SCATTERING OF ELECTROMAGNETIC WAVES IN He^4 *AND IN DEGENERATE SOLUTIONS OF* He^3 *IN* He^4 *AT LOW TEMPERATURES*

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Rayleigh scattering of electromagnetic waves in He^4 at $T < 0.6°K$ and in weak solutions of He^3 at $T \ll T_F$ is considered. The shape and intensity of the Stokes and anti-Stokes satellites are obtained and the total damping decrement of the photon flux in a medium is determined.

THE application of the method of fluctuation theory (see[1]) to the kinetic equation, as was first done by Abrikosov and Khalatnikov for pure He^3 [2], makes it possible to obtain the differential and total extinction coefficients in Rayleigh scattering of electromagnetic waves in He^4 in weak solutions of He^3 in He^4 at low temperatures, to which the present paper is devoted. We consider classical scattering, i.e., $\hbar\Delta\omega \ll kT$, where $\hbar\Delta\omega$ is the change of the energy of the incident photon upon scattering, and T is the temperature of the medium. The incident wave is assumed to be monochromatic. The results can be extended to the quantum case ($\hbar\Delta\omega \gg kT$) in exactly the same manner as is described in[2].

The differential extinction coefficient is given by the well known formula[3]:

$$dh = \frac{\omega^4}{12\pi^2c^4V}\overline{\left|\int \delta\mathscr{E}_{\Delta\omega}(\mathbf{r})e^{-i\mathbf{qr}}\,dV\right|^2}\frac{3}{4}(1+\cos^2\theta)\frac{d\Omega}{4\pi}d\Delta\omega, \quad (1)$$

where $|\mathbf{q}| = (2\omega/c)\sin(\theta/2)$, $\mathbf{q}$ is the scattering vector, θ is the scattering angle, ω is the frequency of the incident wave, c is the velocity of light,

$$\delta\mathscr{E}_{\Delta\omega}(\mathbf{r}) = \lim_{t_0\to\infty}\frac{1}{\sqrt{t_0}}\int_0^{t_0}\delta\mathscr{E}(\mathbf{r},t)e^{i\Delta\omega t}\,dt,$$

and $\delta\mathscr{E}(\mathbf{r},t)$ is the fluctuation of the dielectric constant of the medium. The bar denotes averaging over the fluctuations. The Fourier component of $\delta\mathscr{E}(\mathbf{r},t)$ with respect to time is defined in the same manner as in[2].

1. RAYLEIGH SCATTERING OF ELECTROMAGNETIC WAVES IN He^4 AT $T < 0.6°K$

Owing to the small polarizability of helium, it can be assumed that $\delta\mathscr{E} = (\partial\mathscr{E}/\partial\rho)\delta\rho$, where ρ is the density of the liquid. Then

$$\overline{\left|\int \delta\mathscr{E}_{\Delta\omega}(\mathbf{r})e^{-i\mathbf{qr}}\,dV\right|^2} = \left(\frac{\partial\mathscr{E}}{\partial\rho}\right)^2\overline{|\delta\rho_{\Delta\omega,\mathbf{q}}|^2}. \quad (1.1)$$

Here $\delta\rho_{\Delta\omega,q}$ is the Fourier component of the density fluctuation with respect to r and t. Thus, the problem reduces to a determination of $\overline{|\delta\rho_{\Delta\omega,q}|^2}$. It is known[4] that when $T < 0.6°K$, the He^4 can be regarded as a pure phonon gas. Let us write the kinetic equation for the phonon distribution function $n = n_0 + \delta n$:

$$\dot{\delta n} = \frac{\partial\delta n}{\partial t} + \frac{\partial\delta n}{\partial\mathbf{r}}\frac{\partial H}{\partial\mathbf{p}} - \frac{\partial n_0}{\partial\mathbf{p}}\frac{\partial H}{\partial\mathbf{r}} = I(\delta n) + y(\mathbf{p},\mathbf{r},t). \quad (1.2)$$

Here $H = \epsilon(\mathbf{p}) + \mathbf{p}\cdot\mathbf{v}_s$ is the phonon Hamiltonian[4], $y(\mathbf{p},\mathbf{r},t)$ an arbitrary "extraneous" force, p the phonon momentum, and v_s the velocity of the superfluid motion. The collision integral will be taken in the form that conserves the total energy and the momentum:

$$I(\delta n) = -\frac{1}{\tau}(\delta n - \overline{\delta n} - \overline{3\delta n\cos\vartheta}\cos\vartheta).$$

