SOME PROPERTIES OF LAYER STRUCTURES

E. I. KATS

L. D. Landau Institute of Theoretical Physics, USSR Academy of Sciences

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Fluctuations of the electron density and of the order parameter phase, the magnetic properties and the features of the superconducting transition in layer structures are investigated. A dependence of the critical field on direction characteristic for such systems is derived. The calculations are carried out in the logarithmic approximation.

1. INTRODUCTION

SEVERAL years ago Ginzburg[1] expressed the assumption that a peculiar mechanism of surface conductivity is possible in two-dimensional systems. Such a mechanism allowed one the hope of obtaining high transition temperatures. However, subsequently Rice[2] and Hohenberg[3] showed that a superconducting phase transition in two-dimensional systems is generally speaking impossible on account of the destructive action of fluctuations. Nevertheless, an analogous surface mechanism can take place in thin films with coatings and in layer structures.[4] The situation in this case recalls the case of quasi one-dimensional systems which has already been investigated.[5] There is, however, a number of differences connected in the main with weaker limitations imposed on the transition temperature by phase fluctuations. This in turn renders the nature of the fluctuations three-dimensional in a broader range of temperatures. In this paper we investigate layer structures. The treatment can be applied to such substances as NbB_2 and NbS_2. These are compounds with a layer structure; their conductivity within the layer is metallic, and in the direction perpendicular to the layer there is practically no conductivity. Account of the Coulomb interaction makes it possible, as in [5], to limit the role of electron density fluctuations and to maintain the one-particle nature of the spectrum. However, the Coulomb interaction does not affect the phase fluctuations of the wave function of the superconducting electrons, and to limit these one must take into account the real electron transitions between different layers. With regard to their magnetic properties, layer structures have a specific peculiarity. Depending on the angle of inclination of the external field the superconducting transition can be a first or second-order transition.

2. ELECTRON DENSITY FLUCTUATIONS

Let us first estimate the role of the electron density fluctuations in the system under consideration. The calculations here are completely analogous to those carried out in [5]. Making use of the notation of that paper, one can write

$$T_c = \text{const}\cdot\exp\left\{-\frac{\langle\varphi^2(0)\rangle}{2}\right\}, \tag{1}$$

$$\langle\varphi_{K\omega}\varphi^*_{K\omega}\rangle = \frac{\omega^2 m^2}{n_0^2 K^4}\langle n_{K\omega}n^*_{K\omega}\rangle. \tag{2}$$

The quantity entering in the right-hand side of formula (2) is determined by the Coulomb interaction:

$$\langle n_{K\omega}n_{K\omega}^*\rangle = (V - V_0)\,/\,V_0^2. \tag{3}$$

Let us write down the matrix element of the Coulomb interaction in the system under consideration

$$V_{q,\mathbf{k}} = \sum\int\frac{e^{i\mathbf{k}\boldsymbol{\rho}}}{\sqrt{\rho^2+(z-z')^2}}\,\psi_{q_1}^*\psi_{q_2}\psi_{q_3}^*\psi_{q_4}\,d\boldsymbol{\rho}. \tag{4}$$

Here **k** is the projection of the momentum on the layer, and q is its projection on the direction perpendicular to the layer. In addition, motion within the layer is considered to be free, whereas the motion between layers is considered in the tight-binding approximation. Therefore it is convenient to write ψ_q in the form of an expansion in Wannier functions. If one introduces in formula (4) a cutoff function at the layer thickness, then (4) will correspond to the potential of a system of parallel charged plates. In the region of interest to us $kc \ll 1$ and $qc \ll 1$ (c is the distance between the planes) the Coulomb interaction has the usual three-dimensional character:

$$V_{q,\mathbf{k}} = -4\pi e^2\,/\,(q^2+k^2). \tag{5}$$

In the equation for determining the transition temperature (Fig. 1) this potential enters in such a way that all internal lines corresponding to two-dimensional motion of the electrons do not depend on the wave vector q. Therefore in this projection of the vector there are no

