctuations that appears on the right side of Eq. (2.4). We expand this integral in terms of the turbulence energy U_k or, what is the same thing, in terms of $e^{(1)}_{\mathbf{k}_1}$ taking $e^{(0)}_{\mathbf{k}_1}=0$.

We first limit ourselves to linear terms in U_{k_1}. In accordance with the remarks given above we then find

$$-i(\omega-k_zv_z)\Phi_k^{(1)e}+\frac{e}{m_e}\mathscr{E}_k\frac{\partial\Phi^{(0)e}}{\partial v_z}=\frac{\partial}{\partial v_z}D_0\frac{\partial\Phi_k^{(1)e}}{\partial v_z}+\frac{\partial}{\partial v_z}(\mathscr{E}_k\hat{D}+D_1+\mathscr{E}_kD_2)\frac{\partial\Phi^{(0)e}}{\partial v_z}, \tag{2.10}$$

$$D_0=i\frac{e^2}{m_e^2}\int\frac{U_{k_1}dk_1}{(\omega+\omega_1-(k_z+k_{1z})v_z+i\delta)},\quad\delta\to+0, \tag{2.11}$$

$$\hat{D}=\frac{e^2}{m_e^2}\int\frac{U_{k_1}dk_1}{(\omega+\omega_1-(k_z+k_{1z})v_z+i\delta)}\frac{\partial}{\partial v_z}\frac{1}{(\omega_1-k_{1z}v_z+i\delta)},$$

$$D_1=i\frac{e^2}{m_e^2}\frac{\omega_{pe}^2}{n_0}\int\frac{U_{k_1}dk_1(\omega-k_zv_z)}{\Pi(k+k_1)(k_z+k_{1z})(\omega+\omega_1-(k_z+k_{1z})v_z+i\delta)}\times\frac{1}{(\omega_1-k_{1z}v_z+i\delta)}\int\frac{dv_z'}{(\omega+\omega_1-(k_z+k_{1z})v_z'+i\delta)}\frac{\partial\Phi_k^{(1)e}}{\partial v_z'}, \tag{2.12, 2.13}$$

$$D_2=-\frac{e^2}{m_e^2}\frac{\omega_{pe}^2}{n_0}\int\frac{U_{k_1}dk_1(\omega-k_zv_z)}{\Pi(k+k_1)(k_z+k_{1z})(\omega+\omega_1-(k_z+k_{1z})v_z+i\delta)}\times\int\frac{dv_z'}{(\omega+\omega_1-(k_z+k_{1z})v_z'+i\delta)}\frac{\partial}{\partial v_z'}\frac{1}{(\omega_1-k_{1z}v_z-i\delta)}\frac{\partial}{\partial v_z'}\Phi^{(0)e}(v_z'). \tag{2.14}$$

The various terms in the collision integral (2.10) have a simple physical significance. The diffusion coefficient D_0 describes the change in the quasilinear effects of the relaxation of resonance particles associated with the deviation of the distribution of such particles from the equilibrium distribution $\Phi^{(0)}$. This statement holds for $\omega\ll\omega_1$ and $k\ll k_1$ in which case the denominator in (2.11) can be replaced by a δ-function. In the absence of resonance particles, which is the case when $\omega<\omega_1$ and $k\ll k_1$, the two terms in (2.9) balance each other and the quantity D_0 is small ($W/nT\ll1$) with respect to the first term on the left side of Eq. (2.10). The term with $\hat{D}$ in (2.10), which describes the change in the effects of the induced Compton scattering[6] is of order W/nT and is small with respect to the second term on the left side of Eq. (2.10). The diffusion coefficient D_2 describes the nonlinear induced scattering while D_1 describes the decay interaction.[5)] If the conditions $\omega\ll\omega_1$, $k\ll k_1$ and $\omega_1/k_1\gg v_{Te}$ are satisfied and, if in addition, ω/k

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4)The vector **E** can have an arbitrary orientation with respect to **H**; in particular, the turbulence in the initial state can be regarded as being isotropic.

5)More precisely those turbulence collisions which are decay collisions when $\omega\gg\nu_{turb}$.

$\ll \omega_1/k_1$, the diffusion coefficient D_2 is small compared with D_1, which is given approximately by

$$D_1 = -i(\omega - k_z v_z) n_k^{(1)e} d_1, \quad n_k^{(1)e} = \int \Phi_k^{(1)e} dv_z, \tag{2.15}$$

$$d_1 = -\frac{\omega_{pe}^2}{n_0} \sum_{s=\pm 1} \frac{4\pi e^2}{m_e^2} \int \frac{d\mathbf{k}_1}{(\omega - \mathbf{k}\mathbf{v}^s_{g,\mathbf{k}_1} + i\delta)} \times \left(\frac{\partial}{\partial \omega_1} \omega_1^2 \Pi(k_1) \right)^{-2}_{\omega = \omega_{\mathbf{k}_1,s}} \left(\mathbf{k} \frac{\partial}{\partial \mathbf{k}_1} \right) \frac{W_{\mathbf{k}_1}{}^s}{\omega_{\mathbf{k}_1,s}}; \tag{2.16}$$

$s = \pm 1$ corresponds to the two sides in (2.8) while $v^s_{g,k_1} = \partial\omega_{k,s}/\partial k_1$ is the group velocity for the linear spectra (2.8).