We take the Fourier transforms of (1.2) with respect to r and t, change over to dimensionless variables $\rho' = \delta\rho_{q,\Delta\omega}/\rho$ and $v_s = (v_s)_{q,\Delta\omega}/s$, where s is the speed of sound, and represent $\delta n_{q,\Delta\omega}(\mathbf{p})$ in the form[5]:

$$\delta n_{\mathbf{q},\Delta\omega} = \frac{\partial n_0}{\partial\varepsilon}\left(\frac{\partial\varepsilon}{\partial\rho}\delta\rho_{\mathbf{q},\Delta\omega} + \varepsilon\nu(\cos\vartheta)\right).$$

After simple transformations and after integrating (1.2) with respect to p, with weight ϵp^2, we get

$$(\tilde{z} - \cos\vartheta)\nu(\cos\vartheta) + \tilde{\omega}u\rho' + \cos^2\vartheta v_s + (\tilde{\omega} - \tilde{z})(\nu_0 + \nu_1\cos\vartheta) = Y(\cos\vartheta)/2iqs\pi^2\hbar^3TC_{ph}, \quad (1.3)$$

where

$$Y(\cos\vartheta) = \int y_{\mathbf{q},\Delta\omega}(\mathbf{p})\varepsilon p^2\,dp, \quad \int\frac{\partial n_0}{\partial\varepsilon}\varepsilon^2p^2\,dp = -2\pi^2\hbar^3TC_{ph},$$

$$u = \frac{\rho\partial s}{s\partial\rho}, \quad \tilde{\omega} = \frac{\Delta\omega}{qs}, \quad \tilde{z} = \tilde{\omega}\left(1 - \frac{1}{i\Delta\omega\tau}\right),$$

$$\mathbf{qv} = qv\cos\vartheta, \quad \nu_0 = \overline{\nu}, \quad \nu_1 = \overline{3\nu\cos\vartheta}. \quad (1.4)$$

We now calculate $\overline{Y(\cos\vartheta)Y(\cos\theta')}$ and then, solving the kinetic equation together with the continuity and superfluid-motion equations, we get $\overline{|\delta\rho_{q,\Delta\omega}|^2}$. The entropy per unit volume of the phonon gas is[1]:

$$S = k\int[(1+n)\ln(1+n) - n\ln n]\,d\tau_{\mathbf{p}}. \quad (1.5)$$

Varying (1.5) with respect to n, with allowance for the energy conservation law, and retaining the first nonzero term in powers of δn, we have for the rate of change of the entropy

$$\dot{S} = -k\int\frac{\delta n\dot{\delta n}}{n_0(1+n_0)}d\tau_{\mathbf{p}} = \frac{1}{T}\int\frac{\partial\varepsilon}{\partial n_0}\delta n\dot{\delta n}\,d\tau_{\mathbf{p}}. \quad (1.6)$$

We expand $\delta n(\mathbf{p},\mathbf{r},t)$ and $y(\mathbf{p},\mathbf{r},t)$ in a series of spherical functions

$$\frac{1}{\varepsilon}\frac{\partial\varepsilon}{\partial n_0}\delta n = \sum_{n=0}^{\infty}\sum_{m=-n}^{n}A_n^mP_n^m(\cos\vartheta)e^{im\varphi},$$

$$y = \sum_{n=0}^{\infty}\sum_{m=-n}^{n}y_n^mP_n^m(\cos\vartheta)e^{im\varphi},$$

where ϑ and φ are the polar coordinates of p. We substitute (1.2) in (1.6) and obtain after integrating with respect to τ_p, with allowance for (1.4)

$$\dot{S} = \frac{1}{4\pi^2\hbar^3 T}\Big[\sum_{n=0}^{1}\frac{2}{2n+1}A_n{}^0 Y_n{}^0 + \sum_{n=1}^{\infty}\sum_{m=-n}^{n}\frac{2(n+|m|)!}{(2n+1)(n-|m|)!} \times\Big(2\pi^2\hbar^3 TC_{ph}\frac{A_n{}^m}{\tau}+Y_n{}^m\Big)A_n{}^{-m}\Big] \qquad (m\neq 0 \text{ for } n=1), \quad (1.7)$$