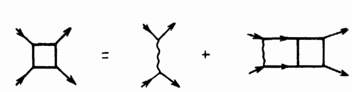

FIG. 1

limitations due to the law of conservation of momentum and all terms of the equation can be integrated independently over q. Since the momentum q enters only into the expression for the potential (5), this integration reduces to the replacement

$$V_{q,\mathbf{k}} \to V_{\mathbf{k}} = -2\pi e^2\,/\,|\mathbf{k}|. \tag{6}$$

Let us now take into account screening. Calculations to first order yield for the polarization operator

$$\Pi(\varepsilon,\mathbf{k}) = \frac{ie^2}{(2\pi)^3}\int\frac{p\,dp\,d\varphi\,d\omega}{(i\omega - p^2/2m + p_0^2/2m)\,(i\omega - i\varepsilon - (\mathbf{p}-\mathbf{k})^2/2m + p_0^2/2m)}$$

$$= \frac{me^2}{\pi}\left(1 - \frac{|\varepsilon|}{\sqrt{\varepsilon^2+v^2k^2}}\right). \tag{7}$$

Introducing the Debye screening radius $\kappa = (me^2/2\pi)^{1/2}$ and assuming $\epsilon \ll vk$, we obtain in place of (6)

$$V = -\frac{2\pi e^2}{\sqrt{k^2+\varkappa^2}}. \tag{8}$$

Below we shall make use of this expression for the Coulomb interaction. We substitute it in formula (3) and obtain from (2)

$$\langle \varphi^2(0) \rangle \sim \frac{m^2\omega_0{}^2 v}{n_0{}^2 e^2} p_0{}^2 \sim 1.$$

Here we have made use of the fact that in a system of charged planes the spectrum of plasma waves is of the following form:

$$\bar{\omega}^2 = \omega_0{}^2 \sin^2\theta + v^2 k^2, \tag{9}$$

where $\omega_0^2 = v^2\kappa^2$. Formula (9) can be readily obtained by substituting in the equation depicted in Fig. 2 expression (5) in place of the wavy line and formula (7) in place of the loop.

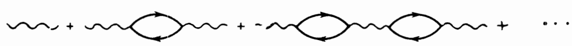

FIG. 2

Thus the plasma spectrum contains an angle-dependent gap (as in the quasi one-dimensional case, however here there is a different angular dependence), i.e. electron density fluctuations in a layer system are of a three-dimensional nature and do not have a destructive effect on the superconducting state. It should be noted that the nature of the density fluctuations in a layer system is such that at low temperatures ($\omega_k/T \gg 1$) (unlike in the one-dimensional case) the fluctuations are finite, even without account of the Coulomb interaction, and there is a logarithmic divergence in the "high-temperature" limit ($\omega_k/T \ll 1$) which is also removed by introduction of the Coulomb interaction. Since the quantity corresponding to the exponent of the exponential in (8) can be written

$$\sum \frac{1}{\omega_k}\left(\langle N_k \rangle + \frac{1}{2} \right) \sim \int \frac{d\mathbf{k}}{\omega_k} \operatorname{cth} \frac{\omega_k}{2T},$$

which yields for $\omega_k/T \gg 1$ a convergent integral $\int \frac{k\,dk}{\omega_k}$, and for $\omega_k/T \ll 1$ the integral

$$\int \frac{k\,dk}{\omega_k} \frac{T}{\omega_k}.$$

There is also another "dangerous" circumstance connected with electron density fluctuations. We are referring to the fact that in the two-dimensional case, if one restricts oneself to the point interaction, one cannot generally assume the one-particle nature of the spectrum: $\epsilon_p = v(|p| - p_0)$ since then the eigenenergy part has a singularity. For instance, to lower order (Fig. 3) $\Sigma \sim \ln(i\omega - \xi)$. Therefore, without account of the Coulomb interaction between the planes we cannot even set up the problem of two-dimensional superconductivity correctly. On the other hand, account of this interaction changes the dangerous eigenenergy part into that shown in Fig. 4 which to lower order has the value

$$\Sigma \sim e^4 (i\omega - \xi) \frac{1}{v} \ln \frac{i\omega - \xi}{v\varkappa}.$$

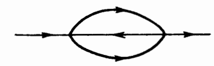

FIG. 3

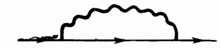

FIG. 4

In the logarithmic approximation one takes into account only terms $\sim e^2 L$ (L is a large logarithm) and the contribution from the eigenenergy parts (Fig. 4) can be discarded.