Attention is directed to the presence of the small factor $\Pi(k_1 + k)$ in the denominator of (2.13). In deriving (2.16) we have made use of the fact that in the linear spectra (2.8) the quantity $\Pi(k_1) = 0$ so that

$$\Pi(k_1 + k) \approx (\omega - \mathbf{k}\mathbf{v}_{g,\mathbf{k}_1}) \frac{\partial \Pi(k_1)}{\partial \omega_1} \Big|_{\omega_1 = \omega_{\mathbf{k}_1}}$$

We first show how the decay instabilities are obtained. Roughly speaking these instabilities can arise when $\omega \gg \nu_{turb}$, in which case the turbulence collisions can be treated by perturbation theory. In the equations

$$-i(\omega - k_z v_z)\Phi_k^{(1)e} = \frac{e}{m_e} \mathscr{E}_k \frac{\partial \Phi^{(0)e}}{\partial v_z} + i n_k^{(1)e} d_1 \frac{\partial}{\partial v_z} (\omega - k_z v_z) \frac{\partial \Phi^{(0)e}}{\partial v_z} \tag{2.17}$$

to a first approximation we can neglect the term with d_1 and find $\Phi_k^{(1)e}$ which is then substituted in $n_k^{(1)}$. Then, when $k_z v_{Ti} \ll \omega \ll k_z v_{Te}$ we can show quite easily that

$$\Pi(k) = \varepsilon(k) + \frac{k_\perp^2}{k_z^2 - \omega^2} = 0,$$

$$\varepsilon(k) = 1 + \frac{\omega_{pe}^2}{k_z^2 v_{Te}^2} - \frac{\omega_{pi}^2}{\omega^2} - \frac{\omega_{pe}^2}{k_z^2 v_{Te}^4} n_0 d_1,$$

which coincides with the nonlinear dielectric constant that describes the decay instabilities.[6] Equation (2.17) allows an exact solution[6)]

$$\varepsilon(k) = \varepsilon_0^{(i)}(k) + \frac{\varepsilon_0^{(e)}(k) - 1}{1 + \omega_{pe}^{-2} k_z^2 n_0 d_1 (\varepsilon_0^{(e)}(k) - 1)}.$$

2. As an example to illustrate the basic modification of the dispersion properties of the plasma at low frequencies we consider uniform turbulence due to plasma oscillations $\omega_{k_1} = \omega_{pe} + 3k_1^2 v_{Te}^2 / 2\omega_{pe}$ in the case in which all fluctuations are directed along H. We have

$$d_1 = -\frac{\omega_{pe}}{4n_0^2 m_e} \int_0^\infty dk_{1z} \left(\omega - \frac{3v_{Te}^2 k_z k_{1z}}{\omega_{pe}} + i\delta \right)^{-1} \left(k_z \frac{\partial}{\partial k_{1z}} W_{k_{1z}} \right).$$

Here, $W_{k_{1z}}$ vanishes when $k_{1z} = 0$ and $k_{1z} > k_{1z}^{max}$; $k_{1z}^{max} \ll \omega_{pe}/v_{Te}$. If $\omega \gg k_z v_g$, then

$$d_1 = \frac{3k_z^2 v_{Te}^2}{4n_0^2 m_e \omega^2} W$$

and when $k_z v_{Ti} \ll \omega \ll k_z v_{Te} j \epsilon_0^{(i)}, \epsilon_0^{(i)}, \epsilon_0^{(e)} \gg 1$ we have

$$\varepsilon(k) \approx -\frac{\omega_{pi}^2}{\omega^2} + \frac{\omega_{pe}^2}{k_z^2 v_{Te}^2} \left(1 + \frac{3Wk_z^2}{n_0 m_e \omega^2} \right)^{-1}; \tag{2.18}$$

$$\omega^2 = k_z^2 v_{\pm 1}^2, \quad v_s^2 = v_{Te}^2 m_e / m_i,$$
$$v_{\pm 1}^2 = \frac{1}{2} v_s^2 \pm \sqrt{\tfrac{1}{4} v_s^4 + 3v_s^2 W / 4n_0 m_e}. \tag{2.19}$$

The solution with the minus sign is aperiodically unstable. This instability is similar to that which has been found in[11] for cold isotropic plasmons (in the present case the plasmons are not cold, that is to say, the plasmon spectrum is neither narrow nor one-dimensional).

If $\omega \ll k_z v_g$ for the entire turbulent spectrum (i.e., eliminating the smallest value of v_g in the spectrum) we have

$$d_1 = \frac{\omega_{pe}^2}{12 n_0^2 m_e v_{Te}^2} \int \frac{W_{k_{1z}} dk_{1z}}{k_{1z}^2}, \tag{2.20}$$

and for the same condition as in (2.18) we have

$$\omega^2 = k_z^2 v_s^2 \left(1 + \frac{\omega_{pe}^2}{12 n_0 T_e} \int \frac{W_{k_{1z}} dk_{1z}}{k_{1z}^2 v_{Te}^2} \right). \tag{2.21}$$

The solution in (2.21) indicates the possibility of acoustic waves in an isothermal plasma; in the absence of turbulence these waves would be highly damped by ion-Landau damping. If $W/nT \gg 12 v_{Te}^2/v_p^2$ where $v_p = \omega_{pe}/k_{1z}$ is the phase velocity of the fluctuations, then the acoustic velocity increases $\omega^2 = k_z^2 \tilde{v}_s^2$,

$$\tilde{v}_s^2 \approx \frac{m_e}{12 m_i} v_p^2 \frac{W}{n_0 T_e},$$

while the damping due to the ions becomes exponentially small. This effect has much in common with acoustic propagation in a plasma subject to a strong radio-frequency field.[15]

The production of isothermal sound (2.21) is characterized by a threshold in terms of W. By virtue of the inequality $\omega \ll k_z v_g$ we have $v_p^2/v_{Te}^2 \ll 9m_i/m_e$ so that

$$W / n_0 T_e \gg 4m_e / 3m_i. \tag{2.22}$$

This condition is satisfied when $W \ll n_0 T_e$.