where $Y_n^m \int y_n^m \epsilon p^2 dp$. Following the general theory of fluctuations, we represent (1.7) in the form

$$\dot{S} = -\sum_{n;m}\dot{X}_n{}^m \dot{x}_n{}^m. \quad (1.8)$$

We put

$$\dot{x}_0{}^0 = Y_0{}^0, \quad \dot{x}_1{}^0 = Y_1{}^0, \quad \dot{x}_n{}^m = 2\pi^2\hbar^3 TC_{ph}\frac{A_n{}^m}{\tau}+Y_n{}^m; \quad (1.9)$$

$$n=1, \quad m\neq 0; \quad n=2,\ldots;$$

it then follows from (1.8), (1.9), and (1.7) that

$$X_n{}^m = -\frac{A_n{}^{-m}(n+|m|)!}{2\pi^2\hbar^3 T(2n+1)(n-|m|)!}, \quad n=0,1,\ldots. \quad (1.10)$$

From the requirement

$$\dot{x}_n{}^m = -\sum_{n',m'}\gamma_{nn'}^{mm'}X_{n'}{}^{m'}+Y_n{}^m$$

we get, comparing (1.9) and (1.10):

$$\gamma_{00}^{00}=0, \quad \gamma_{11}^{00}=0,$$

$$\gamma_{n'n}^{mm'} = \delta_{nn'}\delta_{m,-m'}\frac{(2\pi^2\hbar^3 T)^2 C_{ph}(2n+1)(n-|m|)!}{\tau(n+|m|)!},$$

$$n=1, \quad m\neq 0; \quad n=2,3\ldots.$$

Hence

$$\overline{Y_0{}^0\ Y_0{}^0}=0, \quad \overline{Y_1{}^0\ Y_1{}^0}=0, \quad \overline{Y_n{}^m\ Y_{n'}{}^{m'}} = k(\gamma_{nn'}^{mm'}+\gamma_{n'n}^{m'm}) \times\delta(t-t')\delta(\mathbf{r}-\mathbf{r}') = \frac{2k(2\pi^2\hbar^3 T)^2 C_{ph}(2n+1)(n-|m|)!}{\tau(n+|m|)!} \times\delta_{nn'}\delta_{m,-m'}\delta(t-t')\delta(\mathbf{r}-\mathbf{r}'). \quad (1.11)$$

It is known from the theory of spherical functions that

$$P_n(\cos\gamma) = P_n(\cos\vartheta\cos\vartheta' + \sin\vartheta\sin\vartheta'\cos(\varphi-\varphi')) = \sum_{m=-n}^{n}\frac{(n-|m|)!}{(n+|m|)!}P_n{}^m(\cos\vartheta)P_n{}^m(\cos\vartheta')e^{im(\varphi-\varphi')}, \quad (1.12)$$

$$\sum_{n=0}^{\infty}(2n+1)P_n(\cos\gamma) = 2\delta(\cos\gamma-1). \quad (1.13)$$

Summing (1.11) with the aid of (1.12) and (1.13), we obtain

$$\overline{Y(\cos\vartheta),\ Y(\cos\vartheta')} = (2\pi^2\hbar^3 T)^2\frac{2kC_{ph}}{\tau}[2\delta(\cos\vartheta-\cos\vartheta')-1-3\cos\vartheta\cos\vartheta']. \quad (1.14)$$

We write down the equations of continuity and superfluid motion[4]:

$$\frac{\partial\rho}{\partial t}+\operatorname{div}\Big(\rho\mathbf{v}_s+\int \mathbf{p}n\,d\tau_{\mathbf{p}}\Big)=0, \quad (1.15)$$

$$\frac{\partial\mathbf{v}_s}{\partial t}+\nabla\Big(\mu_0+\int\frac{\partial\varepsilon}{\partial\rho}n\,d\tau_{\mathbf{p}}\Big)=0. \quad (1.16)$$