3. PHASE FLUCTUATIONS

Whereas the introduction of the Coulomb interaction limits the density fluctuations and retains the one-particle nature of the spectrum, the limitation of the effect of phase fluctuations requires an account of the real transitions of electrons between the planes. For estimates we shall utilize the following model dispersion law:

$$\varepsilon = v(|\mathbf{k}| - p_0) + \alpha \cos cq. \tag{10}$$

This is the so-called corrugated cylinder where the amplitude of the corrugation $\alpha \ll vp_0$ (it should be noted that the transition to the limit $\alpha \to 0$ merely signifies the absence of jumps between the planes, whereas a transition to a separate plane occurs for $c \to \infty$). If it turns out that $\alpha \ll T_c$, then with regard to the conductivity the layer structure will behave like a two-dimensional structure, but the nature of the fluctuations in the important region will nevertheless be three dimensional. Therefore in order of magnitude the two-dimensional charge $e^2 \sim \lambda/c$, where λ is the three-dimensional interaction constant, will enter in all formulas. Limitations on the transition temperature are imposed in a different way starting from the exact inequalities of Hohenberg[3] and on account of the fluctuations in the number of Cooper pairs. As has already turned out above, these limitations are of a completely general nature and do not depend on taking into account the Coulomb interaction between the planes. Phase fluctuations bound the superconducting transition temperature from above (see also [4]). Let us use Hohenberg's inequality[3] modified for the case under consideration

$$\int \frac{T\Delta^2}{k^2 + \alpha q^2} dq\, d^2k < \infty. \tag{11}$$

Assuming in order of magnitude $\Delta \sim T \sim T_c$, we obtain from (11)

$$T_c < \varepsilon_F \ln^{-1/3} \alpha. \tag{12}$$

For $T_c > \epsilon_F \ln^{-1/3} \alpha$ the transition is generally speaking impossible on account of the destructive action of the phase fluctuations; for $T_c < \alpha$ the superconducting state is of the usual three-dimensional nature. It is interesting to note that there is also a limitation of the temperature due to fluctuations of the number of Cooper pairs. Let us write down in analogy with (11) the pair correlation function for $\omega = 0$ and a finite temperature

$$K \sim \frac{\Delta^2}{k^2 + \alpha q^2}. \tag{13}$$

Let us now take into account the contribution to the fluctuations K in the expression for the Green's functions. In the lowest order it is given by the diagram shown in

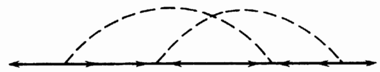

FIG. 5

Fig. 5 where the K is depicted by the dashed line. We have

$$T^2 \int K_1 K_2 G_1 G_2 F. \tag{14}$$

The important region in the integral is $k \sim \alpha^{1/2} p_0$ and in this region we can assume for an estimate that $F \sim 1/\Delta$ and $G \sim T/\Delta^2$. Then (14) yields $T^4 \ln^2 \alpha/\Delta$. It is hence clear that fluctuations in the number of Cooper pairs will begin to affect appreciably the magnitude of the superconducting gap only when

$$T \gtrsim \sqrt{\varepsilon_F \Delta / \ln \alpha}. \tag{15}$$

There exists thus a temperature range in which the superconducting transition is of a two-dimensional nature. In all formulas of this Section α is a dimensionless parameter corresponding to the amplitude of the corrugation. Unlike in the quasi one-dimensional case, inequality (15) provides a stricter limitation:

$$T_c < \sqrt{\varepsilon_F \Delta}\, \ln^{-1/2} \alpha. \tag{16}$$

4. PROPERTIES OF THE SUPERCONDUCTING STATE

A difficulty connected with the doubly logarithmic situation appears in the consistent account of the Coulomb interaction. Thus, even the simplest Cooper diagram (Fig. 6) is equal to

$$\frac{e^4}{\delta p (2\pi)^2 v}\left\{\ln\frac{\omega_0}{2v\varkappa} + \ln\frac{\omega_0}{2v\delta p} - \frac{\delta p}{p_0}\ln\frac{\omega_0}{2vp_0} - \frac{1}{2}\ln^2\frac{\delta p}{\varkappa}\right\}.$$