We note that the instability in (2.19) also has a threshold. By virtue of the relation $\omega^2 \gg k_z^2 v_g^2$, when $W/n_0 T_e \gg m_e/m_i$ we have

$$\frac{W}{n_0 T_e} \gg 108 \frac{m_i}{m_e} \frac{v_{Te}^4}{v_p^4}. \tag{2.23}$$

We also note that when $v_p \gg 3v_{Te}^2/v_{Ti}$ the expansion in terms of the turbulence energy in the collision integrals for the particles and turbulent fluctuations is not valid for plasma oscillations.

3. NONLINEAR PLASMON DRESSING AND CRITERIA FOR APPLICABILITY OF THE EXPANSION OF THE COLLISION INTEGRALS IN TERMS OF THE TURBULENCE ENERGY

1. In the equations that describe the turbulent state of the plasma the plasmon propagation operator $\Pi(k)$ (the Green's function $1/\Pi(k)$) requires nonlinear corrections proportional to the turbulence energy U_k in the first approximation; these correspond to the electromagnetic dressing of the plasmon. In contrast with the usual renormalization of the functional dependence of $\Pi(k)$ on U_k, in the present case we are dealing with a real effect.

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6)This can be found by the formal solution of Eq. (2.17) with respect to $\Phi_k^{(1)e}$ and from the linear equation for $n_k^{(1)e}$.

In Eqs. (2.2) and (2.3) we now expand all quantities in the turbulent field $e^{(k)}$ in the particle collisions and the turbulent fluctuations:

$$\Pi(k_1)U_{k_1} = \frac{e^2}{m_e^2}U_{k_1}\int \tilde{\Sigma}_{k_1,k_2,k_1,-k_2}U_{k_2}dk_2$$

$$+\frac{e^2}{2m_e^2}\int U_{k_2}U_{k_3}dk_2\,dk_3\frac{|S_{k_1,k_2,k_3}|^2}{\Pi(-k_2-k_3)}\delta(k_1-k_2-k_3), \quad (3.1)$$

$$\tilde{\Sigma}_{k,k_1,k_2,k_3} = \Sigma_{k,k_1,k_2,k_3} - \frac{S_{k,k_1,k-k_1}S_{k-k_1,k_2,k_3}}{\Pi(k-k_1)}, \quad (3.2)$$

$$S_{k,k_1,k_2} = \frac{\omega_{pe}^2}{n_0}\int dv_z\frac{\partial\Phi^{(0)e}}{\partial v_z}\frac{1}{(\omega-k_zv_z+i\delta)}$$

$$\times\frac{1}{(\omega_1-k_{1z}v_z+i\delta)(\omega_2-k_{2z}v_z+i\delta)}, \quad (3.3)$$

$$\Sigma_{k,k_1,k_2,k_3} = \frac{\omega_{pe}^2}{n_0}\int dv_z\frac{\partial\Phi^{(0)e}}{\partial v_z}\frac{1}{(\omega-k_zv_z+i\delta)^2}$$

$$\times\frac{1}{(\omega_2-k_{2z}v_z+i\delta)(\omega_3-k_{3z}v_z+i\delta)}\left[\frac{2k_z}{\omega-k_zv_z+i\delta}\right.$$

$$\left.+\frac{k_{2z}+k_{3z}}{\omega_2+\omega_3-(k_{2z}+k_{3z})v_z+i\delta}\right]. \quad (3.4)$$

In obtaining Eq. (3.1) the average over four fields has been taken by pairs and we have used Eq. (2.6), while the average over three fields is reduced to the average over, four fields by use of the approximate relation

$$\Pi(k_1)e^{(0)}_{k_1} = -\frac{ie}{2m_e}\int dk_2\,dk_3S_{k_1,k_2,k_3}\delta(k_1-k_2-k_3)$$
$$\times(e^{(0)}_{k_2}e^{(0)}_{k_3}-\langle e^{(0)}_{k_2}e^{(0)}_{k_3}\rangle). \quad (3.5)$$

For nondecay turbulence, which is the case for Langmuir turbulence, from Eq. (3.1), instead of the relation $\Pi(k_1)=0$ we have

$$\tilde{\Pi}(k_1)U_{k_1}=0, \quad (3.6)$$

$$\tilde{\Pi}(k_1)=\Pi(k_1)-\frac{e^2}{m_e^2}\int U_{k_2}dk_2\tilde{\Sigma}_{k_1,k_2,k_1,-k_2}. \quad (3.7)$$

In the region of spectral transfer where $\mathrm{Im}\,\Pi(k_1)=0$, we can write

$$\omega_1=\omega_{k_1}+\frac{e^2}{m_e^2\partial\Pi(k_1)/\partial\omega_1|_{\omega_1=\omega_{k_1}}}\int U_{k_2}dk_2\,\mathrm{Re}\,\tilde{\Sigma}_{k_1,k_2,k_1,-k_2}. \quad (3.8)$$

In contrast with Eq. (2.9), in the present case there is no unique relation between ω and k. In a number of cases, however, the mean nonlinear frequency shift is larger than the frequency spread and as an approximation we can speak of a linear change in the plasmon spectrum.