Generally speaking it is necessary to introduce in (1.16) an "extraneous" potential, but in the temperature region under consideration its mean value over the fluctuations is equal to zero[6]. Equations (1.3), (1.15), and (1.16) constitute a complete system of equations describing He^4. We take the Fourier transforms of (1.15) and (1.16) with respect to r and t, and change in these transforms to dimensionless variables. After averaging (1.3) over cos ϑ, we get

$$-\tilde{\omega}\rho'+j=0, \quad (1.17)$$

$$-\tilde{\omega}j+\frac{1}{s^2}\Big(\frac{\partial\mathscr{P}}{\partial\rho}\Big)_T\rho'-\frac{\rho_n}{\rho}(3u v_0+\tilde{\omega}v_1)=0, \quad (1.18)$$

$$\tilde{\omega}u\rho'+\frac{1}{3}\tilde{\omega}\rho'-\frac{1}{3}\frac{\rho_s}{\rho}v_1+\tilde{\omega}v_0 = \frac{1}{4i\pi^2\hbar^3 qsTC_{ph}}\int_{-1}^{1}Y(x)\,dx, \quad (1.19)$$

$$[2+(\tilde{\omega}-\tilde{z})\ln\tilde{a}]v_0+\Big(\tilde{\omega}-\tilde{z}\frac{\rho_s}{\rho}\Big)(-2+\tilde{z}\ln\tilde{a})v_1+(u\tilde{\omega}\ln\tilde{a})\rho' + \tilde{z}(-2+\tilde{z}\ln\tilde{a})j = \frac{1}{4i\pi^2\hbar^3 qsTC_{ph}}\int_{-1}^{1}\frac{Y(x)}{\tilde{z}-x}dx, \quad (1.20)$$

where $\mathscr{P}$ is the pressure, and

$$\ln\tilde{a} = \ln\frac{\tilde{z}+1}{\tilde{z}-1}, \quad \rho_s+\rho_n=\rho, \quad j=\Big|\frac{\rho_s}{\rho}v_s+\frac{\rho_n}{\rho}v_n\Big|.$$

The left side of (1.17)–(1.20) coincides, as it should, with the corresponding system in[5]. Solving the obtained system with respect to ρ' and averaging it over the fluctuations with the aid of (1.15), we obtain, after rather laborious calculations,

$$\overline{|\rho'|^2} = \frac{\rho_n kT|\tilde{\omega}^2+u|^2}{6|D|^2(\rho qs^2)^2\tau}\Big|\frac{4}{\tilde{z}^2-1}-\ln^2\tilde{a}-3(-2+\tilde{z}\ln\tilde{a})^2\Big| \quad (1.21)$$

where

$$3D = -(\tilde{\omega}^2-u_{10}{}^2/s^2)\{2+(\tilde{\omega}-\tilde{z})\ln\tilde{a}+3\tilde{\omega}(\tilde{\omega}-\tilde{z})(-2+\tilde{z}\ln\tilde{a})\} + (\rho_n/\rho)\{(\tilde{\omega}^2-u_{10}{}^2/s^2)[2+(\tilde{\omega}-\tilde{z})\ln\tilde{a}-3\tilde{\omega}\tilde{z}(-2+\tilde{z}\ln\tilde{a})] + 3\tilde{\omega}(\tilde{\omega}^2+u)[\tilde{z}(-2+\tilde{z}\ln\tilde{a})+u\ln\tilde{a}]-\tilde{\omega}(3u+1)[\tilde{\omega}(2+(\tilde{\omega}-\tilde{z})\ln\tilde{a}) - 3u(\tilde{\omega}-\tilde{z})(-2+\tilde{z}\ln\tilde{a})]\}$$

is the determinant of the system (1.17)–(1.20), accurate to terms linear in $\rho_n/\rho = 10^{-4}\cdot T^4$; $u_{10}^2 = (\partial\mathscr{P}/\partial\rho)_T$ is the compressibility. Substituting (1.21) in (1), we get finally

$$dh = \frac{\omega^4}{12\pi^2 c^4}\Big(\frac{\partial\mathscr{E}}{\partial\rho}\Big)^2\frac{\rho_n kT|\tilde{\omega}^2+u|^2}{6|D|^2(qs^2)^2\tau}\Big|\frac{4}{\tilde{z}^2-1}-\ln^2\tilde{a} - 3(-2+\tilde{z}\ln\tilde{a})^2\Big|\frac{3}{4}(1+\cos^2\theta)\frac{d\Omega}{4\pi}d\Delta\omega. \quad (1.22)$$

The form of formulas (1.21) and (1.22) greatly simplifies in different limiting cases. It will be shown that (1.21) has δ-like singularities corresponding to scattering by first and second sound. Let us examine these singularities.