Here ω_0 is the total frequency and δp is the momentum transfer. In order to avoid this difficulty partly, we take into account the fact that the interaction (8) is only effective for "small" momentum transfers ($\ll \Lambda$, where Λ is the characteristic cut-off momentum of the theory). If there is in addition some attraction (due, for example, to phonons), then one must add to (8) the appropriate expression. Assuming that the attraction is effective for large momentum transfers ($\gg \Lambda$) and approximating it by a constant, we obtain a potential which depends appreciably on the angles. Expanding it in two-dimensional harmonics (cosines) and retaining only the zeroth term, we obtain the final expression:

$$D(p_1 - p_2) = -g + e^2 \ln \frac{\Lambda}{\sqrt{(p_1 - p_2)^2 + \varkappa^2}} \tag{17}$$

The remaining harmonics contain a small coefficient and can be discarded. Such an expansion makes it in addition possible to avoid a series of difficulties connected with the fact that in the equation for the transition temperature terms with different angular dependences are mixed with one another. We shall solve the equation for the gap with the interaction (17)

$$\Delta = -\frac{m}{2\pi}\int_0^\xi D\Delta\, du. \tag{18}$$

Here u is a logarithmic variable and $\xi = \ln(\Lambda^2/\Delta)$. In the logarithmic approximation this equation refers not

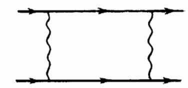

FIG. 6

only to the two-dimensional case; therefore its solution makes it also possible to explain the nature of the momentum dependence of the gap for a broader class of problems with Coulomb interaction. We introduce the following notation:

$$\lambda = \frac{m}{2\pi}, \qquad \eta = \ln\frac{\Lambda}{\sqrt{(p_1 - p_2)^2 + \varkappa^2}}, \qquad \mu = \ln\frac{\Lambda}{\varkappa}, \qquad \nu = \ln\frac{v\varkappa}{\Delta_0}.$$

There are two classes of solutions of Eq. (18). The solutions of one class satisfy the condition $\Delta > v\kappa$, and those of the other satisfy the condition $\Delta < v\kappa$. In the first case we have the detailed form of Eq. (18):

$$\Delta = -\lambda\int_0^\eta (-g + e^2 u)\Delta\, du - \lambda D(\eta)\int_\eta^\xi \Delta\, du. \tag{19}$$

Here account has been taken of the fact that in the logarithmic approximation in the region $u > \eta$ one can assume that $D(u)$ = const. From Eq. (19), differentiating it twice with respect to η, we obtain

$$\Delta_{\eta\eta}'' = \lambda e^2 \Delta. \tag{20}$$

Solving this equation under the conditions that

$$\Delta_\eta'(\xi) = 0, \qquad \Delta(\xi) = \Delta_0, \qquad \Delta(0) = \lambda g \int_0^\xi \Delta\, du, \tag{21}$$

we find that

$$\Delta = \Delta_0 \operatorname{ch} e\sqrt{\lambda}(\xi - \eta), \tag{22}$$

$$\xi = \frac{1}{2\lambda e}\ln\frac{g\sqrt{\lambda} + e}{g\sqrt{\lambda} - e}. \tag{23}$$

The corresponding dependence is shown in Fig. 7 with

$$\Delta_\infty = \Delta_0 \frac{\sqrt{\lambda}\, g}{\sqrt{\lambda g^2 - e^2}}.$$

In the second case the detailed form of Eq. (18) appears as follows:

$$\Delta = -\lambda\int_0^\eta D\Delta\, du - \lambda D(\eta)\int_\eta^\mu \Delta\, du - \lambda D\Delta(\mu)(\xi - \mu),$$

and since $\xi - \mu = \nu$,

$$\Delta_{\eta\eta}'' = \lambda e^2 \Delta. \tag{24}$$

Solving (24) with account of the fact that

$$\Delta(\mu) = \Delta_0, \qquad \Delta'(\mu) = -\lambda e^2 \Delta_0 \nu, \tag{25}$$

$$\Delta(0) = \lambda g \Delta_0 \nu - \lambda g \int_0 \Delta du,$$

we obtain

$$\Delta = \Delta_0 \operatorname{ch} e\sqrt{\lambda}(\mu - \eta) + \lambda e^2 \Delta_0 \nu \operatorname{sh} e\sqrt{\lambda}(\mu - \eta), \tag{26}$$

$$\nu = \frac{\operatorname{ch} e\sqrt{\lambda}\,\mu + e^{-1} g\sqrt{\lambda}\operatorname{sh} e\sqrt{\lambda}\,\mu}{\lambda g - \lambda e^2 \operatorname{sh} e\sqrt{\lambda}\,\mu - \lambda^{3/2} g e(\operatorname{ch} e\sqrt{\lambda}\,\mu - 1)}. \tag{27}$$