For concreteness, we introduce the example of one-dimensional Langmuir turbulence which is treated above. The approximate expression for $\mathrm{Re}\,\tilde{\Sigma}$ when $v_p \gg v_{Te}(m_i/m_e)^{1/4}$ is

$$\mathrm{Re}\,\tilde{\Sigma}_{k_1,k_2,k_1,-k_2}\approx-\frac{\varepsilon_0^{(e)}(k_1-k_2)\varepsilon_0^{(i)}(k_1-k_2)(k_{1z}-k_{2z})^2}{\omega_1^2\omega_2^2(\varepsilon_0^{(e)}(k_1-k_2)+\varepsilon_0^{(i)}(k_1-k_2))}. \quad (3.9)$$

If $|\omega_1-\omega_2| \ll |k_{1z}-k_{2z}|v_{Ti}$ then $\omega_1\approx\omega_{k_1} - \omega_{pe}W/2n_0T_e$. Since the correction to the frequency is independent of k_1 and comes from $\omega_1-\omega_2$ the condition $|\omega_1-\omega_2|\ll|k_{1z}-k_{2z}|v_{Ti}$ means that $v_p \gg 3v_{Te}^2/v_{Ti}$. The effects of nonlinear dispersion can be highly dependent on k for inhomogeneous or nonisotropic turbulence[16] or when binary particle collisions are introduced.[17]

3. We note from Eq. (3.8)

$$\Pi(\omega_1,k_1)\approx(\omega-\omega_{k_1})\frac{\partial\Pi(k_1)}{\partial\omega_1}\Big|_{\omega=\omega_{k_1}}\neq0,$$

and this means that the factor $1/\Pi(k+k_1)$ does not exhibit resonance properties when $k\to0$. Whence we conclude that the condition for the applicability of the results in Sec. 2 is

$$|\Pi(k_1)|\ll|\omega-kv_{g,k_1}|\frac{\partial\Pi}{\partial\omega_1},$$

or more precisely, if account is taken of the compensation of positive frequencies and negative frequencies in Eq. (2.16) (the small factor k/k_1),

$$\frac{k}{k_1}\max(\omega,kv_g)\gg|\omega_1-\omega_{k_1}|; \quad (3.10)$$

where $\omega_1-\omega_{k_1}$ is the nonlinear correction to the frequency (3.8).

The criterion in (3.10) can be obtained if we consider the next order in the turbulence energy ($\sim v_k^2$).[10] Omitting the extremely complicated calculations we have limited outselves to the result.[10]

The correction to D_1 can be written as an additional term δd_1 in Eq. (2.15):[10]

$$\delta d_1=-\frac{\omega_{pe}^2e^4}{n_0m_e^4}\int\frac{U_{k_1}U_{k_2}dk_1\,dk_2(k_{1z}-k_{2z})^2}{\omega_1^4\omega_2^4\Pi(k_1-k)\Pi(k_2-k)} \quad (3.11)$$
$$\times\frac{\varepsilon_0^{(i)}(k_1-k_2)\varepsilon_0^{(e)}(k_1-k_2)}{\varepsilon_0^{(i)}(k_1-k_2)+\varepsilon_0^{(e)}(k_1-k_2)}.$$

Comparing (3.11) with (2.16) taking account of (3.9) for $\mathrm{Re}\,\tilde{\Sigma}$ and (2.7) we obtain the criterion in (3.10). We note that in (3.11) only those frequency regions are important in which $\epsilon_e^{(0)}(k_1-k_2)$ is large, that is to say, ω_1 and ω_2 are of opposite sign so that the positive and negative frequency parts to not balance each other. This feature gives the factor k/k_1 in Eq. (3.10).

4. We now consider the turbulence acoustic wave considered in Sec. 2 from the point of view of the criterion in (3.10). The criterion in (3.10) is of the form

$$\frac{W}{n_0T_e}\frac{m_e}{54m_i}\frac{v_p^4}{v_{Te}^4}\ll\frac{k^2}{k_1^2}\ll1.$$

This corresponds to the criterion for neglecting the nonlinear dispersion and, provisionally, $\omega \ll kv_g$, in which case this acoustic wave is possible. Thus, the turbulent acoustic wave can exist over a relatively wide range of plasma parameters. The conditions for the appearance of the instability (2.19) are very stringent [in accordance with Eqs. (2.19), (2.23), and (3.10)]:

$$\frac{12m_i}{m_e}\ll\frac{v_p^2}{v_{Te}^2}\ll\frac{9T_e}{T_i}\frac{m_i}{m_e},$$
$$\frac{T_i}{T_e}\frac{m_e}{m_i}\ll\frac{3W}{4n_0T_e}\ll\frac{m_e}{m_i}.$$

We note that the limitations imposed on (3.10) become less stringent for turbulence fluctuations whose frequency difference is larger than the Langmuir frequency. This is the case for non-electrostatic fluctuations.[10]

4. INTEGRAL EQUATIONS FOR SUMMING THE SERIES IN THE TURBULENCE ENERGY IN THE COLLISION INTEGRALS FOR PARTICLES AND TURBULENCE FLUCTUATIONS

1. When the nonlinear frequency correction is intro-

duced the renormalized group velocity is a physical quantity. Consequently, the collision integral for the plasma particles in the turbulence fluctuations must contain the complete plasmon Green's function $1/\tilde{\Pi}(k_1 + k)$ rather than $1/\Pi(k_1 + k)$ as in Eq. (2.13). It will be evident that when $k \to 0$ the quantity $\tilde{\Pi}(k_1 + k)$ is of order ν_k. Consequently, all terms of this order must be retained. In the present section we expand the collision integrals for the particles and turbulence fluctuations in the small parameter W/nT, assuming that in the first approximation. $\Pi(k_1 + k)$ is a quantity of order U_k.