First sound. Let at first $\Delta\omega\tau \gg 1$. When $\tilde{\omega}\approx 1$, which corresponds to the first sound, (1.21) takes on the dispersion form

$$\overline{|\rho'|^2} = \Big(\frac{2kT}{qs^3\rho}\Big)\frac{3\pi\rho_n(u+1)^2/4\rho}{(\tilde{\omega}^2-u_{1\infty}{}^2/s^2)^2+(3\pi\rho_n(u+1)^2/4\rho)^2} \approx \frac{\pi kT}{\rho s^2}[\delta(\Delta\omega-u_{1\infty}q)+\delta(\Delta\omega+u_{1\infty}q)]. \quad (1.23)$$

We note that when T = 0.5°K we have $\rho_n/\rho \approx 10^{-5}$. In (1.23) we have

$$u_{1\infty} = u_{10}+s(\rho_n/\rho)\{{}^3/_4(u+1)^2\ln(2\Delta\omega\tau)-3u-2\}.$$

The line width is determined by the quantity

$(3/4)\pi(\rho_n/\rho)(u+1)^2$. We recall that $u = (\rho/s)\partial s/\partial\rho \approx 2.7$.

Let now $\Delta\omega\tau \ll 1$, and then

$$\overline{|\rho'|^2} = \left(\frac{2kT}{qs^3\rho}\right)\frac{3/5(\rho_n/\rho)(u+1)^2\Delta\omega\tau}{(\tilde\omega^2 - u_1^2/s^2)^2 + [3/5(\rho_n/\rho)(u+1)^2\Delta\omega\tau]^2} \approx \frac{\pi kT}{\rho s^2}[\delta(\Delta\omega - u_1 q) + \delta(\Delta\omega + u_1 q)], \quad (1.24)$$

where

$$u_1 = u_{10} + \frac{s}{4}(3u+1)^2\frac{\rho_n}{\rho}.$$

Second sound. Near $\tilde\omega = 1/\sqrt{3}$ and when $\Delta\omega\tau \ll 1$, which corresponds to second sound, (1.21) takes the form

$$\overline{|\rho'|^2} = \left(\frac{\sqrt{3}\rho_n kT(3u+1)^2}{8qs^3\rho^2}\right)\frac{4\Delta\omega\tau/15}{(\tilde\omega^2 - u_2^2/s^2) + (4\Delta\omega\tau/15)^2} \approx \left(\frac{3\pi\rho_n kT}{\rho^2 s^2}\right)\left(\frac{3u+1}{4}\right)^2[\delta(\Delta\omega - u_2 q) + \delta(\Delta\omega + u_2 q)], (1.25)$$

where

$$u_2 = \frac{s}{\sqrt{3}}\left[1 - \frac{3}{4}(3u^2 + 2u + 1)\frac{\rho_n}{\rho}\right].$$

Comparing (1.23) and (1.24) with (1.25), we see that scattering by second sound is weaker by a factor ρ_n/ρ than by first sound, and in the calculation of the total extinction coefficient it can be neglected. Let us integrate (1.22) approximately:

$$h = \frac{\omega^4}{6\pi c^4}\left(\frac{\partial\mathscr{E}}{\partial\rho}\right)^2\frac{\rho kT}{s^2}. \quad (1.26)$$

From the condition $\hbar\Delta\omega \ll kT$ it follows that $\omega \ll 10^{17}$. Putting $\omega = 10^{16}$ and $T = 0.5°K$, we have $h \approx (\partial\mathscr{E}/\partial\rho)^2\rho^2 \times 10^{-4}\ cm^{-1}$.