For $e\sqrt{\lambda}\,\mu < 1$ and $g\sqrt{\lambda}/e < 1$ we obtain in the logarithmic region the usual result

$$\Delta_0 \sim v\varkappa \exp\left\{-\frac{1}{g - e^2 \ln(p_0/\varkappa)}\right\}. \tag{28}$$

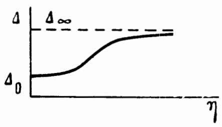

FIG. 7

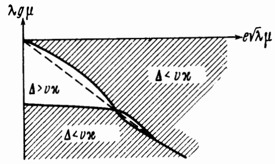

FIG. 8

Figure 8 shows the regions within which class-I and class-II solutions exist. For the former the following relations must be fulfilled:

$$z \operatorname{cth} z > -t > \frac{z^2 \operatorname{sh} z}{\mu + z(\operatorname{ch} z - 1)}$$

or

$$z \operatorname{cth} z < -t < \frac{z^2 \operatorname{sh} z}{\mu + z(\operatorname{ch} z - 1)}, \tag{29}$$

where $z = e\sqrt{\lambda}\,\mu$ and $t = \lambda g \mu$.

Formulas analogous to (20)–(27) are obtained in determining the residue near the pole of the vertex function. To this end, one must solve the inhomogeneous equation corresponding to the homogeneous Eq. (18)

$$\Gamma = D - \lambda \int D\Gamma\, du. \tag{30}$$

The solution of (30) in the logarithmic approximation is given in the Appendix. Here we should note that the superconducting transition temperature of a "two-dimensional" metal becomes zero in the approximation following the logarithmic approximation. Consequently, the logarithmic approximation is not sufficient for an exact solution of the problem of surface superconductivity. However, even account of the following terms in Eq. (18) $\sim e^4 L \sim e^2$ (where L is a large logarithm) result in so far insurmountable difficulties because the corrections to Γ and D are of the same order $\sim \kappa^2/p_0^2 \sim e^2$.

5. MAGNETIC PROPERTIES

Before we proceed directly to the investigation of the magnetic properties of layer structures, let us point out that without account of phase fluctuations there exists in the two-dimensional case a region of applicability of the Ginzburg-Landau equation (there is no such temperature region in the one-dimensional case). Let us write down the free-energy expansion

$$F_s = F_n + m \int d^2\rho \left(a\tau |\Delta|^2 + \frac{b}{2T_c^2} |\Delta|^4 + c\xi_0^2 |\nabla\Delta|^2 \right). \tag{31}$$

We shall calculate the thermodynamic fluctuations Δ about the equilibrium value Δ_0, $\Delta = \Delta_0 + \Delta_1$. In order of magnitude $\Delta_0 \sim T_c \tau^{1/2}$ where $\tau = (T - T_c)/T_c$ is the proximity to the transition point. Then,

$$\langle \Delta_1^2 \rangle \sim \frac{T_c}{m} \int \frac{d^2 k}{a\tau + c\xi_0^2 k^2} \sim \frac{T_c}{m\xi_0^2} \ln \tau \sim \frac{T_c^3}{mv^2} \ln \tau$$

and the condition that the fluctuations be small [the condition of applicability of expansion (31)] yields

$$\langle \Delta_1^2 \rangle / \Delta_0^2 \sim T_c \ln \tau / \varepsilon_F \tau \ll 1$$

and, thus we have a temperature range within which the Ginzburg-Landau equation is valid

$$T_c / \varepsilon_F \ll \tau \ll 1. \tag{32}$$

Additional limitations imposed by the phase fluctuations have already been investigated in Sec. 2. The Ginzburg-Landau equation is readily derived in the logarithmic region (28) by the standard method [7] using the model dispersion law (10) and introducing the effective interaction constant

$$\tilde{g} = g - e^2 \ln (p_0 / \varkappa).$$

It is more convenient to evaluate the corresponding integrals not in the coordinate but in the momentum representation, since the coordinate form of the Green's function in a layer system is inconvenient. As a result of the calculations we have