We now obtain the equation for the kernel for the collision integral for the particles and turbulence fluctuations. We show that in a very natural way this equation can be obtained within the framework of the weak-correlation approximation for the fields $e_k^{(0)}$ (between themselves and between the perturbation field $e_k^{(1)}$) that is to say, in the approximation usually used in the theory of weak turbulence, in particular, in obtaining Eq. (3.1). It is now convenient to write Eq. (2.5) in somewhat different form, introducing the notation

$$\tilde{\varphi}_k^{(1)\alpha} = \varphi_k^{(1)\alpha} - \frac{e_\alpha}{im_\alpha} e_k^{(1)} \frac{1}{(\omega - k_z v_z + i\delta)} \frac{\partial \Phi^{(0)\alpha}}{\partial v_z},$$
$$\tilde{\varphi}_k^{(0)\alpha} = \varphi_k^{(0)\alpha} - \frac{e_\alpha}{im_\alpha} e_k^{(0)} \frac{1}{(\omega - k_z v_z + i\delta)} \frac{\partial \Phi^{(0)\alpha}}{\partial v_z}. \tag{4.1}$$

We have

$$-i(\omega - k_z v_z)\Phi_k^{(1)} + \frac{e_\alpha}{m_\alpha}\mathscr{E}_k \frac{\partial \Phi^{(0)\alpha}}{\partial v_z} - \frac{\partial}{\partial v_z}\langle D^*\rangle \frac{\partial \Phi^{(0)\alpha}}{\partial v_z}$$
$$= -\frac{\partial}{\partial v_z}\frac{e_\alpha}{m_\alpha}\int dk_1\, dk_2\, \delta(k - k_1 - k_2)\langle e_{k_1}^{(1)}\tilde{\varphi}_{k_2}^{(0)\alpha} + e_{k_1}^{(0)}\tilde{\varphi}_{k_2}^{(1)\alpha}\rangle, \tag{4.2}$$

$$\Pi(k) e_k^{(1)} = -\frac{4\pi i}{k_z}\sum_\alpha e_\alpha \int \tilde{\varphi}_k^{(1)\alpha}\, dv_z, \tag{4.3}$$

$$-i(\omega - k_z v_z)\tilde{\varphi}_k^{(1)\alpha} + \frac{e_\alpha}{m_\alpha}\int dk_1\, dk_2\, \delta(k - k_1 - k_2)$$
$$\times e_{k_1}^{(0)} \frac{\partial \Phi_{k_2}^{(1)\alpha}}{\partial v_z} - \frac{\partial}{\partial v_z}(D^* - \langle D^*\rangle)\frac{\partial \Phi^{(0)\alpha}}{\partial v_z}$$
$$\times \frac{e_\alpha}{m_\alpha}\frac{\partial}{\partial v_z}\int dk_1\, dk_2\, \delta(k - k_1 - k_2)\,(\mathscr{E}_{k_1}\varphi_{k_2}^{(0)\alpha}$$
$$\times e_{k_1}^{(1)}\tilde{\varphi}_{k_2}^{(0)\alpha} + e_{k_1}^{(0)}\tilde{\varphi}_{k_2}^{(1)\alpha} - \langle e_{k_1}^{(1)}\tilde{\varphi}_{k_2}^{(0)\alpha} + e_{k_1}^{(0)}\tilde{\varphi}_{k_2}^{(1)\alpha}\rangle), \tag{4.4}$$

$$-i(\omega - k_z v_z)\tilde{\varphi}_k^{(0)\alpha} = -\frac{e_\alpha}{m_\alpha}\frac{\partial}{\partial v_z}\int dk_1\, dk_2$$
$$\times \delta(k - k_1 - k_2)\,(e_{k_1}^{(0)}\varphi_{k_2}^{(0)\alpha} - \langle e_{k_1}^{(0)}\varphi_{k_2}^{(0)\alpha}\rangle). \tag{4.5}$$

Here

$$D^* = i(\omega - k_z v_z)\frac{e_\alpha^2}{m_\alpha^2}\int \frac{dk_1\, dk_2\, e_{k_1}^{(0)} e_{k_2}^{(1)}\delta(k - k_1 - k_2)}{(\omega_1 - k_{1z}v_z + i\delta)(\omega_2 - k_{2z}v_z + i\delta)}.$$

We see from Eq. (4.2) that for low-frequencies $k \to 0$ the collision integral that contains D^* is much larger than the other integrals on the right side of Eq. (4.2).

Actually, when $k \to 0$, $k_2 \to -k_1$ and by virtue of the fact that $e_{k_1}^{(0)}$ is close to the linear field we find that $e_{k_2}^{(1)}$, which contains the quantity $1/\Pi(k_2)$ by virtue of Eq. (4.3), is a large quantity. On the right side of Eq. (4.2) the quantity $\tilde{\varphi}_k^{(1)}$ does not contain the large factor that has been indicated and, in accordance with Eq. (4.5), is nonlinear in $e_k^{(0)}$, that is to say, the quantity $\langle e_{k_1}^{(1)}\tilde{\varphi}_{k_2}^{(0)}\rangle$ is proportional to a higher power of the turbulence energy. Hence, we start the calculation by forming the equation for D^*. The right side of Eq. (4.2) will not be used below but the use of this method for unknown $\langle D^*\rangle$ can also be used to compute these integrals.