2. RAYLEIGH SCATTERING OF ELECTROMAGNETIC WAVES IN WEAK DEGENERATE SOLUTIONS OF He^3 IN He^4

He^3 forms a Fermi liquid when $T \ll T_F$, where $kT_F \equiv \mu$ is the chemical potential of He^3 at 0°K. For a 5% solution of He^3 in He^4, for example, $T_F \approx 0.3°K$. The phenomenological theory of such a Fermi liquid is given in[7]. Calculation shows that the presence of phonons can be neglected accurate to terms of order $\rho_n/\rho = 10^{-4}T^4$. Let us write an expression for $\delta\mathscr{E}$:

$$\delta\mathscr{E} = \left(\frac{\partial\mathscr{E}}{\partial\rho_4}\right)_{\rho_3}\delta\rho_4 + \left(\frac{\partial\mathscr{E}}{\partial\rho_3}\right)_{\rho_4}\delta\rho_3 = \left(\frac{\partial\mathscr{E}}{\partial\rho_4}\right)_{\rho_3}\left(\delta\rho_4 - \left(\frac{\partial\rho_4}{\partial\rho_3}\right)_{\mathscr{E}}\delta\rho_3\right).$$

According to the experimental data $\partial\rho_4/\partial\rho_3 \approx 1.6$. We assume that $\rho_3 \ll \rho_4$ (φ_3 and ρ_4 are the densities of the He^3 and He^4 particles). We can therefore assume that $\delta\mathscr{E} \approx (\partial\mathscr{E}/\partial\rho_4)_{\rho_3}\delta\rho_4$.

Let us write the kinetic equation for the distribution of the Fermi particles $n = n_0 + \delta n$:

$$\dot{\delta n} = \frac{\partial\delta n}{\partial t} + \frac{\partial\delta n}{\partial\mathbf{r}}\frac{\partial\varepsilon}{\partial\mathbf{p}} - \frac{\partial n_0}{\partial\mathbf{p}}\int f(\mathbf{p},\mathbf{p}')\frac{\partial\delta n(\mathbf{p}')}{\partial\mathbf{r}}d\tau_{\mathbf{p}'} = I(\delta n) + y. \quad (2.1)$$

Here $f(\mathbf{p}, \mathbf{p}')$ is the Landau function, and we assume for simplicity that $f = f_0$. According to[7], the energy spectrum is

$$\varepsilon(\mathbf{p}) = \varepsilon_0(\rho_3, \rho_4) + \frac{p^2}{2m^*} + \frac{\Delta m}{m^*}\mathbf{p}\mathbf{v}_s + \int f(\mathbf{p},\mathbf{p}')\delta n(\mathbf{p}')d\tau_{\mathbf{p}'},$$

where m^* is the effective mass of the Fermi excitation, $\Delta m = m^* - m_3$.

We take the collision integral in the form

$$I(\delta n) = -\frac{1}{\tau}(\delta n - \overline{\delta n} - 3\overline{\delta n\cos\vartheta}\cos\vartheta) \quad (2.2)$$

(ϑ is the angle between $\mathbf{p}$ and the scattering vector $\mathbf{q}$). The fluctuation of the random force is calculated in the same manner as in[2], the only difference being that the collision integral is taken in the form (2.2). For the rate of change of the entropy we have

$$\dot S = -k\left\{\int\frac{\delta n[I(\delta n) + y]}{n_0(1-n_0)}d\tau_{\mathbf{p}}dV + \frac{1}{kT}\int f(\mathbf{p},\mathbf{p}')\delta(\mathbf{r}-\mathbf{r}')\delta n I(\delta n')d\tau_{\mathbf{p}}d\tau_{\mathbf{p}'}dV\,dV'\right\}, \quad (2.3)$$

$$n_0(1-n_0) = kT\delta(\varepsilon - \mu).$$

All the quantities change near the Fermi boundary, and we therefore put

$$\delta n = \delta n^\varepsilon(\vartheta,\varphi)\delta(\varepsilon-\mu), \quad y = y^\varepsilon(\vartheta,\varphi)\delta(\varepsilon-\mu)$$

(ϑ, φ—polar angles of the momentum vector $\mathbf{p}$). We expand $\delta n^\epsilon(\vartheta,\varphi)$ and $y^\epsilon(\vartheta,\varphi)$ in series of spherical polynomials:

$$\delta n^\varepsilon(\vartheta,\varphi) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n}A_n{}^m P_n{}^m(\cos\vartheta)e^{im\varphi}, \quad (2.4)$$

$$y^\varepsilon(\vartheta,\varphi) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n}y_n{}^m P_n{}^m(\cos\vartheta)e^{im\varphi}. \quad (2.5)$$