$$\left\{ \frac{1}{4m} \left(\frac{\partial}{\partial \rho} + 2ie\mathbf{A}_\rho \right)^2 + \frac{\alpha^2 c^2}{16\varepsilon_F} \left(\frac{\partial}{\partial z} + 2ieA_z \right)^2 + \frac{6(\pi T_c)^2}{7\zeta(3)\varepsilon_F} \left[\frac{T_c - T}{T_c} - \frac{7\zeta(3)}{8(\pi T_c)^2} |\Delta(\mathbf{r})|^2 \right] \right\} \Delta^*(\mathbf{r}) = 0, \tag{33}$$

where ρ is the component of the radius vector in the layer, or in the usual notation

$$\sum_{j,k} \left(\frac{1}{2m} \right)_{jk} (\nabla_j + 2ieA_j)(\nabla_k + 2ieA_k)\Delta^* + \frac{1}{\eta} \left\{ \frac{T_c - T}{T_c} - \frac{7\zeta(3)}{8(\pi T_c)^2} |\Delta|^2 \right\} \Delta^* = 0. \tag{34}$$

Let us now consider the critical field of such layer superconductors. The calculations can be carried out analogously as in [5]. However, here it is simpler to use Eq. (34) directly. The result is shown in Fig. 9 where the dependence of the critical field is plotted as a function of the angle with the plane of the layer. The corresponding values of the critical fields are

$$H_{c1}{}^{\perp} \sim \frac{c}{e} \frac{\Delta_0^2}{v^2}, \quad H_{cs}{}^{\parallel} \sim \frac{\Delta_0}{\mu_B}$$

$$H_c \sim \Delta_0 \sqrt{m p_0}, \quad H_{c2}{}^{\parallel} \sim \frac{\Delta_0}{\mu_B} \frac{\Delta_0}{\alpha}$$

and the characteristic angles are

$$\cos\theta_1 \sim H_{c1}{}^{\perp} / H_c, \quad \operatorname{tg}\theta_2 \sim H_{cs}{}^{\parallel} / H_{c1}{}^{\perp},$$

where μ_B is the Bohr magneton and the other notation is that of [5]. The dashed curve in the region $\theta < \theta_1$ corresponds to the supercooling field. The form of the corresponding curves is given by the formula (T = 0)

$$H_{cr} \sim \left\{ \frac{T_c^2/\varepsilon_F}{m^{-1}\sin^2\theta + \varepsilon_F^{-1}\alpha^2 c^2 \cos^2\theta} \right\}^{1/2}. \tag{35}$$

From this formula it follows that for

$$\theta \leqslant \theta_1 \quad \varkappa_{GL} < 1, \tag{36}$$

and for

$$\theta > \theta_1 \quad \varkappa_{GL} > 1,$$

which is also readily obtained from (34).

The point θ_1 is characterized by the fact that the curves of first and second-order transitions intersect

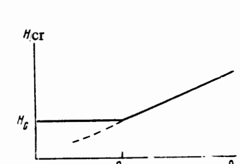

FIG. 9

at it. The physical reason for such a form of the curve is contained in the different mechanisms of the destruction of superconductivity in parallel and perpendicular fields.

Whereas for the perpendicular field there is no need for electron jumps between layers, account of this phenomenon is essential for describing the destruction of superconductivity in a parallel field.

6. CONCLUSION

A number of superconductors with a layer structure have recently been investigated. Unfortunately only measurements of the specific heat of NbB_2 in the temperature range $0.6-2.8°$ K and of NbS_2 in the $1.7-6.4°$ K have been carried out. The behavior of the specific heat is in qualitative agreement with the results which follow from this work. As regards a more detailed comparison, it is difficult, on the one hand, on account of the incomplete nature of the experiments which have been carried out, and, on the other hand, because of the inadequacy of the logarithmic approximation. The anomalously small jump in the specific heat at the transition temperature (~ 0.12 instead of 1.43 assumed from the BCS theory) can be explained both as an effect of the anisotropy as well as by the fact that at low temperatures the lattice part of the specific heat of a layer structure contains a term linear in the temperature (which is absent in the three-dimensional case); this term is not separable from the electronic contribution to the specific heat, since

$$c_n(T) = c_{el} + c_{ph}, \quad c_{ph} = N\frac{\pi}{12}\nu\frac{T}{\Theta},$$

where Θ is the Debye temperature and ν is the modulus of the displacement (this contribution to the specific heat is due to bending oscillations of the layers). A comparison of the dependence of the critical field on the angle with the plane of the layer obtained in this paper with experiment would be considerably more interesting. However, such experiments have so far not been carried out.