Using Eqs. (4.3) and (4.4) we can form expressions for $\langle e_{k_1}^{(0)} e_{k_2}^{(1)}\rangle$:

$$\Pi(k_2)\langle e_{k_1}^{(0)} e_{k_2}^{(1)}\rangle + i\frac{e}{m_e}\int dk_1'\, dk_2'\, \delta(k_2 - k_1' - k_2')$$
$$\times S_{k_2, k_1', k_2'}\langle e_{k_1}^{(0)} e_{k_1'}^{(0)} e_{k_2'}^{(1)}\rangle + \frac{\omega_{pe}^2}{n_0 k_{2z}}\int \frac{dv_z\, dk_1'\, dk_2'}{\omega_2 - k_{2z}v_z + i\delta}\frac{\partial}{\partial v_z}$$
$$\times(\langle \varphi_{k_1'}^{(0)e} e_{k_1}^{(0)} e_{k_2'}^{(1)}\rangle + \langle e_{k_1}^{(0)} e_{k_1'}^{(0)} \varphi_{k_2'}^{(1)e}\rangle)\,\delta(k_2 - k_1' - k_2')$$
$$= -\frac{\omega_{pe}^2}{n_0}\int \frac{U_{k_1}\delta(k_1 + k_2 - k')\, dk'}{k_{2z}(\omega_2 - k_{2z}v_z + i\delta)}\frac{\partial \Phi_{k'}^{(1)e}}{\partial v_z}\, dv_z$$
$$-\frac{\omega_{pe}^2}{n_0 k_{2z}}\int dv_z\, dk_1'\, dk_2'\, \delta(k_2 - k_1' - k_2')\frac{\mathscr{E}_{k_1'}}{\omega_2 - k_{2z}v_z + i\delta}.$$
$$\times \frac{\partial}{\partial v_z}\langle e_{k_1}^{(0)}\varphi_{k_2'}^{(0)}\rangle \equiv G. \tag{4.6}$$

The right side of Eq. (4.6) G does not contain $e_k^{(1)}$ and can be computed as a summation in powers of U_{k_1} in the standard way by means of Eq. (4.5). In the first approximation we have

$$G = -\frac{\omega_{pe}^2 U_{k_1}}{n_0 k_{2z}}\int \frac{dv_z\, dk'\, \delta(k_1 + k_2 - k')}{(\omega_2 - k_{2z}v_z + i\delta)}$$
$$\times\left[\frac{\partial \Phi_{k'}^{(1)e}}{\partial v_z} + \frac{e}{m_e} i\mathscr{E}_{k'}\frac{\partial}{\partial v_z}\frac{1}{(\omega_1 - k_{1z}v_z + i\delta)}\frac{\partial \Phi^{(0)e}}{\partial v_z}\right]. \tag{4.7}$$

In the transformation for the average $\langle e_{k_1}^{(0)} e_{k_1'}^{(0)} e_{k_2'}^{(0)}\rangle$ in the average over the four fields we use the weak-field correlation approximation. In the expression for $e_{k_1}^{(0)}$, we make use of Eq. (3.5) for the quadratic combinations of the fields and for $e_{k_1}^{(1)}$ the relation obtained from Eqs. (4.3) and (4.4) if we limit ourselves to linear and quadratic terms in $e_k^{(1)}$ and $e_k^{(0)}$. The linear terms in this relation contain only $e_k^{(0)}$ and in $\langle e_{k_1}^{(0)} e_{k_1'}^{(0)} e_{k_2'}^{(1)}\rangle$ they give terms $\sim U_k^2$ which must be referred to the right side of Eq. (4.6) (being independent of $e_k^{(1)}$) and which are neglected in the approximation in (4.7).

Hence, in obtaining $e_{k_2}^{(1)}$ it is sufficient to use the relation

$$\Pi(k)\frac{e}{m_e}e_k^{(1)} = \frac{\omega_{pe}^2}{n_0 k_z}\int \frac{dv_z}{\omega - k_z v_z}\frac{\partial}{\partial v_z}(D^* - \langle D^*\rangle)\frac{\partial \Phi^{(0)e}}{\partial v_z}.$$

Similar considerations indicate that in calculating $\langle \tilde{\varphi}_k^{(1)e} e_{k_1}^{(0)} e_{k_1'}^{(0)}\rangle$ we can make use of the following relations for $\tilde{\varphi}_k^{(1)e}$

$$-i(\omega - k_z v_z)\tilde{\varphi}_k^{(1)e} = \frac{\partial}{\partial v_z}(D^* - \langle D^*\rangle)\frac{\partial \Phi^{(0)e}}{\partial v_z}.$$

Finally, in computing $\langle \varphi_{k'}^{(0)} e_{k_1}^{(0)} e_{k_2'}^{(1)}\rangle$ it is sufficient to use the first approximation for $\tilde{\varphi}_k^{(0)}$ from Eq. (4.5). As a result of these calculations the left side of Eq. (4.6) is reduced to a form that contains only the average over the four turbulent fields which, as an approximation, can be divided into possible products of the averages of two fields. We find

$$G=\Pi(k_2)\langle e_{k_1}^{(0)} e_{k_2}^{(1)}\rangle-\frac{e^2}{m_e^2}U_{k_1}\int dk_2'\,dk_2''$$

$$\times\delta(k_1+k_2-k_2'-k_2'')\tilde{\Sigma}_{k_2',k_2',-k_1,k_2''}(\langle e_{k_2'}^{(0)}e_{k_2''}^{(1)}+e_{k_2''}^{(0)}e_{k_2'}^{(1)}\rangle)$$

$$+\frac{e^2}{m_e^2}\int dk_2'\,dk_2''\delta(k_1+k_2-k_2'-k_2'')S_{k_2,k_3''-k_1,k_2'}$$

$$\times U_{k_2''-k_1}\frac{1}{\Pi(k_1)}S_{k_1,-k_2''+k_1,k_2''}\langle e_{k_2''}^{(0)}e_{k_2'}^{(1)}\rangle. \tag{4.8}$$

Here, $\tilde{\Sigma}$ and S correspond to the definitions (3.2) and (3.3). We note that the operator $\tilde{\Pi}(k_2)$ appears automatically in Eq. (4.8) as do additional terms $\sim U_k$ which are of the same order as $\Pi(k_2)$ or $\tilde{\Pi}(k_2)$.