Substituting (2.4) and (2.5) in (2.3) and integrating with respect to $d\tau_p$ and the unit volume, we obtain as the result

$$\dot S = \frac{1}{T}\left(\frac{d\tau_p}{d\varepsilon}\right)_{\varepsilon=\mu}\left\{\sum_{n=1}^{\infty}\sum_{m=-n}^{n}\frac{(n+|m|)!}{(2n+1)(n-|m|)!}\left(\frac{A_n{}^m}{\tau} - y_n{}^m\right)A_n{}^{-m} - (1+F_0)y_0{}^0A_0{}^0 - 1/3\,y_1{}^0A_1{}^0\right\}, \quad F_0 = \left(\frac{d\tau_p}{d\varepsilon}\right)_{\varepsilon=\mu}f_0 \quad (m \neq 0 \text{ при } n = 1).$$

Proceeding in exactly the same manner as in Sec. 1, we get

$$\overline{y_{\mathbf{q},\Delta\omega}(\vartheta)y_{\mathbf{q},\Delta\omega}(\vartheta')} = \frac{2kT}{\tau}\left(\frac{d\varepsilon}{d\tau}\right)_{\varepsilon=\mu} \times [2\delta(\cos\vartheta - \cos\vartheta') - 1 - 3\cos\vartheta\cos\vartheta']. \quad (2.6)$$

To obtain the complete system of equations describing the solution, we add to (2.1) the equations of continuity and of superfluid motion[4,7]:

$$\frac{\partial\rho}{\partial t} + \operatorname{div}\left(\rho_4\mathbf{v}_s + \int\mathbf{p}n\,d\tau_{\mathbf{p}}\right) = 0, \quad (2.7)$$

$$\frac{\partial\mathbf{v}_s}{\partial t} + \nabla\left(\mu_{04} + \int n\frac{\partial\varepsilon}{\partial\rho_4}d\tau_{\mathbf{p}}\right) = 0. \quad (2.8)$$

Averaging (2.1), (2.7), and (2.8) over ϑ and taking their Fourier transforms with respect to $\mathbf{r}$ and t, we obtain after some calculations

$$\left[1 + \frac{1}{\xi\sigma} - \left(F_0 - \frac{1}{\xi\sigma}\right)W\right]v_0 + \frac{W}{\sigma}v_1 + 2u^2\frac{m_4}{m^*}\alpha W\rho' + 2\frac{\Delta m}{m^*}\xi W v_s = \frac{1}{2i\varepsilon_F q v_F}\int_{-1}^{1}\frac{y^\varepsilon_{\mathbf{q},\Delta\omega}(z)}{z-\xi}dz. \quad (2.9)$$

$$\left(\frac{1}{\sigma} + \xi\right)v_0 - \frac{1}{3}v_1 + \frac{2}{3}\frac{\Delta m}{m^*}v_s = -\frac{1}{2i\varepsilon_F q v_F}\int_{-1}^{1}y^\varepsilon_{\mathbf{q},\Delta\omega}(z)dz, \quad (2.10)$$

$$\frac{3}{2}\frac{m_3}{m_4}x\tilde\omega v_0 - \frac{1}{2}\frac{m^*}{m_4}v_1 - \tilde\omega\rho' + v_s = 0, \quad (2.11)$$

$$-3/2\,u^2 x\alpha v_0 + u^2(1+\beta x)\rho' - \tilde\omega v_s = 0. \quad (2.12)$$

Here

$$\alpha = \frac{\rho_4}{m_4 s^2}\left(\frac{\partial\varepsilon_0}{\partial\rho_4}\right), \quad \beta = \frac{\rho_4^2}{m_4 s^2}\left(\frac{\partial^2\varepsilon_0}{\partial\rho_4^2}\right), \quad \sigma = i\tau q v_F,$$

$$\tilde\omega = \frac{\Delta\omega}{q v_F}, \quad \xi = \tilde\omega\left(1 - \frac{1}{i\Delta\omega\tau}\right), \quad u^2 = \frac{s^2}{v_F^2}, \quad x = \frac{\rho_3}{(m_3/m_4)\rho_4 + \rho_3},$$