The dependence of the gap on the momentum obtained in this paper, connected with the Coulomb interaction, is reflected, for instance, in the relation between the size of the gap and the superconducting transition temperature. The usual relation $\Delta = \pi T_c/\gamma$ (γ is the Euler constant) will now no longer be fulfilled.

The author expresses his deep gratitude to I. E. Dzyaloshinskiĭ for directing the work.

APPENDIX

Let us write down Eq. (30) in detail. Denoting the vertex for $P_2 < P_1$ by Γ_1 and for $P_2 > P_1$ by Γ_2, we obtain instead of Eq. (30) the following system:

$$\Gamma_1 = D(\eta) - \lambda D(\eta)\int_\mu^\xi \Gamma_2\,du - \lambda D(\eta)\int_\eta^\mu \Gamma_1\,du - \lambda\int_0^\eta D\Gamma_1\,du, \quad (A.1)$$

$$\Gamma_2 = D(\mu) - \lambda D(\nu)\int_\nu^\xi \Gamma_2\,du - \lambda\int_\mu^\nu D\Gamma_2\,du - \lambda\int_0^\mu D\Gamma_1\,du;$$

whence we obtain by differentiation

$$\Gamma''_{1\eta\eta} = \lambda e^2\Gamma_1; \quad \Gamma''_{2\nu\nu} = \lambda e^2\Gamma_2. \quad (A.2)$$

Taking into account that

$$\Gamma_{1\eta}'(\mu) = e^2 - \lambda e^2\int_\mu^\xi \Gamma_2\,du;$$

$$\Gamma_1(\mu) = D(\mu) - \lambda D(\mu)\int_\mu^\xi \Gamma_2\,du - \lambda\int_0^\mu D\Gamma_1\,du = \Gamma_2(\mu);$$

$$\Gamma_1(0) = -g + \lambda g\int_\mu^\xi \Gamma_2\,du + \lambda g\int_0^\mu \Gamma_1\,du, \quad (A.3)$$

as well as

$$\Gamma_{2\nu}'(\xi) = 0; \quad \Gamma_2(\mu) = D(\mu) - \lambda D(\mu)\int_\mu^\xi \Gamma_2\,du - \lambda\int_0^\mu D\Gamma_1\,du, \quad (A.4)$$

we obtain the solutions of this system

$$\Gamma_1(\eta,\mu) = \frac{\epsilon(e\lambda^{-1/2}\,\mathrm{sh}\,e\sqrt{\lambda}\mu - g\,\mathrm{ch}\,e\sqrt{\lambda}\mu)}{e\,\mathrm{ch}\,e\sqrt{\lambda}\xi - g\sqrt{\lambda}\,\mathrm{sh}\,e\sqrt{\lambda}\xi}\,\mathrm{ch}\,e\sqrt{\lambda}(\xi-\eta) - \frac{e}{\sqrt{\lambda}}\,\mathrm{sh}\,e\sqrt{\lambda}(\mu-\eta),$$

$$\Gamma_2(\eta,\mu) = \Gamma_1(\mu,\eta). \quad (A.5)$$

Near the pole the constants g and e are related by the relationship

$$g = \frac{e}{\sqrt{\lambda}}\,\mathrm{cth}\,e\sqrt{\lambda}\xi.$$

Therefore the solutions

$$\Gamma_1(\eta,\mu) = \frac{-ge\,\mathrm{ch}\,e\sqrt{\lambda}\eta + \lambda^{-1/3}e^2\,\mathrm{sh}\,e\sqrt{\lambda}\eta}{-g\sqrt{\lambda}\,\mathrm{sh}\,e\sqrt{\lambda}\xi + e\,\mathrm{ch}\,e\sqrt{\lambda}\xi}\,\mathrm{ch}\,e\sqrt{\lambda}(\xi-\mu),$$

$$\Gamma_2(\eta,\mu) = \Gamma_1(\mu,\eta).$$

It is hence seen that near the pole the residue

$$\mathrm{res}\,\Gamma \sim \mathrm{ch}\,e\sqrt{\lambda}(\xi-\mu)\,\mathrm{ch}\,e\sqrt{\lambda}(\xi-\eta).$$

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