In the case of nondecay turbulence the last term in Eq. (4.8) vanishes and the sought equation assumes the form

$$\tilde{\Pi}(k-k_1)\langle e_{k_1}^{(0)}e_{k-k_1}^{(1)}\rangle=-\frac{\omega_{pe}^2U_{k_1}}{n_0(k_z-k_{1z})}$$

$$\times\int dv_z\left[\frac{\partial\Phi_k^{(1)e}}{\partial v_z}+i\frac{e}{m_e}\mathscr{E}_k\frac{\partial}{\partial v_z}\frac{1}{(\omega_1-k_{1z}v_z+i\delta)}\frac{\partial\Phi^{(0)e}}{\partial v_z}\right]$$

$$\times\frac{1}{\omega-\omega_1-(k_z-k_{1z})v_z+i\delta}+\frac{e^2}{m_e^2}U_{k_1}\int dk_1'\langle e_{k_1'}^{(0)}e_{k-k_1'}^{(1)}\rangle$$

$$\times(\tilde{\Sigma}_{k-k_1,k_1',-k_1,k-k_1'}+\tilde{\Sigma}_{k-k_1,k-k_1',-k_1,k_1'}). \tag{4.9}$$

if the integral equation (4.9) is solved it is a simple matter to obtain the collision integral that contains D^*.

Finally, the collision integrals on the right side of Eq. (4.2), which contain $\langle e_{k_1}^{(1)}\tilde{\varphi}_{k_2}^{(0)\alpha}\rangle$ can, by means of Eq. (4.5), be reduced to a form that contains the smallest two $e_k^{(0)}$. Consequently, by these same methods these can be reduced to terms of the form $U_{k_1'}\langle e_{k_1}^{(0)}e_{k-k_1}^{(1)}\rangle$ that is to say, they can be computed from the solution of (4.9). These, however, are of order W/nT with respect to the term with D^* because in the transformation of the average of the three turbulence fields only the expression $1/\Pi(k_1-k_1')\ll 1$ arises. In the integral with $\langle e_{k_1}^{(0)}\tilde{\varphi}_{k_2}^{(1)}\rangle$ on the right side of Eq. (4.2) there also only arise terms $U_{k_1'}\langle e_{k_1}^{(0)}e_{k-k_1}^{(1)}\rangle$ with small factors $1/\Pi(k_1-k_1')$ and linear terms in U_k which give the terms D_0 and $\hat{D}$ in Eq. (2.10). This indicates that the method can be used for taking account of higher order terms in U_k. In the first approximation being used here the right side of Eq. (4.2) vanishes and the summation over U_k required for lower frequencies in perturbation theory reduces to the solution of the integral equation (4.9).

2. We now solve the integral equation that has been obtained in the limiting case $|\omega_1-\omega_1'|\ll|k_{1z}-k_{1z}|v_{Ti}$; $\omega\ll k_{1z}v_{Ti}$ which, for the example considered above, corresponds to the case of one-dimensional Langmuir turbulence $v_p\gg 3v_{Te}^2/v_{Ti}$. This is the limit in which the expansion of the collision integral in terms of U_k yields questionable results. From Eqs. (3.2) and (3.4) we have

$$\tilde{\Sigma}_{k-k_1,k_1',-k_1,k-k_1'}\approx-\frac{\omega_{pe}^2T_e}{(T_e+T_i)v_{Te}^2\omega_1\omega_1'(\omega_1-\omega)(\omega_1'-\omega)}. \tag{4.10}$$

This result holds if the frequencies ω_1 and ω_1' are of opposite sign; if the frequencies ω_1 and ω_1' are the same sign then the expression in (4.10) vanishes. The quantity $\tilde{\Sigma}_{k-k_1,k_1-k_1'-k_1,k_1'}$ does not vanish when ω_1 and ω_1' are of the same sign, being given by (4.10).

We now divide (4.9) by $\tilde{\Pi}(k-k_1)$ and form the equation for

$$S_\pm(k)=\int dk_{1\pm}\frac{\langle e_{k_1}^{(0)}e_{k-k_1}^{(1)}\rangle}{\omega_1(\omega_1-\omega)},$$

where in S_+ the integration is carried out over the region of positive frequencies; in S_- it is carried out over negative frequencies. We now obtain a system of linear algebraic equations for $S_\pm$; the solution has the form

$$S_+(k)+S_-(k)=-\left[1+\frac{\omega_{pe}^4}{4\pi n_0(T_e+T_i)}\int\frac{U_{k_1}dk_1}{\omega_1^2(\omega_1+\omega)^2\tilde{\Pi}(k_1+k)}\right]^{-1}$$

$$\times\frac{\omega_{pe}^2}{n^0}\int\frac{U_{k_1}dk_1}{\tilde{\Pi}(k+k_1)(k_{1z}+k_z)}\int dv_z\frac{1}{(\omega+\omega_1-(k_{1z}+k_z)v_z+i\delta)}$$

$$\times\left[\frac{\partial\Phi_k^{(1)e}}{\partial v_z}-i\frac{e}{m_e}\mathscr{E}_k\frac{\partial}{\partial v_z}\frac{1}{(\omega_1-k_{1z}v_z+i\delta)}\frac{\partial\Phi^{(0)e}}{\partial v_z}\right].$$