$$\nu(\cos\vartheta) = \frac{1}{\varepsilon_F}\left(\frac{\partial\varepsilon}{\partial n_0}\right)\delta n_{q,\Delta\omega};$$

and we have introduced the dimensionless variables $\rho' = (\delta\rho_4)_{q,\Delta\omega}$, $v_s = (v_s)_{q,\Delta\omega}$, $\nu_0 = \overline{\nu}$, and $\nu_1 = 3\overline{\nu\cos\vartheta}$. In addition,

$$W = -1 + \frac{\xi}{2}\ln\frac{\xi+1}{\xi-1}.$$

We solve the system (2.9)–(2.12) with respect to ρ' and average the result over the fluctuations with the aid of (2.6). After rather laborious calculations we obtain

$$\overline{|\rho'|^2} = \frac{3xkT}{m_4 v_F^2 qs\varepsilon_F}\left(\frac{d\varepsilon}{d\tau}\right)_{\varepsilon=\mu}\frac{4xm_4(\alpha+\Delta m/m_4)^2\Delta\omega\tau/15m^*(\Delta\omega^2\tau^2+1)}{(\widetilde{\omega}^2 - u_1^2/v_F^2)^2 + A^2}. \quad (2.13)$$

where

$$A = 4xm_4(\alpha+\Delta m/m_4)^2\Delta\omega\tau/15m^*(\Delta\omega^2\tau^2+1).$$

The numerator of the fraction in (2.13) determines the width of the line both when $\Delta\omega\tau \ll 1$ and when $\Delta\omega\tau \gg 1$. In both cases

$$\overline{|(\delta\rho_4)_{q,\Delta\omega}|^2} = \frac{\pi\rho_4 kT}{s^2}[\delta(\Delta\omega - u_1 q) + \delta(\Delta\omega + u_1 q)], \quad (2.14)$$

where

$$u_1 = s + x\frac{s}{2}\left[\frac{m_4}{m^*}\left(\alpha+\frac{\Delta m}{m_4}\right)^2 + \beta - \frac{\Delta m}{m_4}\right].$$

Substituting (2.14) in (1), we get

$$dh = \frac{\omega^4\rho_4 kT}{12\pi c^4 s^2}\left(\frac{\partial\mathscr{E}}{\partial\rho_4}\right)^2[\delta(\Delta\omega - u_1 q) + \delta(\Delta\omega + u_1 q)]\cdot \frac{3}{4}(1+\cos^2\theta)\frac{d\Omega}{4\pi}d\Delta\omega. \quad (2.15)$$

Integrating (2.15), we return to (1.26). We note that (1.26) coincides with the results obtained in[8] and[9] for the hydrodynamic case and in[9] for a nondegenerate solution of He^3 in He^4.

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[1] L. D. Landau and E. M. Lifshitz, Statisticheskaya fizika (Statistical Physics), Nauka, 1964 [Addison-Wesley, 1958].

[2] A. A. Abrikosov and I. M. Khalatnikov, Zh. Eksp. Teor. Fiz. **33**, 110 (1957) [Sov. Phys.-JETP **6**, 84 (1958)].

[3] L. D. Landau and E. M. Lifshitz, Elektrodinamika sploshnykh sred (Electrodynamics of Continuous Media), Fizmatgiz, 1959 [Addison-Wesley, 1961].

[4] I. M. Khalatnikov, Vvedenie v teoriyu sverkhtekuchesti (Introduction to the Theory of Superfluidity), Nauka, 1965.

[5] I. M. Khalatnikov and D. M. Chernikova, Zh. Eksp. Teor. Fiz. **49**, 1957 (1965) [Sov. Phys.-JETP **22**, 1336 (1966)].

[6] I. M. Khalatnikov, Zh. Eksp. Teor. Fiz. **33**, 809 (1957) [Sov. Phys.-JETP **6**, 624 (1958)].

[7] I. M. Khalatnikov, Zh. Eksp. Teor. Fiz. **55**, 1919 (1968) [Sov. Phys. **28**, 1014 (1969)].

[8] V. L. Ginzburg, Zh. Eksp. Teor. Fiz. **13**, 243 (1943).

[9] L. P. Gor'kov and L. P. Pitaevskiĭ, Zh. Eksp. Teor. Fiz. **33**, 634 (1957) [Sov. Phys. **6**, 406 (1958)].

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181