Here the integration is carried over all frequencies. To the accuracy required here we find

$$\langle D^*\rangle=-i(\omega-k_zv_z)\frac{e^2}{m_e^2}[S_+(k)+S_-(k)];$$

which allows us to obtain the coefficients D_1 and D_2 in Eq. (2.10), for example

$$D_1=-i\frac{\omega_{pe}^2}{n_0}(\omega-k_zv_z)n_k^{(1)}\frac{e^2}{m_e^2}\int\frac{U_{k_1}dk_1}{\tilde{\Pi}(k_1+k)\omega_1(\omega_1+\omega)^3}$$

$$\times\left[1+\frac{\omega_{pe}^4}{4\pi n_0(T_e+T_i)}\int\frac{U_{k_1}dk_1}{\omega_1^2(\omega_1+\omega)^2\tilde{\Pi}(k_1+k)}\right]^{-1}. \tag{4.11}$$

This expression differs from the one obtained above by expansion in U_k in that the denominator contains an expression that differs from unity while the plasmon Green's function in the numerator $1/\Pi(k)$ is replaced by $1/\tilde{\Pi}(k)$. Thus, the result reduces to the renormalization of the plasmon propagator and the renormalization of the effective electron charge. The denominator, like the denominator in (4.11), also appears in D_2 and the estimate $D_2/D_1\ll 1$ obtained above for $\omega/k\ll\omega_1/k_1$, $\omega<\omega_1$, $k\ll k_1$ remains valid in the present case.

From Eqs. (2.10) and (4.11) we can obtain the dielectric function

$$\varepsilon(k)=\varepsilon_0^{(i)}(k)+(\varepsilon_0^{(e)}(k)-1)(1+n_0m_ed^2(T_e+T_i)^{-1}) \tag{4.12}$$

$$\times\left(1+\frac{(\varepsilon_0^{(e)}(k)-1)n_0k_z^2}{\omega_{pe}^2}d_1+\frac{n_0m_e}{T_e+T_i}d_2\right)^{-1}.$$

$$d_1=\frac{e^2\omega_{pe}^2}{m_e^2n_0}\int\frac{U_{k_1}dk_1}{\tilde{\Pi}(k+k_1)\omega_1(\omega_1+\omega)^3},$$

$$d_2=\frac{e^2\omega_{pe}^2}{m_e^2n_0}\int\frac{U_{k_1}dk_1}{\tilde{\Pi}(k+k_1)\omega_1^2(\omega_1+\omega)^2}. \tag{4.13}$$

3. We now consider one-dimensional Langmuir turbulence. In the region $v_p\gg 3v_{Te}^2/v_{Ti}$ in accordance with Sec. 3 the group velocities of the plasmons are not affected. Hence, in Eq. (4.13) $\tilde{\Pi}$ can be replaced by Π.[7] Further, in the limit $\omega\ll\omega_1$, $k\ll k_1$ $\omega/k\ll\omega_1/k_1$, we find $d_2\approx d_1$ and both give in Eq. (2.16).

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[7]This is not evident from expansion of d_1 in terms of U_k since the corrections $\sim U_k^2$ coincide with (3.11).

If however $k_z v_{Ti} \ll \omega \ll k_z v_{Te}$, then Eq. (4.12) yields

$$\varepsilon(k) = -\frac{\omega_{Ei}^2}{\omega^2} + \frac{\omega_{Ee}^2}{k_z^2 v_{Te}^2}\left(1 + \frac{3k_z^2 T_e W}{4\omega^2 n_0 m_e (T_e + T_i)}\right) \times \left(1 + \frac{3k_z^2 W (2T_e + T_i)}{4\omega^2 n_0 m_e (T_e + T_i)}\right)^{-1},$$

$$\omega^2 = k_z^2 \tilde{v}_\pm^2, \tag{4.14}$$

$$\tilde{v}_\pm^2 = \frac{1}{2}\left(v_s^2 - v_\sim^2 \frac{T_e}{T_e + T_i}\right) \pm \left\{\frac{1}{4}\left(v_s^2 - v_\sim^2 \frac{T_e}{T_e + T_i}\right)^2 + v_\sim^2 v_s^2 \frac{2T_e + T_i}{T_e + T_i}\right\}^{1/2}.$$

Here, $v_\sim^2 = 3W/4n_0 m_e$. The instability in (4.14) is qualitatively different from that in (2.19). When $v_\sim^2 \gg v_s^2$ the square of the growth rate (4.14) is proportional to W while the square of the growth rate (2.19) is proportional to $\sqrt{W}$. Finally, when $\omega \ll k_z v_{Ti}$, $\omega \ll k_z v_g$ we have

$$\omega^2 = -2k_z^2 v_\sim^2 (T_e + T_i) T_e^{-1}.$$

In conclusion, we wish to emphasize the following points.

1. The appearance of new modes of oscillation in the presence of turbulence is of a rather general nature and the possibility is not excluded that these are possible in other turbulent media (for example, fluids).

2. The theory developed here takes account of the effect of correlation between turbulence fluctuations and makes it possible to trace their effect on the low-frequency properties of the turbulent plasma.

3. In taking account of renormalization effects the plasmon Green's function has a singularity when $k \to 0$. Taking account of this singularity in the proper way leads to the need for summing the perturbation theory series in the turbulence energy.

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